Chapter – 3

Pair of Linear Equations in Two Variables

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Linear equations can be used to represent almost any situation involving an unknown quantity.

We apply linear equations in various real-life situations like weather predictions, ingredients of a recipe, our monthly expenditure, predicting profit in business and Government surveys.

We know that an equation of the form $ax^2 + bx + c = 0$ where a, b and c are real numbers and a and b are both not equal to zero, is called a linear equation in two variables x and y.

Let us consider an example of a linear equation in two variables, $x + 3y = 7$

$LHS = x + 3y$

If we put $x = 1$ and $y = 2$ in the LHS of the equation we get,

$LHS = x + 3y = 1 + 3(2) = 7$

Here, LHS=RHS

So, we can say that $x = 1$ and $y = 2$ is a solution of the equation $x + 3y = 7$ Let us consider one more value of x and y.

Again putting $x = 2$ and $y = 1$ in the LHS of the equation we get,

$LHS = x + 3y = 2 + 3(1) = 5$

Now, LHS $\neq$ RHS

Therefore, we say that $x = 2$ and $y = 1$ is not a solution of the equation $x + 3y = 7$.

We know that the graph of a linear equation in two variables is a straight line.

If we say that $x = 1$ and $y = 2$ is a solution of the equation $x + 3y = 7$, then it means that the point (1,2) will lie on the line representing the equation, $x + 3y = 7$.

Now, we had considered one more case, where the value of $x = 2$ and $y = 1$ was not the solution of the equation, $x + 3y = 7$.

Therefore, we say that the point (2,1) does not lie on the line representing the equation, $x + 3y = 7$.

So, every solution of the equation is a point on the line representing it.

Each solution (x, y) of a linear equation in two variables $ax^2 + bx + c = 0$, corresponds to a point on the line representing the equation, and vice versa.

Let us consider one more Linear Equation in two variables, $x + 5y = 9$

Now, the two equations are $x + 3y = 7$ and $x + 5y = 9$

These two equations are in the same variable x and y. Therefore, we say that they are a pair of linear equations in two variables.

Now consider a situation here, Rhea goes to a market to buy coloured pens and papers, required for a science project she has to submit in the school the next day. She has Rs. 100 with her. The number of coloured pens she bought is 2 times the number of coloured papers. The cost of one coloured paper and one coloured pen is Rs.10 and Rs. 20 respectively. If we denote the number of coloured pens by x and the number of coloured paper by y, we get two sets of equations.

$$x = 2y$$

$$10x + 20y = 100$$

These equations are a pair of linear equations in two variables.

These equations taken together give complete information about Rhea at the market.

The general form for a pair of linear equations in two variables x and y is

$a_1x + b_1y + c_1 = 0$
$a_2x + b_2y + c_2 = 0$

Where, a_1, b_1, c_1, a_2, b_2, c_2 are all real numbers and $a_1^2 + b_1^2 \neq 0$, $a_22 + b_2^2 \neq 0$

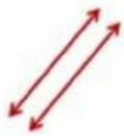
Graphical Representation of Linear Equation

We know that the geometrical representation of linear equations in two variables is a straight line, but when we have a pair of linear equations then there will be two straight lines, which are considered together.

When there are two lines in a plane, one of the following possibilities may arise.

1) The two lines will intersect

2) The two lines are parallel



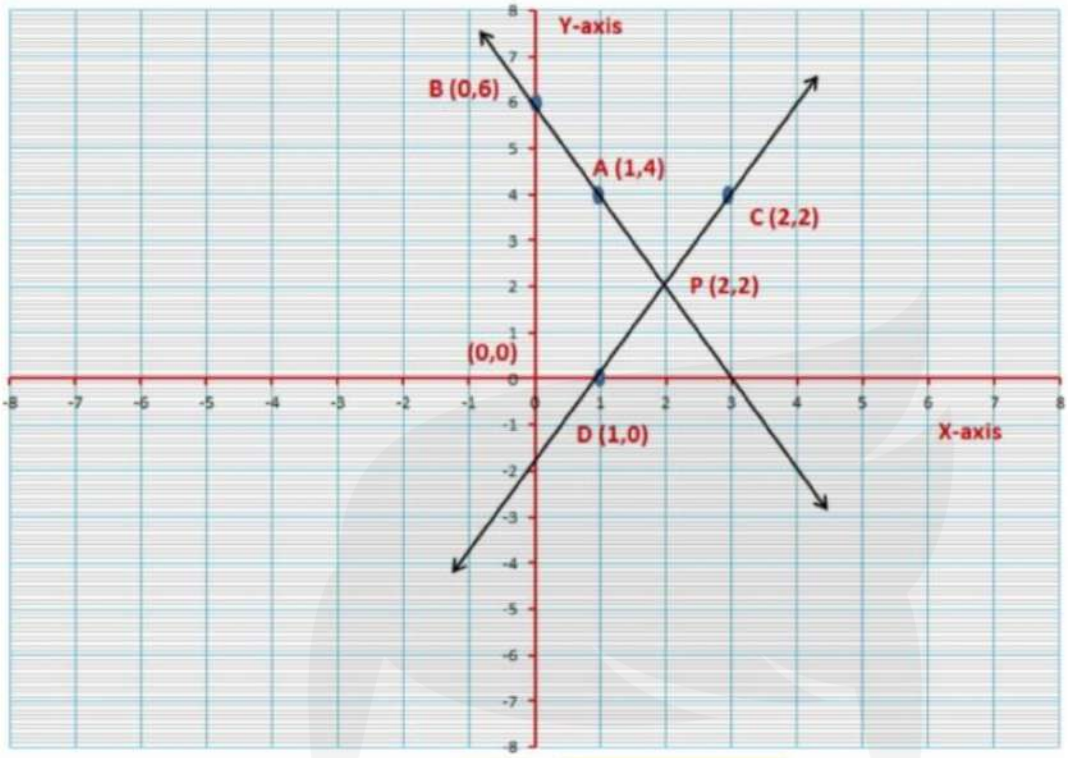
3) Two lines are coincident

Let us consider some examples here.

Example: Aditya is walking along with the line joining points(1,4) and (0,6). Aditi is walking along the line joining the points (3,4) and (1,0). Represent on the graph and find the point where both of them cross each other.

Let the points be A(1,4), B(0,6), C(3,4) and D(1,0).

We will plot these points on the graph paper as shown.



If we join points A and B, we get the path travelled by Aditya.

Similarly, on joining the points C and D, we get the path traveled by Aditi.

We can see that the two lines are intersecting at one point, that is, point P. This point P is the point where both Aditi and Aditya will cross each other.

