# Mathematics

**(Chapter – 7) (Integrals) (Exercise 7.1)**
**(Class – XII)**

Find an anti-derivative (or integral) of the following functions by the method of inspection.

**Question 1:**

$\sin 2x$

**Answer 1:**

The anti-derivative of $\sin 2x$ is a function of $x$ whose derivative is $\sin 2x$. It is known that,

$$\frac{d}{dx}(\cos 2x) = -2\sin x \qquad \Rightarrow \sin 2x = -\frac{1}{2}\frac{d}{dx}(\cos 2x)$$

$$\therefore \sin 2x = \frac{d}{dx}\left(-\frac{1}{2}\cos 2x\right)$$

Therefore, the anti-derivative of $\sin 2x$ $is -\frac{1}{2}\cos 2x$

**Question 2:**

$cos3x$

**Answer 2:**

The anti-derivative of $cos\ 3x$ is a function of x whose derivative is cos 3x.
It is known that,

$$\frac{d}{dx}(sin3x) = 3cos3x \qquad \Rightarrow cos3x = \frac{1}{3}\frac{d}{dx}(sin3x)$$

$$\therefore cos3x = \frac{d}{dx}\left(\frac{1}{3}sin3x\right)$$

Therefore, the anti-derivative of $cos3x$ $is \frac{1}{3}sin3x$.

**Question 3:**

$e^{2x}$

**Answer 3:**

The anti-derivative of e2x is the function of x whose derivative is $e^{2x}$.
It is known that,

$$\frac{d}{dx}(e^{2x}) = 2e^{2x}$$

$$\Rightarrow e^{2x} = \frac{1}{2}\frac{d}{dx}(e^{2x})$$

$$\therefore e^{2x} = \frac{d}{dx}\left(\frac{1}{2}e^{2x}\right)$$

Therefore, the anti-derivative of $e^{2x}$ is $\frac{1}{2}e^{2x}$.

**Question 4:**

$(ax + +b)^2$

**Answer 4:**

The anti-derivative of $(ax + b)^2$ is the function of x whose derivative is $(ax + b)^2$.
It is known that,

$$\frac{d}{dx}(ax+b)^2 = 3a(ax+b)^2$$

$$\Rightarrow (ax+b)^2 = \frac{1}{3a}\frac{d}{dx}(ax+b)^3$$

$$\therefore (ax+b)^2 = \frac{d}{dx}\left(\frac{1}{3a}(ax+b)^3\right)$$

Therefore, the anti-derivative of $(ax+b)^2$ is $\frac{1}{3a}(ax+b)^3$.

**Question 5:**

$sin2x - 4e^{3x}$

**Answer 5:**

The anti-derivative of $sin\ 2x - 4e^{3x}$ is the function of x whose derivative is $sin\ 2x - 4e^{3x}$.

It is known that,

$$\frac{d}{dx}\left(-\frac{1}{2}cos2x - \frac{4}{3}e^{3x}\right) = sin2x - 4e^{3x}$$

Therefore, the anti derivative of $(sin2x - 4e^{3x})$ is $\left(-\frac{1}{2}cos2x - \frac{4}{3}e^{3x}\right)$.

**Find the following integrals in Exercise 6 to 20**

**Question 6:**

$$\int (4e^{3x} + 1)dx$$

**Answer 6:**

$\int (4e^{3x} + 1)dx \quad = 4\int e^{3x}dx + \int 1\,dx$

$= 4\left(\frac{e^{3x}}{3}\right) + x + C \qquad = \frac{4}{3}e^{3x} + x + C$

**Question 7:**

$$\int x^2\left(1 - \frac{1}{x^2}\right)dx$$

**Answer 7:**

$\int x^2\left(1 - \frac{1}{x^2}\right)dx \quad = \int (x^2 - 1)dx$

$= \int x^2dx - \int 1\,dx \quad \frac{x^3}{3} - x + C$

**Question 8:**

$$\int (ax^2 + bx + c)dx$$

**Answer 8:**

$\int (ax^2 + bx + c)dx \quad = a\int x^2dx + b\int xdx + c\int 1.dx$

$= a\left(\frac{x^3}{3}\right) + b\left(\frac{x^2}{2}\right) + cx + C$

$= \frac{ax^3}{3} + \frac{bx^2}{2} + cx + C$

**Question 9:**

$$\int (2x^2 + e^x)dx$$

**Answer 9:**

$\int (2x^2 + e^x)dx \quad = 2\int x^2dx + \int e^xdx$

$= 2\left(\frac{x^3}{3}\right) + e^x + C$

$= \frac{2}{3}x^3 + e^x + C$

**Question 10:**

$$\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 dx$$

**Answer 10:**

$$\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 dx \quad =\int\left(x+\frac{1}{x}-2\right)dx$$

$$=\int x dx+\int\frac{1}{x}dx-2\int 1.dx \quad =\frac{x^2}{2}+log|x|-2x+C$$

**Question 11:**

$$\int\frac{x^3+5x^2-4}{x^2}\,dx$$

**Answer 11:**

$$\int\frac{x^3+5x^2-4}{x^2}\,dx \quad =\int(x+5-4x^{-2})dx$$

$$=\int x dx+5\int 1.dx-4\left(\frac{x^{-1}}{-1}\right)+C \quad =x^2+5x+\frac{4}{x}+C$$

**Question 12:**

$$\int\frac{x^3+3x+4}{\sqrt{x}}dx$$

**Answer 12:**

$$\int\frac{x^3+3x+4}{\sqrt{x}}dx \quad =\int(x^{\frac{5}{2}}+3x^{\frac{1}{2}}+4x^{-\frac{1}{2}})\,dx$$

$$=\frac{x^{\frac{7}{2}}}{\frac{7}{2}}+\frac{3\left(x^{\frac{3}{2}}\right)}{\frac{3}{2}}+\frac{4\left(x^{\frac{1}{2}}\right)}{\frac{1}{2}}+C \quad =\frac{2}{7}x^{\frac{7}{2}}+2x^{\frac{3}{2}}+8x^{\frac{1}{2}}+C \quad =\frac{2}{7}x^{\frac{7}{2}}+2x^{\frac{3}{2}}+8\sqrt{x}+C$$

**Question 13:**

$$\int\frac{x^3-x^2+x-1}{x-1}dx$$

**Answer 13:**

$$\int\frac{x^3-x^2+x-1}{x-1}dx$$

On dividing, we obtain

$$=\int(x^2+1)dx \;=\int x^2dx+\int 1dx \;=\frac{x^3}{3}+x+C$$

**Question 14:**

$$\int(1-x)\sqrt{x}dx$$

**Answer 14:**

$$\int(1-x)\sqrt{x}dx$$

$$=\int\left(\sqrt{x}-x^{\frac{3}{2}}\right)dx=\int x^{\frac{1}{2}}dx-\int x^{\frac{3}{2}}dx$$

$$=\frac{x^{\frac{3}{2}}}{\frac{3}{2}}-\frac{x^{\frac{5}{2}}}{\frac{5}{2}}+C=\frac{2}{3}x^{\frac{3}{2}}-\frac{2}{5}x^{\frac{5}{2}}+C$$

**Question 15:**

$$\int \sqrt{x}(3x^2 + 2x + 3)dx$$

**Answer 15:**

$$\int \sqrt{x}(3x^2 + 2x + 3)dx$$

$$= \int \left(3x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + 3x^{\frac{1}{2}}\right)dx \ = 3\int x^{\frac{5}{2}}dx + 2\int x^{\frac{3}{2}}dx + 3\int x^{\frac{1}{2}}dx$$

$$= 3\left(\frac{x^{\frac{7}{2}}}{\frac{7}{2}}\right) + 2\left(\frac{x^{\frac{3}{2}}}{\frac{5}{2}}\right) + 3\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right) + C \ = \frac{6}{7}x^{\frac{7}{2}} + \frac{4}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + C$$

**Question 16:**

$$\int (2x - 3cosx + e^x)dx$$

**Answer 16:**

$$\int (2x - 3cosx + e^x)dx \ = 2\int x^2dx - 3\int sinxdx + 5\int x^{\frac{1}{2}}dx$$

$$= \frac{2x^2}{2} - 3(sinx) + e^x + C \ = x^2 - 3sinx + e^x + C$$

**Question 17:**

$$\int \left(2x^2 - 3sinx + 5\sqrt{x}\right)dx$$

**Answer 17:**

$$\int \left(2x^2 - 3sinx + 5\sqrt{x}\right)dx \ = 2\int x^2dx - 3\int sinxdx + 5\int x^{\frac{1}{2}}dx$$

$$= \frac{2x^3}{3} - 3(-cosx) + 5\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right) + C \ \ = \frac{2}{3}x^3 + 3cosx + \frac{10}{3}x^{\frac{3}{2}} + C$$

**Question 18:**

$$\int \sec x\,(\sec x + \tan x)dx$$

**Answer 18:**

$$\int \sec x\,(\sec x + \tan x)dx \ = \int (sec^2x + \sec x \tan x)dx$$

$$= \int sec^2x\,dx + \int \sec x \tan x\,dx \ = \tan x + \sec x + C$$

**Question 19:**

$$\int \frac{sec^2x}{cosec^2x}dx$$

**Answer 19:**

$$\int \frac{sec^2x}{cosec^2x}dx = \int \frac{\frac{1}{cos^2x}}{\frac{1}{sin^2x}}dx \ \ = \int \frac{sin^2x}{cos^2x}dx$$

$$= \int tan^2xdx \ \ = \int (sec^2x - 1)dx \ = \int sec^2xdx - \int 1dx = \tan x - x + C$$

## Question 20:

$$\int \frac{2-3\sin x}{cos^2 x} dx$$

## Answer 20:

$$\int \frac{2-3\sin x}{cos^2 x} dx = \int \left(\frac{2}{cos^2 x} - \frac{3sinx}{cos^2 x}\right) dx = \int 2sec^2 x dx - 3\int \tan x \sec x\, dx = 2\tan x - 3\sec x + C$$

Choose the correct answer in Exercises 21 and 22

## Question 21:

The anti-derivative of $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$ equals

(A) $\frac{1}{3}x^{\frac{1}{3}} + 2x^{\frac{1}{2}} + C$ (B) $\frac{2}{3}x^{\frac{2}{3}} + \frac{1}{2}x^2 + C$ (C) $\frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$ (D) $\frac{3}{2}x^{\frac{3}{2}} + \frac{1}{2}x^{\frac{1}{2}} + C$

## Answer 21:

$$\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx$$

$$= \int x^{\frac{1}{2}} dx + \int x^{-\frac{1}{2}} dx$$

$$= \int \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C = \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$$

Hence, the correct answer is C.