Example: The cost of 2 kg of apples and 1 kg of grapes on a day was found to be Rs.160. After a month, the cost of 4 kg of apples and 2 kg of grapes is Rs.300. Represent the situation algebraically and geometrically.

(REFERENCE: NCERT)

Let the cost of 1 kg of apples and grapes be Rs. x and Rs. y respectively.

Cost of 2 kg of apples and 1 kg of grapes = $2x + y$

$$2x + y = 160$$

Cost of 4 kg of apples and 2 kg of grapes = $4x + 2y$

$$4x + 2y = 300$$

Then the algebraic representation is given by the equations,

$$2x + y = 160$$

$$4x + 2y = 300$$

Now we will represent these equations graphically and for that, we need two solutions for each equation.

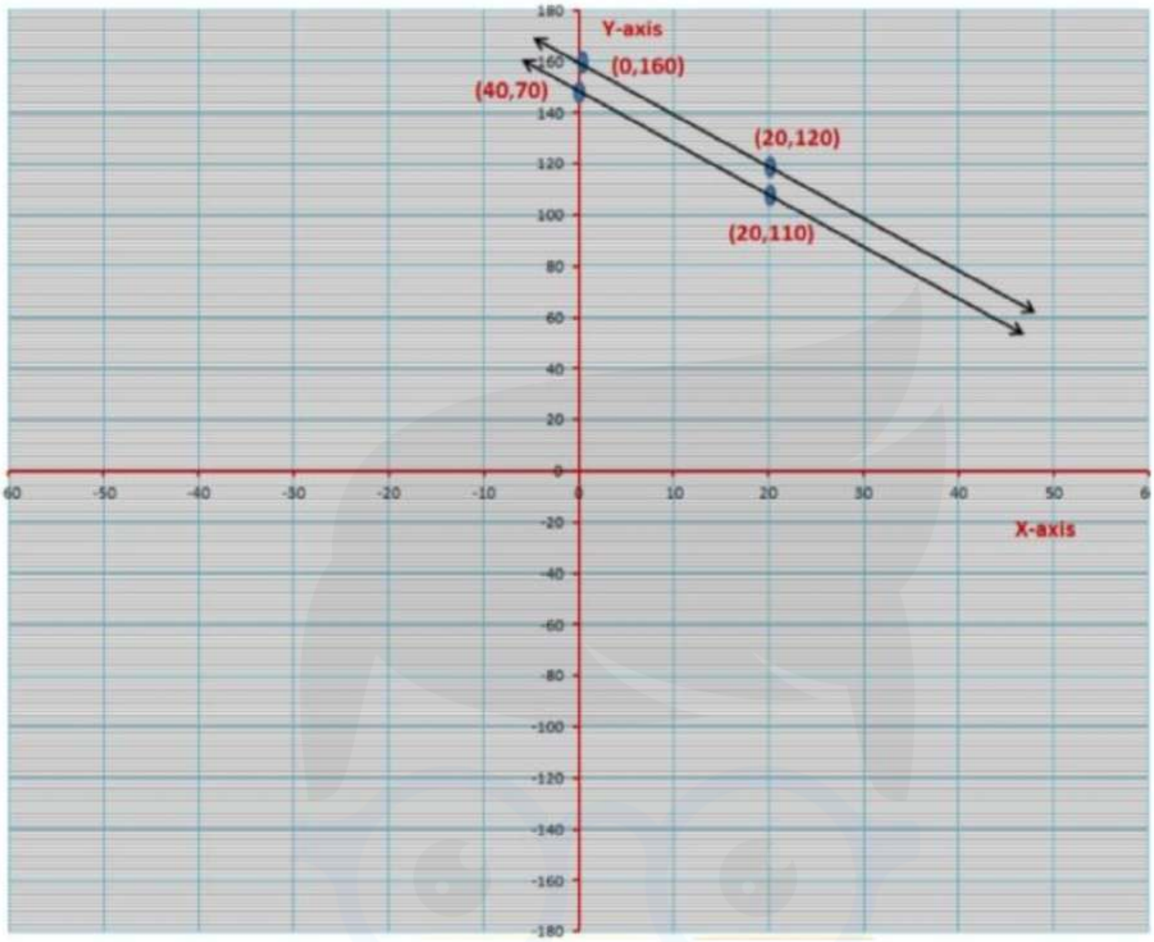
x	0	20
$y = 160 - 2x$	160	120

x	0	20
$y = \frac{300 - 4x}{2}$	150	110

Let the points be A1(0, 160), B1(20, 120). These are the solutions of $2x + y = 160$.

Similarly, the solution of $4x + 2y = 300$ are represented by the points A2(0, 150), B2(20, 110).

Now we will plot these points on a graph paper.



We see that the two lines are not intersecting anywhere, that is, the two lines are parallel.

Solving Pair of Linear Equations - Graphical Method

Graphical Method of Solution of a Pair of Linear Equations

We know that a pair of linear equations is represented graphically by two straight lines and these lines may be parallel, may intersect or may coincide.

Now, we will consider certain cases here,

i) If the two lines are intersecting each other at one point only, then we say that there is one and only one solution, that is, a unique solution exists for this pair of linear equations in two variables. Such a pair of linear equations is called a consistent pair of Linear equations.

ii) If the two lines are coincident then we say that the pair of linear equations has infinitely many solutions. Such a pair of linear equations is called an inconsistent pair of Linear equations.

iii) If the two lines are parallel to each other, that is, they do not meet at all, and then we say that the two linear equations have no common solution. Such a pair of linear equations is called a dependent pair of Linear equations.

If the lines represented by the equation,

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$ are,

i) Intersecting, then $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

ii) Coincident, then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

iii) Parallel, then $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Compare the ratios	Graphical Representation	Algebraic Expressions	Consistency
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting Lines	Exactly one solution	System is consistent
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident Lines	Infinitely many Solutions	System is consistent (dependent)
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel Lines	No solution	System is inconsistent

Let us consider some examples here, to understand better the above relations.

Example: Solve graphically the pair of linear equations $3x - 4y + 3 = 0$ and $3x + 4y - 21 = 0$. Find the coordinate of the vertices of the triangular region formed by these lines and X-axis. Also, calculate the area of this triangle.

Firstly we will find two solutions for each equation.