## Question 22:

If $\frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}$ such that $f(2) = 0$, then $f(x)$ is

(A) $x^4 + \frac{1}{x^3} - \frac{129}{8}$ (B) $x^3 + \frac{1}{x^4} + \frac{129}{8}$ (C) $x^4 + \frac{1}{x^3} + \frac{129}{8}$ (D) $x^3 + \frac{1}{x^4} - \frac{129}{8}$

## Answer 22:

It is given that, $\frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}$

Anti-derivative of $4x^3 - \frac{3}{x^4} = f(x)$

$$\therefore f(x) = \int 4x^3 - \frac{3}{x^4} dx$$

$$= 4\int x^3 dx - 3\int (x^{-4})\, dx$$

$$= 4\left(\frac{x^4}{4}\right) - \left(\frac{x^{-3}}{-3}\right) + C = x^4 + \frac{1}{x^3} + C$$

Also, $f(2) = 0$, $\therefore f(2) = (2)^4 + \frac{1}{(2)^3} + C = 0$

$$\Rightarrow 16 + \frac{1}{8} + C = 0$$

$$\Rightarrow C = -\left(16 + \frac{1}{8}\right)$$

$$\Rightarrow C = -\frac{129}{8}$$

$$\therefore f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8}.$$

Hence, the correct answer is A.

# Mathematics

**(Chapter – 7) (Integrals) (Exercise 7.2)**
**(Class – XII)**

Integrate the functions in Exercises 1 to 37:

**Question 1:**

$$\frac{2x}{1+x^2}$$

**Answer 1:**

Let $1+x^2=t \quad \Rightarrow 2x\,dx=dt$

$$I=\int\frac{2x}{1+x^2}dx=\int\frac{1}{t}dt \quad \Rightarrow log(t)+C \quad \Rightarrow log|1+x^2|+C$$

**Question 2:**

$$\frac{(\log x)^2}{x}$$

**Answer 2:**

Let $log\ x\ = t \Rightarrow \frac{1}{x}dx=dt$

$$I=\int\frac{(\log x)^2}{x}dx=\int t^2dt \ =\frac{t^3}{3}+C \ =\frac{(\log x)^2}{x}+C$$

**Question 3:**

$$\frac{1}{x+x\log x}$$

**Answer 3:**

$$\frac{1}{x+x\log x}=\frac{1}{x(1+\log x)}$$

Let $(1+\log x)=t \ \Rightarrow \frac{1}{x}dx=dt$

$$I=\int\frac{1}{x+x\log x}dx=\int\frac{1}{t}dt \ =log(t)+C \ =log(1+\log x)+C$$

**Question 4:**

$$\sin x.\sin(\cos x)$$

**Answer 4:**

Let $cos\,x\ = t \Rightarrow -sin\,x\,dx\ = dt$

$$I=\int\sin x.\sin(\cos x)dx=-\int\sin t\,dt \ =-[-\cos t]+C=\cos t+C=cos(cos\,x)+C$$

**Question 5:**

$$sin(ax+b)\,cos(ax+b)$$

**Answer 5:**

$$sin(ax+b)\,cos(ax+b)=\frac{2sin(ax+b)\,cos(ax+b)}{2}=\frac{sin2(ax+b)}{2}$$

Let $2(ax+b)=t \quad \Rightarrow 2adx=dt$

$$I=\int\frac{sin2(\mathrm{ax}+\mathrm{b})}{2}dx=\frac{1}{2}\int\frac{\sin t\,dt}{2a} \ =\frac{1}{4a}[-\cos t]+C \ =-\frac{1}{4a}cos\,2(ax+b)+C$$

**Question 6:**

$$\sqrt{ax+b}$$

**Answer 6:**

Let $ax+b=t \Rightarrow a\,dx=dt \quad \Rightarrow \; dx=\frac{1}{a}dt$

$$I=\int (ax+b)^{\frac{1}{2}}dx=\frac{1}{a}\int t^{\frac{1}{2}}dt=\frac{1}{a}\left(\frac{t^{\frac{1}{2}}}{\frac{3}{2}}\right)+C$$

$$=\frac{2}{3a}\;(ax+b)^{\frac{3}{2}}+C$$

**Question 7:**

$$x\sqrt{x+2}$$

**Answer 7:**

Let $x+2=t \Rightarrow dx=dt \;\Rightarrow x=t-2$

$$I=\int x\sqrt{x+2}\,dx=\int (t-2)\sqrt{t}dt \quad =\int\left(t^{\frac{3}{2}}-2t^{\frac{1}{2}}\right)dt$$

$$=\int t^{\frac{3}{2}}dt-2\int t^{\frac{1}{2}}dt=\frac{t^{\frac{5}{2}}}{\frac{5}{2}}-2\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right)+C \;=\frac{2}{5}t^{\frac{5}{2}}-\frac{4}{3}t^{\frac{3}{2}}+C$$

$$=\frac{2}{5}(x+2)^{\frac{5}{2}}-\frac{4}{3}(x+2)^{\frac{3}{2}}+C$$

**Question 8:**

$$x\sqrt{1+2x^2}$$

**Answer 8:**

Let $1+2x^2=t \quad \Rightarrow 4xdx=dt$

$$I=\int x\sqrt{1+2x^2}dx=\frac{1}{4}\int \sqrt{t}dt=\frac{1}{4}\int t^{\frac{1}{2}}dt$$

$$=\frac{1}{4}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right)+C=\frac{1}{6}(1+2x^2)^{\frac{3}{2}}+C$$