1) Table for $3x - 4y + 3 = 0$

x	3	-1
$y = \frac{3x+3}{4}$	3	0

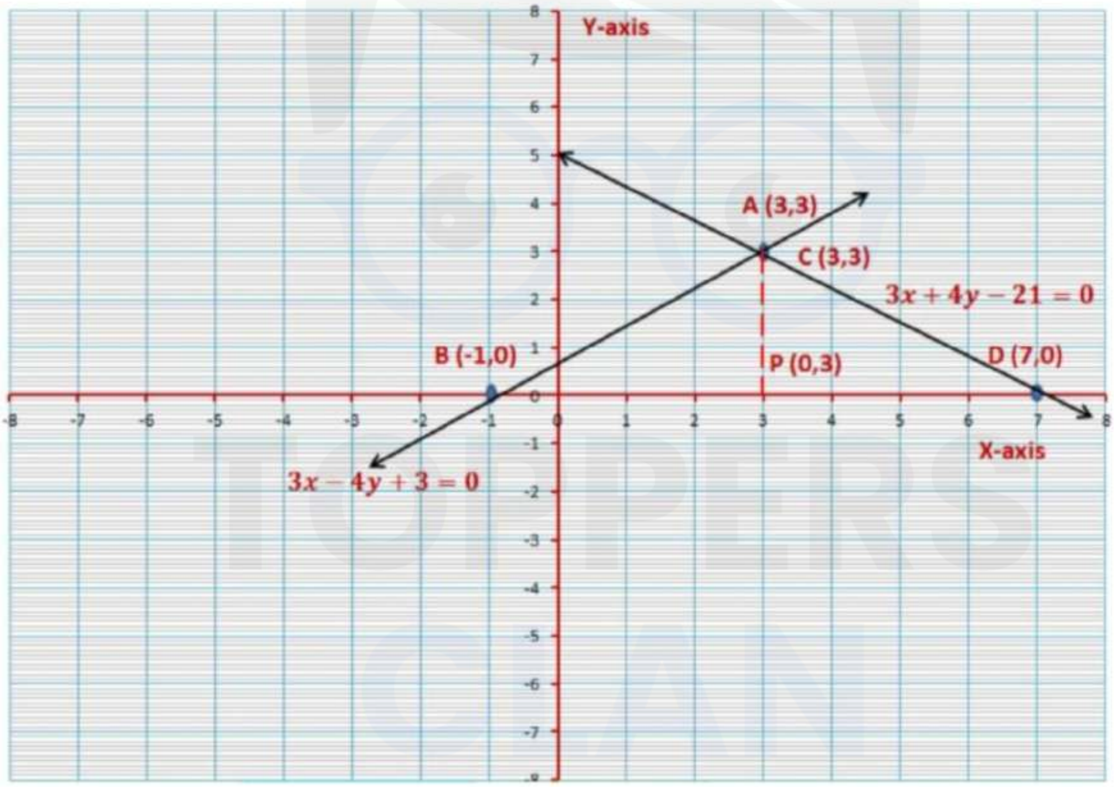
Let the points be A (3, 3) and B (-1,0)

2) Table for $3x + 4y - 21 = 0$

x	3	7
$y = \frac{21-3x}{4}$	3	0

Let the points be C (3, 3) and D (7,0)

Now we will plot these points on a graph paper.



We will join points A and B, A and D respectively.

(Point A and Point C have the same coordinate)

When we join these points we will get a triangle Δ ABD.

The coordinates of the vertices of Δ ABD are,

A (3,3), B (-1,0) and D (7,0)

Next, we have to find the area Δ ABD.

We know,

Area of a triangle = $\frac{1}{2}$ × Base × Height

In Δ ABD,

Base = 8 units

Height = 3 units

Area of Δ ABD = $\frac{1}{2}$ ×BD×AP

=12× 8 ×3 = 12 sq. units

Example: Solve the following system of linear equations graphically:

$$2x - y - 4 = 0$$
$$x + y + 1 = 0$$

The pair of linear equations is,

$$2x - y - 4 = 0$$
$$x + y + 1 = 0$$

First, we will find two solutions of each equation.

1) Table for $2x - y - 4 = 0$

x	0	2
$y = 2x - 4$	−4	0

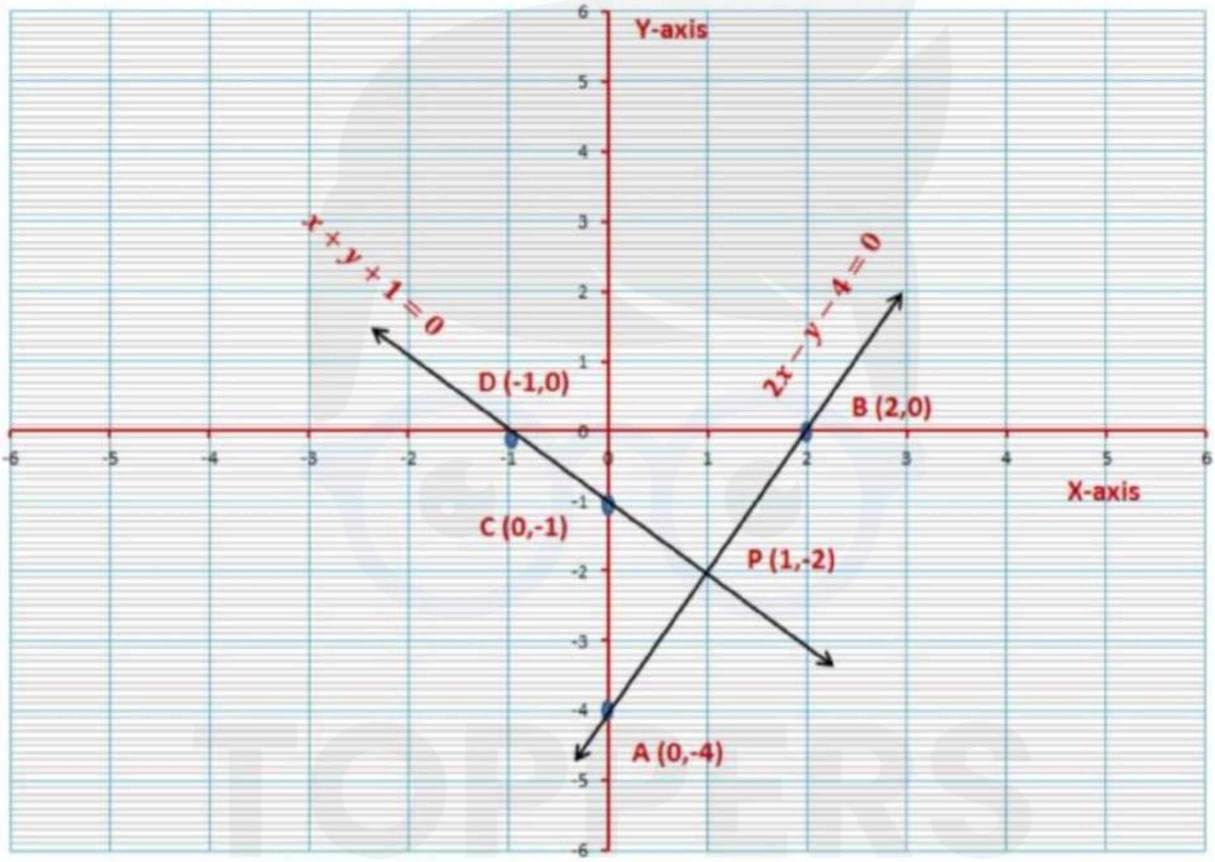
Let the points be A(0,-4) and B(2,0)

2) Table for x + y + 1 = 0

x	0	-1
$y = 1 - x$	-1	0

Let the points be C (0,-1) and D (-1,0)

Now we plot these points on a graph paper.



We have joined points A and B, C and D respectively.

We see that the two lines are intersecting at point P (1, -2).

So the solution of the given pair of equations is x = 1 andy =−2.

Example: Form a pair of linear equations in two variables using the following information and solve it graphically. Five years ago, Mayank was twice as old as Rajat. Ten years later, Mayank's age will be ten years more than Rajat's age. Find their present ages.

Let Mayank's and Rajat's present age be x years and y years respectively.

5 years ago,

Mayank's age was $(x - 5)$ years

Rajat's age was $(y - 5)$ years

Now it is given that 5 years ago Mayank was twice as old as Rajat.

$$\therefore (x - 5) = 2(y - 5)$$

$$x - 5 = 2y - 10$$

$$x - 2y = -10 + 5$$

$x - 2y = -5 \rightarrow$ Eq.1

10 years later,

Mayank's age will be $(x + 10)$ years

Rajat's age was $(y + 10)$ years

It is given that 10 years later, Mayank's age will be ten years more than Rajat's age.

$$\therefore (x + 10) = (y + 10) + 10$$

$$x + 10 = y + 20$$

$x - y = 10 \rightarrow$ Eq.2

We get a pair of linear equations,

$$x - 2y + 5 = 0$$

$$x - y - 10 = 0$$

Now we draw the graph of Equations 1 and 2.

We will find two solutions of the two equations first

Table for $x - 2y + 5 = 0$

x	5	-5
$y = \frac{x+5}{2}$	5	0

Let the points be A (5, 5) and B(-5,0)

Table for $x - y - 10 = 0$

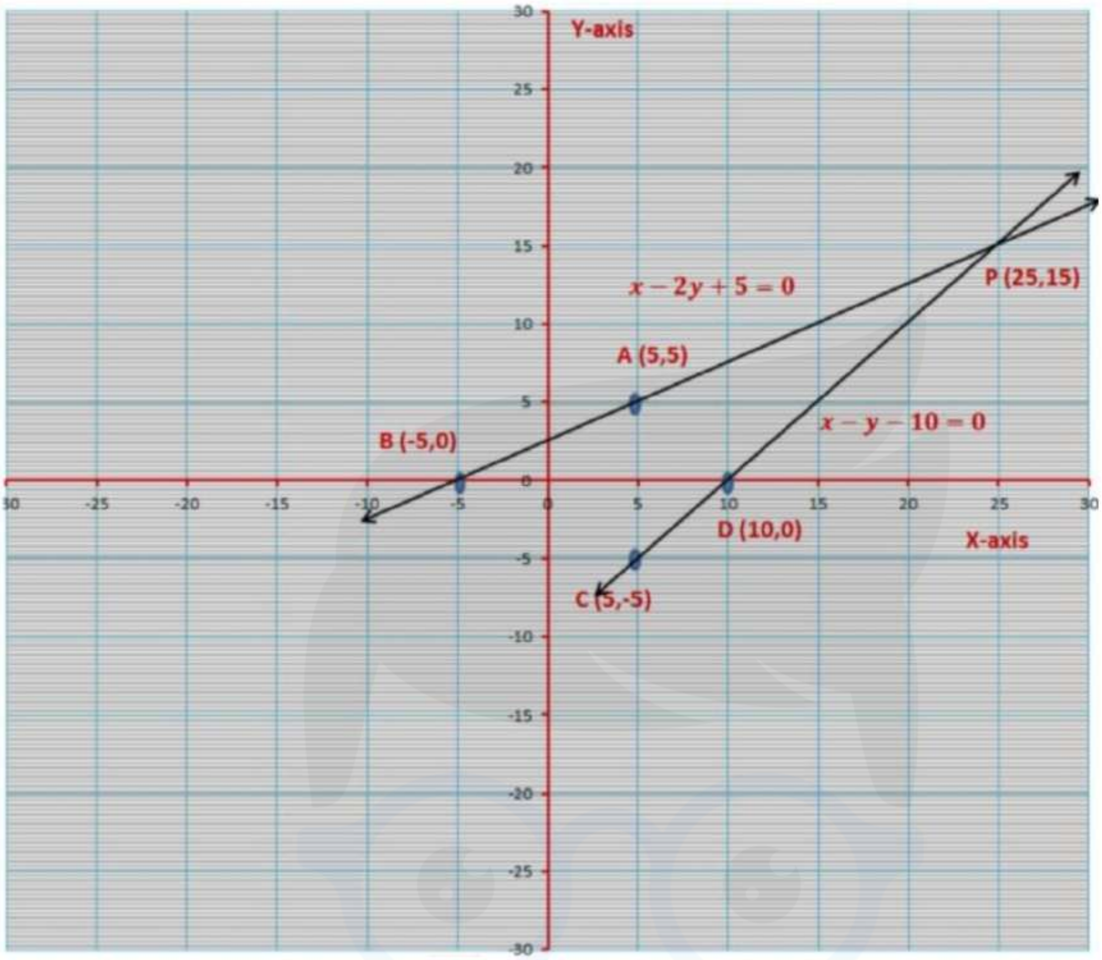
x	5	10
$y = x - 10$	-5	0

Let the points be C (5,-5) and D (10,0)

Now we plot these points on a graph paper.

Join points A and B to get, line AB

Join points C and D to get, line CD



We see that the lines AB and CD are intersecting at point P. The coordinate of points of P is (25, 15), that is, x = 25 years and y = 15 years

Mayank's age = 25 years

Rajat's age = 15 years

Example: Check whether the given pair of linear equations is consistent or inconsistent. $x + 2y = 4$ and $3x + 6y = 12$

The pair of linear equations is

$$x + 2y = 4$$
$$3x + 6y = 12$$

The standard form of pair of linear equations is

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

On comparing the linear equations with the standard form of equations we get,

$a_1 = 1$, $b_1 = 2$ and $c_1 = 4$

$a_2 = 3$, $b_2 = 6$ and $c_2 = 12$

Here, $\frac{a_1}{a_2} = \frac{1}{3}$

$\frac{b_1}{b_2} = \frac{2}{3} = \frac{1}{3}$

$\frac{c_1}{c_2} = \frac{4}{12} = \frac{1}{3}$

We see that $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{3}$

Therefore, the given pair of linear equations is consistent.