**Question 9:**

$$(4x+2)\sqrt{x^2+x+1}$$

**Answer 9:**

Let $x^2+x+1=t \quad \Rightarrow (2x+1)dx=dt$

$$I=\int (4x+2)\sqrt{x^2+x+1}dx=\int 2\sqrt{t}\,dt$$

$$=2\int \sqrt{t}\,dt=2\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right)+C=\frac{4}{3}(x^2+x+1)^{\frac{3}{2}}+C$$

**Question 10:**

$$\frac{1}{x-\sqrt{x}}$$

**Answer 10:**

$$\frac{1}{x-\sqrt{x}}=\frac{1}{\sqrt{x}(\sqrt{x}-1)}$$

Let $(\sqrt{x}-1)=t \quad \Rightarrow \frac{1}{2\sqrt{x}}dx=dt$

$$I=\int \frac{1}{\sqrt{x}(\sqrt{x}-1)}dx=\int \frac{2}{t}dt \; =2\log|t|+C=2\log\left|\sqrt{x}-1\right|+C$$

**Question 11:**

$$\frac{x}{\sqrt{x+4}}, x>0$$

**Answer 11:**

$Let\ x+4=t \quad \Rightarrow dx=dt$

$$I=\int \frac{x}{\sqrt{x+4}}dx=\int \frac{t-4}{\sqrt{t}}\,dt \;=\frac{t^{\frac{3}{2}}}{\frac{3}{2}}-4\left(\frac{t^{\frac{1}{2}}}{\frac{1}{2}}\right)+C=\frac{2}{3}t^{\frac{3}{2}}-8t^{\frac{1}{2}}+C$$

$$=\frac{2}{3}t.t^{\frac{1}{2}}-8t^{\frac{1}{2}}+C \;=\frac{2}{3}t^{\frac{1}{2}}(t-12)+C=\frac{2}{3}(x+4)^{\frac{1}{2}}(x+4-12)+C$$

$$=\frac{2}{3}\sqrt{(x+4)}(x-8)+C$$

**Question 12:**

$$(x^3-1)^{\frac{1}{3}}x^5$$

**Answer 12:**

$Let\ x^3-1=t \Rightarrow 3x^2dx=dt$

$$I=\int (x^3-1)^{\frac{1}{3}}x^5dx=\int (x^3-1)^{\frac{1}{3}}x^3.x^2dx$$

$$=\int t^{\frac{1}{3}}(t+1)\frac{dt}{3} \;=\frac{1}{3}\int \left(t^{\frac{4}{3}}+t^{\frac{1}{3}}\right)dt=\frac{1}{3}\left[\frac{t^{\frac{7}{3}}}{\frac{7}{3}}+\frac{t^{\frac{4}{3}}}{\frac{4}{3}}\right]+C=\frac{1}{3}\left[\frac{3}{7}t^{\frac{7}{3}}+\frac{3}{4}t^{\frac{4}{3}}\right]+C$$

$$=\frac{1}{7}(x^3-1)^{\frac{7}{3}}+\frac{1}{4}(x^3-1)^{\frac{4}{3}}+C$$

**Question 13:** $\frac{x^2}{(2+3x^3)^3}$

**Answer 13:**

Put, $2+3x^3=t$

$\therefore 9x^2dx=dt$

$$\Rightarrow \int \frac{x^2}{\left(2+3x^3\right)^3}dx=\frac{1}{9}\int \frac{dt}{(t)^3}$$

$$=\frac{1}{9}\left[\frac{t^{-2}}{-2}\right]+C$$

$$=-\frac{1}{18}\left(\frac{1}{t^2}\right)+C$$

$$=\frac{-1}{18\left(2+3x^3\right)^2}+C$$

**Question 14:**

$$\frac{1}{x(\log x)^m}, x>0, m\neq 1$$

**Answer 14:**

Put $\log x = t$

$$\therefore \frac{1}{x}dx = dt$$

$$\Rightarrow \int \frac{1}{x(\log x)^m}dx = \int \frac{dt}{(t)^m}$$

$$= \left(\frac{t^{-m+1}}{1-m}\right) + C$$

$$= \frac{(\log x)^{1-m}}{(1-m)} + C$$

**Question 15:**

$$\frac{x}{9-4x^2}$$

**Answer 15:**

$Let\ 9-4x^2 = t \quad \Rightarrow -8x = dt$

$$I = \int \frac{x}{9-4x^2}dx = \frac{-1}{8}\int \frac{1}{t}dt = \frac{-1}{8}log|t| + C = \frac{-1}{8}log|9-4x^2| + C$$