Example: Find the value of 'k' for which the system of equations $kx - 5y = 2$ and $6x + 2y = 7$ has no solution.
The given pair of linear equations is,

$$kx - 5y = 2$$
$$6x + 2y = 7$$

We know the standard form of pair of linear equations is

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

On comparing the linear equations with the standard form of equations we get,

$a_1 = k$, $b_1 = -5$and $c_1 = 2$
$a_2 = 6$, $b_2 = 2$and $c_2 = 7$

Here, $\frac{a_1}{a_2} = \frac{k}{6}$

$$\frac{b_1}{b_2} = \frac{-5}{2}$$

$$\frac{c_1}{c_2} =$$

We know that a pair of linear equations has no common solution when,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{k}{6} = \frac{-5}{2} \neq \frac{2}{7}$$

$$\frac{k}{6} = \frac{-5}{2}$$

$$k = \frac{-5X6}{2}$$

k = -15

Solving Pair of Linear Equations - Substitution Method

Substitution Method

Algebraic Methods for solving a Pair of Linear Equations

The graphical method for solving a pair of linear equations is more suitable for integers but for non-integers, it may not be as accurate as we need. So we use other algebraic methods to solve the pair of linear equations.

Some of the algebraic methods which we are going to study now are

i) Substitution Method
ii) Elimination Method
iii) Cross-Multiplication Method

Substitution Method
In the substitution method we find out the value of one variable in terms of the other variable and then we substitute this value in

other equations to get an equation in one variable. Now, this equation can be solved easily.

Let us consider some examples to understand the substitution method better.

Example: Solve the following pair of linear equations by the substitution method.

$0.2x + 0.3y = 1.3$ and $0.6x + 0.5y = 2.3$

The given pair of linear equations is

$0.2x + 0.3y = 1.3 \rightarrow$ Eq 1

$0.6x + 0.5y = 2.3 \rightarrow$ Eq 2

First, we find the value of variable y in terms of other variables, i.e. x.

From equation (1),

$$y = \frac{1.3 - 0.2x}{0.3}$$

Next, we substitute the value of y in equation (2) From equation (2),

$$0.6x + 0.5\frac{0.5(1.3 - 0.2)}{0.3} = 2.3$$

$$0.6x + \frac{5(1.3 - 0.2)}{0.3} = 2.3$$

$1.8x + (6.5 - x) = 2.3 \times 3$

$1.8x + 6.5 - x = 6.9$

$0.8x = 6.9 - 6.5$

$0.8x = 0.4$

$x = 0.5$

Putting the value of x in equation 1, we get

$0.2 \times 0.5 + 0.3y = 1.3$

$0.3y = 1.3 - 0.01$

$0.3y = 1.29$

$y = 4.3$

Therefore, $x = 0.5$ and $y = 4.3$

Example: Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of m for which $y = mx + 3$.

(REFERENCE: NCERT)

The given pair of linear equations is

$2x + 3y = 11$ →Eq 1

$2x - 4y = -24$ →Eq 2

We will first find the value of variable y in terms of variable, x.

From equation (1),

$$y = \frac{11 - 2x}{3}$$

Next substituting this value of y in equation (2), we get

$$2x - 4\frac{(11 - 2x)}{3} = -24$$

$$2x - \left(\frac{11 - 2x}{3}\right) = -24$$

$6x - (44 - 8x) = -24 \times 3$

$6x + 8x - 44 = -72$

$14x = -72 + 44$

$14x = -28$

$x = -2$

Putting the value of x in equation 1, we get

$2 \times (-2) - 4y = -24$

$-4 - 4y = -24$

$-4y = -24 + 4$

$-4y = -20$

$y = 5$

Therefore, $x = -2$ and $y = 5$

Substituting the value of x and y in the equation

$y = mx + 3$, we get

$$5 = -2m + 3$$
$$m = -1$$

Example: The difference between the two numbers is 75 and on number is four times the other.

Let the two numbers be x and y.

$$x > y$$

It is given that the difference between x and y is 75.

$x - y = 75 \rightarrow$ Eq 1

One number is four times the second number.

$x = 4y \rightarrow$Eq 2

Substituting the value of x from Eq 2 in Eq 1, we get

$$4y - y = 75$$

$$3y = 75$$

$$y = 25$$

Substituting the value of y in Eq 2, we get

$$x = 4 \times (25)$$

$$x = 100$$

The two numbers are 100 and 25.

Solving Pair of Linear Equations - Elimination Method

Elimination Method

Elimination Method is another method of eliminating one variable but in this method, one variable out of two variables is eliminated by making the coefficient of that variable

equal in both the equations.

Now, we will consider some examples.

Example: Use the elimination method to find all possible solutions of the following pair of linear equations.

$ax + by - a + b = 0$ and $bx - ay - a - b = 0$

The given pair of linear equations is

$ax + by - a + b = 0 \rightarrow$ Eq 1

$bx - ay - a - b = 0 \rightarrow$ Eq 2

Now, we will multiply Eq 1 by b and Eq 2 by a to make the coefficient of x equal

$bax + b^2y - ab + b^2 = 0 \rightarrow$ Eq 3

$abx - a^2y - a^2 - ab = 0 \rightarrow$ Eq 4

Subtract Eq 3 from Eq 4 to eliminate x, as the coefficients of x are the same

$bax + b^2y - ab + b^2 - (abx - a^2y - a^2 - ab) = 0$

$(ba - ab)x + (b^2 + a^2)y - ab + b^2 + a^2 + ab = 0$

$(b^2 + a^2)y = -(b^2 + a^2)$

$y =-(b^2 + a^2)b^2 + a^2$

$y = -1$

Putting $y = 1$ in Eq 2 we get,

$bx - a(-1) - a - b = 0$

$bx + a - a - b = 0$

$bx - b = 0$

$x = 1$

Therefore, $x = 1$, and $y = -1$ which is the required unique solution of the given pair of linear equations.

Example: The sum of the digits of a two-digit number is 6. Also, seventeen times this number is five times the number obtained by reversing the order of the digits. Find the

number.

Let the digit at the unit place be x and the digit at tens place be y.

The Original two-digit number is $10y + x$

The sum of the digits of the original number = 6

$\therefore y + x = 6$

$x + y - 6 = 0 \rightarrow$ Eq 1

On reversing the order of the two-digit number, we get y at units place and x at the tens place.

$\therefore$ The new two-digit number= $10x + y$

Seventeen times of the original number is equal to five the reversed number

$17(10y + x) = 5(10x + y)$

$170y + 17x = 50x + 5y$

$170y - 5y = 50x - 17x$

$165y = 33x$

$x = 5y$

$x - 5y = 0 \rightarrow \text{Eq } 2$

On subtracting Eq 2 from Eq 1, we get

$x + y - 6 - (x - 5y) = 0$
$x + y - 6 - x + 5y = 0$

$6y = 6$
$y = 1$

Substituting $y = 1$ in Eq 1

$x + 1 - 6 = 0$
$x = 5$

Therefore, the required two-digit number $= 10y + x$

$= 10 \times 1 + 5 = 15$

Example: A fraction becomes $\frac{5}{7}$, if 2 is subtracted to both the numerator and the denominator. If 3 is subtracted to both the numerator and denominator, it becomes $\frac{2}{3}$. Find the fraction.

Let the fraction be $\frac{x}{y}$.

Subtracting 2 to both the numerator and denominator, we get new fraction = $\frac{x-2}{y-2}$

It is given that the new fraction obtained = $\frac{5}{7}$

$$\therefore \frac{x-2}{y-2} = \frac{5}{7}$$

$$7(x-2) = 5(y-2)$$

$$7x - 14 = 5y - 10$$

$$7x - 5y = 14 - 10$$

$$7x - 5y = 4$$

$$7x - 5y - 4 = 0 \rightarrow \text{Eq } 1$$

Next, we subtract 3 to both the numerator and denominator New fraction $= \frac{x-3}{y-3}$

It is given that the new fraction obtained = $\frac{2}{3}$

$$\therefore \frac{x-3}{y-3} = \frac{2}{3}$$

$$3(x-3) = 2(y-3)$$

$$3x - 9 = 2y - 6$$

$$3x - 2y = 3$$

$$3x - 2y - 3 = 0 \rightarrow \text{Eq } 2$$

$$7x - 5y - 4 = 0 \rightarrow \text{Eq } 1$$

Multiplying Eq 1 by 3 and Eq 2 by 7 to make the coefficient of x equal, we get

$$21x - 15y - 12 = 0 \rightarrow \text{Eq } 3$$

$$21x - 14y - 21 = 0 \rightarrow \text{Eq } 4$$

Now, on subtracting, we get

$21x - 15y - 12 - (21x - 14y - 21) = 0$

$21x - 15y - 12 - 21x + 14y + 21 = 0$

$-y + 9 = 0$

$y - 9 = 0$

$y = 9$

Putting the value of y in Eq 1, we get

$7x - 5(9) - 4 = 0$

$7x - 45 - 4 = 0$

$7x = 49$

$x = 7$

Therefore the required fraction is $\frac{7}{9}$.

Example: The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10km, the charge paid is Rs. 110 and for a

journey of 15 km the charge paid is Rs.160. What are the fixed charges and the charge per kilometer? How much does a person have to pay for traveling a distance of 25

km?

Let fixed charge be Rs. x and charge per kilometer be Rs. y For 10 km distance,

Fixed charge + charge per kilometer = Rs. 110

$$x + 10y = 110$$

$x + 10y - 110 = 0 \rightarrow$ Eq 1

For 15 km distance,

Fixed charge + charge per kilometer = Rs. 16

$$x + 15y = 160$$

$x + 15y - 160 = 0 \rightarrow$ Eq 2

Now subtracting Eq 2 from Eq 1 we get,

$x + 15y - 160 - (x + 10y - 110) = 0$

$x + 15y - 160 - x - 10y + 110 = 0$

$15y - 10y - 50 = 0$

$5y - 50 = 0$

$y = 10$

Substituting the value of y in Eq 1 we get,

$x + 10y - 110 = 0$

$x + 10(10) - 110 = 0$

$x + 100 - 110 = 0$

$x = 10$

Therefore, the fixed charge is Rs. 5 and the charge per kilometer is Rs. 10.

For 25 km distance,

Fixed charge + charge per kilometer = Rs. $x + 25y$

$= \text{Rs } 10 + 25\,(10)$

$= \text{Rs } 260$

Solving Pair of Linear Equations - Cross Elimination Method

Cross Multiplication Method

Cross Multiplication Method is another useful method of solving a linear pair of linear equations.

To solve a pair of linear equations by cross multiplication method following steps are followed,

We know the standard form of pair of linear equations is

$a_1x + b_1y + c_1 = 0 \rightarrow$ Eq 1

$a_2x + b_2y + c_2 = 0 \rightarrow$ Eq 2

Now we multiply Eq 1 by b2 and Eq 2 by b1 to get,

$$b_2a_1x + b_2b_1y + b_2c_1 = 0 \rightarrow \text{Eq 3}$$

$$b_1a_2x + b_1b_2y + b_1c_2 = 0 \rightarrow \text{Eq 4}$$

Next, we subtract Eq 4 from Eq 3

$$b_2a_1x + b_2b_1y + b_2c_1 - (b_1a_1 + b_1b_2y + b_1c_2) = 0$$

$$b_2a_1x + b_2b_1y + b_2c_1 - b_1a_1 - b_1b_2y - b_1c_2 = 0$$

$$(b_2a_1 - b_1a_2)x + (b_2b_1 - b_1b_2)y + b_2c_1 - b_1c_2 = 0$$

$$(b_2a_1 - b_1a_2)x + b_2c_1 - b_1c_2 = 0$$

$x = \dfrac{b_1c_2 - b_2c_1}{(a_1b_2 - a_2b_1)}$, provided $(a_1b_2 - a_2b_1) \neq 0 \rightarrow$ Eq 5

Now we substitute the value of x in Eq 1to get,

$$\frac{a_1(b_1c_2 - b_2c_1)}{(a_1b_2 - a_2b_{1)}} + b_1y + c_1 = 0$$

$$\frac{a_1(b_1c_2 - a_1b_2c_1)}{(a_1b_2 - a_2b_{1)}} + b_1y + c_1 = 0$$

$$b_1 y = c_1 - \frac{a_1 b_1 c_2 - a_1 b_2 c_1}{(a_1 b_2 - a_2 b_1)}$$

$$b_1 y = \frac{-(a_1 b_1 c_2 - a_1 b_2 c_1) - c_1(a_1 b_2 - a_2 b_1)}{(a_1 b_2 - a_2 b_1)}$$

$$y = \frac{-a_1 b_1 c_2 + a_1 b_2 c_1 - a_1 b_2 c_1 + a_2 b_1 c_1}{b_1(a_1 b_2 - a_2 b_1)}$$

$$y = \frac{a_2 b_1 c_1 - a_1 b_1 c_2}{b_1(a_1 b_2 - a_2 b_1)}$$

$$y = \frac{b_1(a_2 c_1 - a_1 c_2)}{b_1(a_1 b_2 - a_2 b_1)}$$

$$y = \frac{(c_1 a_2 - c_2 a_1)}{a_1 b_2 - a_2 b_1} \rightarrow \text{Eq } 6$$

The above solution is commonly written as,

$$\frac{x}{b_1 c_2 - b_2 c_1} = \frac{y}{c_1 a_2 - c_2 a_1} = \frac{1}{a_1 b_2 - a_2 b_1}$$

Now two different cases may arise,

Case 1: $(a_1 b_2 - a_2 b_1) \neq 0$.