**Question 16:**

$$e^{2x+3}$$

**Answer 16:**

$Let\ 2x+3 = t \quad \Rightarrow 2dx = dt$

$$I = \int e^{2x+3}dx = \frac{1}{2}\int e^t dt = \frac{1}{2}(e^t) + C = \frac{1}{2}e^{(2x+3)} + C$$

**Question 17:**

$$\frac{x}{e^{x^2}}$$

**Answer 17:**

$Let\ x^2 = t \quad \Rightarrow 2x\,dx = dt$

$$I = \int \frac{x}{e^{x^2}}dx = \frac{1}{2}\int \frac{1}{e^t}dt = \frac{1}{2}\left(\frac{e^{-t}}{-1}\right) + C$$

$$= -\frac{1}{2}e^{-x^2} + C = \frac{-1}{2e^{x^2}} + C$$

**Question 18:**

$$\frac{e^{tan^{-1}x}}{1+x^2}$$

**Answer 18:**

$$Let\ tan^{-1}x = t \quad \Rightarrow \frac{1}{1+x^2}dx = dt$$

$$I = \int \frac{e^{tan^{-1}x}}{1+x^2}dx = \int e^t dt = e^t + C = e^{tan^{-1}x} + C$$

**Question 19:**

$$\frac{e^{2x}-1}{e^{2x}+1}$$

**Answer 19:**

$$\frac{e^{2x}-1}{e^{2x}+1}$$

Dividing numerator and denominator by $e^x$, we obtain

$$\frac{\frac{(e^{2x}-1)}{e^x}}{\frac{(e^{2x}+1)}{e^x}}=\frac{e^x-e^{-x}}{e^x+e^{-x}}$$

$Let\ \ e^x+e^{-x}=t \qquad \Rightarrow (e^x-e^{-x})dx=dt$

$$I=\int\frac{e^{2x}-1}{e^{2x}+1}dx=\int\frac{e^x-e^{-x}}{e^x+e^{-x}}dx=\int\frac{dt}{t}=log|t|+C=log|\ e^x+e^{-x}|+C$$

**Question 20:**

$$\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$$

**Answer 20:**

$Let\ e^{2x}+e^{-2x}=t \quad \Rightarrow (2e^{2x}-2e^{-2x})dx=dt \quad \Rightarrow\ 2(e^{2x}-e^{-2x})dx=dt$

$$I=\int\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}dx=\int\frac{dt}{2t}=\frac{1}{2}\int\frac{1}{t}dt=\frac{1}{2}log|t|+C=\frac{1}{2}log|e^{2x}+e^{-2x}|+C$$

**Question 21:**

$$tan^2(2x-3)$$

**Answer 21:**

$$tan^2(2x-3)\ =\ sec^2(2x-3)-1$$

$Let\ 2x-3=t \qquad \Rightarrow 2dx=dt$

$$I=\int tan^2(2x-3)dx=\int\ [sec^2(2x-3)-1]\,dx\ =\frac{1}{2}\int(sec^2t)dt\ -\int 1dx$$

$$=\frac{1}{2}\tan t-x+C=\frac{1}{2}\tan(2x-3)-x+C$$

**Question 22:**

$$sec^2(7-4x)$$

**Answer 22:**

$Let\ 7-4x=t\Rightarrow -4\ dx=dt$

$$I=\int sec^2(7-4x)dx=\frac{-1}{4}\int sec^2t\ dt$$

$$=\frac{-1}{4}(\tan t)+C$$

$$=\frac{-1}{4}\tan(7-4x)+C$$

**Question 23:**

$$\frac{sin^{-1}x}{\sqrt{1-x^2}}$$

**Answer 23:**

$Let \quad sin^{-1}x = t \qquad \Rightarrow \dfrac{1}{\sqrt{1-x^2}}dx = dt$

$$I = \int \frac{sin^{-1}x}{\sqrt{1-x^2}}dx = \int t\,dt \;= \frac{t^2}{2} + C \;= \frac{(sin^{-1}x)^2}{2} + C$$

**Question 24:**

$$\frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}$$

**Answer 24:**

$$\frac{2\cos x - 3\sin x}{6\cos x + 4\sin x} = \frac{2\cos x - 3\sin x}{2(3\cos x + 2\sin x)}$$

$Let \quad (3\cos x + 2\sin x) = t \qquad \Rightarrow (-3\sin x + 2\cos x)dx = dt$

$$I = \int \frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}dx = \int \frac{dt}{2t} = \frac{1}{2}\int \frac{1}{t}dt = \frac{1}{2}log|t| + C$$

$$= \frac{1}{2}log|3\cos x + 2\sin x)| + C$$

**Question 25:**

$$\frac{1}{cos^2x(1-\tan x)^2}$$

**Answer 25:**

$$\frac{1}{cos^2x(1-\tan x)^2} = \frac{sec^2x}{(1-\tan x)^2}$$

$Let \quad (1-\tan x) = t \Rightarrow -sec^2x\,dx = dt$

$$I = \int \frac{sec^2x}{(1-\tan x)^2}dx = \int \frac{-dt}{t^2} \;= -\int t^{-2}dt = \frac{1}{t} + C$$

$$= \frac{1}{1-\tan x} + C$$

**Question 26:**

$$\frac{\cos\sqrt{x}}{\sqrt{x}}$$

**Answer 26:**

$Let \;\; \sqrt{x} = t \quad \Rightarrow \dfrac{1}{2\sqrt{x}}dx = dt$

$$I = \int \frac{\cos\sqrt{x}}{\sqrt{x}}\;\mathrm{dx} = 2\int \cos t dt \;\; = 2\sin \mathrm{t} + \mathrm{C} \;= 2\sin\sqrt{x} + \mathrm{C}$$