$a_1 b_2 \neq a_2 b_1$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

In this case, the pair of linear equations has a unique solution.

Case 2: $(a_1 b_2 - a_2 b_1) = 0$

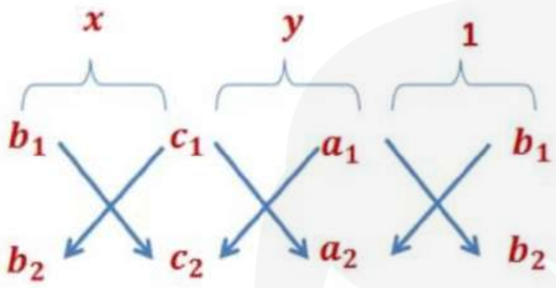
$a_1 b_2 = a_2 b_1$,

$\frac{a_1}{a_2} = \frac{b_1}{b_2}$, then there are two conditions

i) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ then the pair of linear equations has infinitely many solutions.

ii) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ then the pair of linear equations has no solutions.

The diagram given below is useful in memorizing the method of cross-multiplication.



The arrow between the numbers indicates that they are to be multiplied and the second product is to be subtracted from the first.

Therefore, we get,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{(c_1a_2 - c_2a_1)} = \frac{1}{(a_1b_2 - a_2b_1)}$$

Example: Solve the following pair of linear equations by Cross- Multiplication Method.

$$8x + 5y = 9,\ 3x + 2y = 4$$

(REFERENCE: NCERT)
The given pair of linear equation is

$$8x + 5y = 9$$

$8x + 5y - 9 = 0 \rightarrow$ Eq 1

$$3x + 2y = 4$$

$3x + 2y - 4 = 0 \rightarrow$ Eq 2

We know the standard form of pair of linear equations is

$$a_1x + b_1y + c_1 = 0 \rightarrow \text{Eq } 3$$

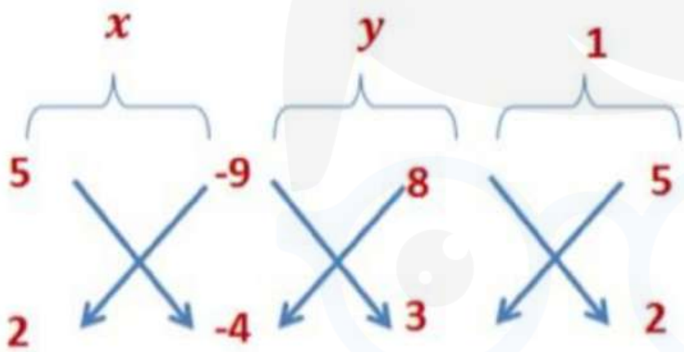
$$a_2x + b_2y + c_2 = 0 \rightarrow \text{Eq } 4$$

Next we compare the given pair of linear equations with the standard form of linear equation.

So, we compare Eq 1 with Eq 3 and Eq 2 with Eq 4 respectively to get,

$$a_1 = 8,\ a_2 = 3,\ b_1 = 5,\ b_2 = 2,\ c_1 = -9,\ c_2 = -4$$

To solve the equations by cross multiplication method, we draw the diagram given below



Therefore, we get,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{(c_1a_2 - c_2a_1)} = \frac{1}{(a_1b_2 - a_2b_1)}$$

$$\frac{x}{5(-4) - 2(-9)} = \frac{y}{(-9)3(-4)8} = \frac{1}{8(2) - 3(5)}$$

$$\frac{x}{-20+18} = \frac{y}{-27+32} = \frac{1}{16-15}$$

$$\frac{x}{-2} = \frac{y}{5} = 11$$

Therefore, $x = -2$ and $y = 5$

Example 2: For which values of a and b does the following pair of linear equations have an infinite number of solutions.

$2x + 3y = 7$ and $(a - b)x + (a + b)y = 3a + b - 2$

(REFERENCE: NCERT)

The given pair of linear equation is

$$2x + 3y = 7$$

$2x + 3y - 7 = 0 \rightarrow$ Eq 1

$$(a - b)x + (a + b)y = 3a + b - 2$$

$(a - b)x + (a + b)y - 3a - b + 2 = 0 \rightarrow$ Eq 2

Next, we compare the given pair of linear equations with the standard form of linear equation,

$$a_1x + b_1y + c_1 = 0 \rightarrow \text{Eq } 3$$
$$a_2x + b_2y + c_2 = 0 \rightarrow \text{Eq } 4$$

On comparing Eq 1with Eq 3 and Eq 2 with Eq 4, we get

$a_1 = 2$, $a_2 = (a - b)$, $b_1 = 3$, $b_2 = (a + b)$, $c_1 = -7$, $c_2 = -3a - b + 2$

For an infinite number of solutions we have

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ then the pair of linear equations has infinitely many solutions.

$$\frac{2}{(a-b)} = \frac{3}{a+b} = \frac{-7}{-3a-b+2}$$

(1) (2) (3)

Taking 1 and 2 terms we get,

$$\frac{2}{(a-b)} = \frac{3}{(a+b)}$$

$2(a + b) = 3(a - b)$

$2a + 2b = 3a - 3b$

$2a - 3a = -3b - 2b$

$-a = -5b$

$a = 5b \rightarrow$ Eq 3

Taking 2 and 3 we get,

$$\frac{3}{(a+b)} = \frac{-7}{-3a - b + 2}$$

$3(-3a - b + 2) = -7(a + b)$

$-9a - 3b + 6 = -7a - 7b$

$-9a + 7a - 3b + 7b = -6$

$-2a + 4b = -6$

Dividing the above equation by 2 we get,

$-a + 2b = -3 \rightarrow$ Eq 4

Putting $a = 5b$ in the above equation we get,

$$-5b + 2b = -3$$

$$-3b = -3$$

$$b = 1$$

Putting the value of b in $\rightarrow$ Eq 3 we get

$$a = 5(1) = 5$$

Therefore, $a = 5$, and $b = 1$

Example: The ratio of incomes of two persons is 9:7 and the ratio of their expenditures is 4:3. If each of them saves Rs 200 per month, find their monthly incomes.