**Question 27:**

$$\sqrt{\sin 2x}\ \cos 2x$$

**Answer 27:**

Let $\sin 2x = t \Rightarrow \cos 2x dx = dt$

$$I = \int \sqrt{sin2x} = \frac{1}{2}\int \sqrt{t}\ dt = \frac{1}{2}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C = \frac{1}{3}t^{\frac{3}{2}} + C = \frac{1}{3}(\sin 2x)^{\frac{3}{2}} + C$$

**Question 28:**

$$\frac{\cos x}{\sqrt{1+\sin x}}$$

**Answer 28:**

*Let* $1 + \sin x = t \Rightarrow \cos x\, dx = dt$

$$I = \int \frac{\cos x}{\sqrt{1+\sin x}} dx = \int \frac{dt}{\sqrt{t}} = \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C = 2\sqrt{t} + C = 2\sqrt{1+\sin x} + C$$

**Question 29:**

$$\cot x \log \sin x$$

**Answer 29:**

*Let* $\log \sin x = t \Rightarrow \frac{1}{\sin x}.\cos x\, dx = dt \Rightarrow \cot x\, dx = dt$

$$I = \int \cot x \log \sin x\, dx = \int t\, dt = \frac{t^2}{2} + C = \frac{(\log \sin x)^2}{2} + C$$

**Question 30:**

$$\frac{\sin x}{1+\cos x}$$

**Answer 30:**

*Let* $1 + \cos x = t \Rightarrow -\sin x dx = dt$

$$I = \int \frac{\sin x}{1+\cos x} dx = \int -\frac{dt}{t} = -log|t| + C = -log|1+\cos x| + C$$

**Question 31:**

$\frac{\sin x}{(1+\cos x)^2}$

**Answer 31:**

*Let* $1 + \cos x = t \Rightarrow -\sin x dx = dt$

$$I = \int \frac{\sin x}{(1+\cos x)^2} dx = \int -\frac{dt}{t^2} = -\int t^{-2} dt$$

$$= \frac{1}{t} + C = \frac{1}{1+\cos x} + C$$

**Question 32:**

$$\frac{1}{1+\cot x}$$

**Answer 32:**

$$I=\int \frac{1}{1+\cot x}dx \ =\int \frac{1}{1+\frac{\cos x}{\sin x}}dx \ =\int \frac{\sin x}{\sin x+\cos x}dx \ =\frac{1}{2}\int \frac{2\sin x}{\sin x+\cos x}dx$$

$$=\frac{1}{2}\int \frac{(\sin x+\cos x)+(\sin x-\cos x)}{\sin x+\cos x}dx=\frac{1}{2}\int 1\,dx+\frac{1}{2}\int \frac{(\sin x-\cos x)}{\sin x+\cos x}dx$$

$$=\frac{1}{2}(x)+\frac{1}{2}\int \frac{(\sin x-\cos x)}{\sin x+\cos x}dx$$

Let $\sin x+\cos x=t \qquad \Rightarrow (\cos x-\sin x)\,dx=dt$

$$I=\frac{x}{2}+\frac{1}{2}\int \frac{-dt}{t}=\frac{x}{2}-\frac{1}{2}\log|t|+C=\frac{x}{2}-\frac{1}{2}\log|\sin x+\cos x|+C$$

**Question 33:**

$$\frac{1}{1-\tan x}$$

**Answer 33:**

$$I=\int \frac{1}{1-\tan x}dx=\int \frac{1}{1-\frac{\sin x}{\cos x}}dx \ =\int \frac{\cos x}{\cos x-\sin x}dx \ =\frac{1}{2}\int \frac{2\ \cos x}{\cos x-sinx}dx$$

$$=\frac{1}{2}\int \frac{(\cos x-\sin x)+(\cos x+\sin x)}{(\cos x-\sin x)}dx$$

$$=\frac{1}{2}\int 1\,dx+\frac{1}{2}\int \frac{(\sin x+\cos x)}{(\cos x-sinx\ )}dx$$

$$=\frac{1}{2}(x)+\frac{1}{2}\int \frac{\cos x+sins\ x}{\cos x-\sin x}dx$$

Let $\cos x-\sin x\ =t \qquad \Rightarrow (-\sin x-\cos x)dx=dt$

$$I=\frac{x}{2}+\frac{1}{2}\int \frac{-dt}{t} \ =\frac{x}{2}-\frac{1}{2}\log|\cos x-\sin x|+C$$