(REFERENCE: NCERT)

Let the monthly income of the two persons be 9x and 7x respectively.

Let their monthly expenditure be 4y and 3y respectively.

Monthly Savings = Monthly Income – Monthly expenditure

Monthly Savings of the first person=$9x - 4y$
Monthly Savings of the second person= $7x - 3y$

$$9x - 4y = 200$$

$$9x - 4y - 200 = 0 \rightarrow \text{Eq } 1$$

$$7x - 3y = 200$$

$$7x - 3y - 200 = 0 \rightarrow \text{Eq } 2$$

The standard form of pair of linear equations is

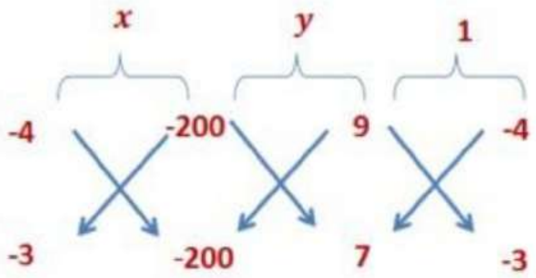
$$a_1x + b_1y + c_1 = 0 \rightarrow \text{Eq } 3$$
$$a_2x + b_2y + c2 = 0 \rightarrow \text{Eq } 4$$

Next, we compare the given pair of linear equations with the standard form of linear equations.

So, we compare Eq 1 with Eq 3 and Eq 2 with Eq 4 respectively to get,

$a_1 = 9, a_2 = 7, b_1 = -4, b2 = -3, c_1 = -200, c_2 = -200$.

By using cross multiplication method we get,



$$\frac{x}{-4(-200)-(-3)(-200)} = \frac{y}{(-200)7-(-200)9}$$

$$= \frac{1}{9(-3)-7(-4)}$$

$$\frac{x}{800-600} = \frac{y}{1400+1800} = \frac{1}{-27+28}$$

$$\frac{x}{200} = \frac{y}{400} = \frac{1}{1}$$

x = 200

y = 400

Monthly income of first person = 9x = 9 × 200 = Rs.1800

Monthly income of second person = 7x = 7 × 200 = Rs. 1400

Equations Reducible to Linear Equations in Two Variables

Equations Reducible to a pair of Linear Equations in Two Variables

Some equations are not linear but can be reduced to a pair of linear equations by making suitable substitutions. Let us consider some examples.

Example: Solve for x and y.

x + y = 18xy and x + 2y = 21xy

The given pair of linear equations is

x + y = 18xy

$x + 2y = 21xy$

On dividing Eq 1 and Eq 2 by xy we get,

$\frac{1}{y} + \frac{1}{x} = 18 \rightarrow$ Eq 1

$\frac{1}{y} + \frac{2}{x} = 21 \rightarrow$ Eq 2

These equations are not in the form$ax + by + c = 0$. If we substitute $\frac{1}{y} = p$ and $\frac{1}{x} = q$ in Eq 1 and Eq 2 we get,

$p + q = 18 \rightarrow$ Eq 3

$p + 2q = 21 \rightarrow$ Eq 4

On subtracting Eq 3 from Eq 4 we get,

$$p + 2q - (p + q) = 21 - 18$$
$$p + 2q - p - q = 21 - 18$$
$$p - p + 2q - q = 3$$
$$q = 3$$
$$q = 3$$

Putting $q = 3$ in Eq 3 we get,

$$p + (3) = 18$$
$$p = 18 - 3$$
$$p = 15$$

Now, $\frac{1}{y} = p$

$\frac{1}{y} = 15 \Rightarrow y = \frac{1}{15}$

$\frac{1}{x} = q$

$\frac{1}{x} = 3 \Rightarrow x = \frac{1}{3}$

Therefore, $x = \frac{1}{3}$ and $y = \frac{1}{15}$ is the required solution.

Example: If 2 times the area of a smaller square is subtracted from the area of a larger square, the result is 16 m^2. The sum of the area of the two squares is 208 m^2. Determine the side of the two squares.

Let the area of the smaller square be x^2 and the area of the larger square be y^2.

On subtracting the area of the smaller square from the area of the larger square we get,

$y^2 - 2x^2 = 16$ m2 → Eq 1

On adding the area of the two squares we get,
$y^2 + x^2 = 208\ m^2$ → Eq 2.

To convert Eq 1 and Eq 2 to linear equations we assume that,

$x^2 = p$ and $y^2 = q$.

Eq 1 becomes,

$$q - 2p = 16$$

$q - 2p - 16 = 0$ → Eq 3

Eq 2 becomes,

$$q + p = 208$$

$q + p - 208 = 0$ → Eq 4

Now, subtracting Eq 3 from Eq 4, we get

$$q + p - 208 - (q - 2p - 16) = 0$$

$$q + p - 208 - q + 2p + 16 = 0$$

$$3p - 192 = 0$$

$$p = 64$$

Putting the value of p in Eq 4 we get,

$$q + 64 - 208 = 0$$

$$q - 144 = 0$$

$$q = 144$$

Now we know that $x^2 = p$ and $y^2 = q$

$x^2 = p \Rightarrow x^2 = 64$

$x = 8$

$y^2 = q \Rightarrow y^2 = 144$

$y = 12$

Therefore, the side of the smaller square is 8 m and the side of the larger square is 12 m.

Example: Ritu can row downstream 20 km in 2 h and upstream 4 km in 2 h. Find her speed of rowing in still water and the speed of the current.

(REFERENCE: NCERT)

Let the speed of rowing in still water be x km\hr and the speed

of current be y km\hr.

Downstream speed = $(x + y)$ km\hr

Upstream speed = $(x - y)$ km\hr

When Ritu is rowing downstream then,

Time taken = 2 h

Distance covered = 20 km

Speed= $(x + y)$ km\hr

We know, Time = $\frac{Distance}{Speed}$

$2 = \frac{20}{x+y}$

2 (x + y) = 20

x + y = 10 → Eq 1

When Ritu is rowing upstream then,

Time taken = 2 h

Distance covered = 4 km

Speed= (x − y) km\hr

We know, Time = $\frac{Distance}{Speed}$

$2 = \frac{4}{x-y}$

2 (x − y) = 4

x − y = 2 → Eq 2

Subtracting Eq 1 from Eq 2 we get,

$$x - y - (x + y) = 2 - 10$$

$$x - y - x - y = -8$$

$$x - x - y - y = -8$$

$$-2y = -8$$

$$y = 4$$

Putting y = 4 in Eq 2 we get,

$$x - 4 = 2$$

$$x = 6$$

Hence, the speed of rowing in still water is 6 km\hr and the speed of the current is 4 km\hr.