**Question 34:**

$$\frac{\sqrt{tanx}}{\sin x\cos x}$$

**Answer 34:**

$$I=\int \frac{\sqrt{\tan x}}{\sin x\cos x}dx=\int \frac{\sqrt{\tan x}\ \times \cos x}{\sin x\cos x\times \cos x}dx=\int \frac{\sqrt{tanx}}{\tan x cos^2x}dx=\int \frac{sec^2x}{\sqrt{\tan x}}dx$$

$Let\ \tan x=t \quad \Rightarrow \quad sec^2xd=dt$

$$I=\int \frac{dt}{\sqrt{t}}=2\sqrt{t}+C=2\sqrt{\tan x}+C$$

**Question 35:**

$$\frac{(1+\log x)^2}{x}$$

**Answer 35:**

$Let\ (1+\log x)=t \Rightarrow \quad \frac{1}{x}dx=dt$

$$I=\int \frac{(1+\log x)^2}{x}dx=\int t^2 dt=\frac{t^3}{3}+C$$

$$=\frac{(1+\log x)^3}{3}+C$$

**Question 36:**

$$\frac{(x+1)(x+\log x)^2}{x}$$

**Answer 36:**

$$\frac{(x+1)(x+\log x)^2}{x}=\frac{(x+1)}{x}(x+\log x)^2=\left(1+\frac{1}{x}\right)(x+\log x)^2$$

$Let\ (x+\log x)=t \quad i.e.\ \left(1+\frac{1}{x}\right)dx=dt$

$$I=\int\left(1+\frac{1}{x}\right)(x+\log x)^2 dx=\int t^2 dt$$

$$=\frac{t^3}{3}+C=\frac{(x+\log x)^3}{3}+C$$

**Question 37:**

$$\frac{x^3 sin(tan^{-1}x^4)}{1+x^8}$$

**Answer 37:**

$Let\ x^4=t$

$\Rightarrow \quad 4x^3dx=dt$

$$I=\int \frac{x^3 sin(tan^{-1}x^4)}{1+x^8}dx=\frac{1}{4}\int \frac{sin(tan^{-1}t)}{1+t^2} \quad \ldots (1)$$

$Let\ \ tan^{-1}t=u$

$$\Rightarrow \frac{1}{1+t^2}dt=du$$

From (1), we obtain

$$I=\int \frac{x^3 sin(tan^{-1}x^4)}{1+x^8}dx=\frac{1}{4}\int \sin u\, du\ =\frac{1}{4}(-\cos u)+C$$

$$=\frac{-1}{4}cos(tan^{-1}t)+C\ =\frac{-1}{4}cos(tan^{-1}x^4)+C$$

Choose the correct answer in Exercises 38 and 39.

**Question 38:**

$$\int \frac{10x^9 + 10^x log_e 10}{x^{10} + 10^x} dx \quad \text{equals}$$

(A) $10^x - x^{10} + C$ (B) $10^x + x^{10} + C$

(C) $(10^x - x^{10})^{-1} + C$ (D) $log(10^x + x^{10}) + C$

**Answer 38:**

$Let\ \ x^{10} + 10^x = t$

$\Rightarrow (10x^9 + 10^x log_e 10)dx = dt$

$$I = \int \frac{10x^9 + 10^x log_e 10}{x^{10} + 10^x} dx \ = \int \frac{dt}{t} = \log t + C$$

$= \log(10^x + x^{10}) + C$

Hence, the correct answer is (D).

**Question 39:**

$$\int \frac{dx}{sin^2 x\, cos^2 x} \quad \text{equals}$$

(A) $\tan x + \cot x + C$ (B) $\tan x - \cot x + C$

(C) $\tan x \cot x + C$ (D) $\tan x \cot 2x + C$

**Answer 39:**

$$I = \int \frac{dx}{sin^2 x\, cos^2 x}$$

$$= \int \frac{1}{sin^2 x\, cos^2 x} dx$$

$$= \int \frac{sin^2 x + cos^2 x}{sin^2 x\, cos^2 x} dx$$

$$= \int \frac{sin^2 x}{sin^2 x\, cos^2 x} dx + \int \frac{cos^2 x}{sin^2 x\, cos^2 x} dx$$

$$= \int sec^2 x\, dx + \int cosec^2 x\, dx$$

$= \tan x - \cot x + C$

Hence, the correct answer is (B).

# Mathematics

**(Chapter – 7) (Integrals) (Exercise 7.3)**
**(Class – XII)**

Find the integrals of the functions in Exercises 1 to 22:

**Question 1:**

$$sin^2(2x+5)$$

**Answer 1:**

$$sin^2(2x+5) = \frac{1-cos2(2x+5)}{2} = \frac{1-cos(4x+10)}{2}dx$$

$$I = \int sin^2(2x+5)dx = \int \frac{1-cos(4x+10)}{2}dx = \frac{1}{2}\int 1\,dx - \frac{1}{2}\int cos(4x+10)dx$$

$$= \frac{1}{2}x - \frac{1}{2}\left(\frac{\sin(4x+10)}{4}\right) + C = \frac{1}{2}x - \frac{1}{8}\sin(4x+10) + C$$

**Question 2:**

$$\sin 3x\,.\cos 4x$$

**Answer 2:**

It is known that, $\sin A\cos B = \frac{1}{2}\{\sin(A+B)+\sin(A-B)\}$

$$I = \int \sin 3x\,.\cos 4x\,dx = \frac{1}{2}\int \{\sin(3x+4x)+\sin(3x-4x)\}dx$$

$$= \frac{1}{2}\{\sin 7x + \sin(-x)\}dx = \frac{1}{2}\{\sin 7x - \sin x\}dx = \frac{1}{2}\int \sin 7x\,dx - \frac{1}{2}\int \sin x\,dx$$

$$= \frac{1}{2}\left(\frac{-\cos 7x}{7}\right) - 1\frac{1}{2}(-\cos x) + C$$

$$= -\frac{\cos 7x}{14} + \frac{\cos x}{2} + C$$

**Question 3:**

$$\cos 2x\ \ \cos 4x\cos 6x$$

**Answer 3:**

It is known that, $\cos A\cos B = \frac{1}{2}\{\cos(A+B)+\cos(A-B)\}$

$$I = \int \cos 2x\ (\cos 4x\cos 6x)dx = \int \cos 2x\left[\frac{1}{2}\{\cos 2x(4x+6x)+\cos(4x-6x)\}\right]dx$$

$$= \frac{1}{2}\int\{\cos 2x cos\ 10x + \cos 2x\cos(-2x)\}dx$$

$$= \frac{1}{2}\int\{\cos 2x cos\ 10x + cos^2 2x\}dx$$

$$= \frac{1}{2}\int\left[\frac{1}{2}\{\cos(2x+10x)+\cos(2x-10x)\} + \left\{\frac{1+\cos 4x}{2}\right\}\right]dx$$

$$= \frac{1}{4}\int(\cos 12x + \cos 8x + 1 + \cos 4x)\,dx$$

$$= \frac{1}{4}\left[\frac{\sin 12x}{12} + \frac{\sin 8x}{8} + x + \frac{\sin 4x}{4}\right] + C$$

**Question 4:**

$$sin^3(2x+1)$$

**Answer 4:**

$$I=\int sin^3(2x+1)\ dx=\int sin^3(2x+1)\,dx=\int sin^2(2x+1).\sin(2x+1)dx$$

$$=\int \{1-cos^2(2x+1)\}.\sin(2x+1)dx$$

Let $\cos(2x+1)=t \quad \Rightarrow -2\sin(2x+1)dx=dt \quad \Rightarrow \sin(2x+1)dx=\frac{-dt}{2}$

$$I=\frac{-1}{2}\int(1-t^2)dt=\frac{-1}{2}\left\{t-\frac{t^3}{3}\right\}+C=\frac{-1}{2}\left\{\cos(2x+1)-\frac{\cos(2x+1)^3}{3}\right\}+C$$

$$=-\frac{\cos(2x+1)}{2}+\frac{\cos(2x+1)^3}{3}+C$$

**Question 5:**

$$sin^3xcos^3x$$

**Answer 5:**

$$Let\ I=\int sin^3xcos^3x\,dx=\int cos^3x.sin^2x\ .\sin x\,x\,dx=\int cos^3x(1-cos^2x).\sin x\,dx$$

$Let\ \cos x=t \quad \Rightarrow -\sin x\,.dx=dt$

$$I=-\int t^3(1-t^2)dt=-\int(t^3-t^5)dt=-\left\{\frac{t^4}{4}-\frac{t^6}{6}\right\}+C$$

$$=-\left\{\frac{cos^4x}{4}-\frac{cos^6x}{6}\right\}+C=\frac{cos^4x}{4}-\frac{cos^6x}{6}+C$$

**Question 6:**

$$\sin x\sin 2x\sin 3x$$

**Answer 6:**

It is known that $\sin A\sin B=\frac{1}{2}\{\cos(A-B)-\cos(A+B)\}$

$$I=\int \sin x\sin 2x\sin 3x\ dx=\int\left[\sin x.\frac{1}{2}\{\cos(2x-3x)-\cos(2x+3x)\}\right]dx$$

$$=\frac{1}{2}\int\{\sin x\cos(-x)-\sin x\cos 5x\}\,dx=\frac{1}{2}\int\{\sin x\cos\ x-\sin x\cos 5x\}\,dx$$

$$=\frac{1}{2}\int\frac{\sin 2x}{2}dx-\frac{1}{2}\int\sin x\cos 5x\ dx=\frac{1}{4}\left[\frac{-\cos 2x}{2}\right]-\frac{1}{2}\int\left\{\frac{1}{2}\sin(x+5x)+\sin(x-5x)\right\}dx$$

$$=\frac{-\cos 2x}{8}-\frac{1}{4}\int\{\sin 6x+\sin(-4x)\}dx=\frac{-\cos 2x}{8}-\frac{1}{4}\left[\frac{-\cos 6x}{6}+\frac{\cos 4x}{4}\right]+C$$

$$=\frac{-\cos 2x}{8}-\frac{1}{8}\left[\frac{-\cos 6x}{3}+\frac{\cos 4x}{2}\right]+C$$

$$=\frac{1}{8}\left[\frac{\cos 6x}{3}-\frac{\cos 4x}{2}-\cos 2x\right]+C$$

**Question 7:**

$$\sin 4x \sin 8x$$

**Answer 7:**

$$\text{It is known that} \quad \sin A \sin B = \frac{1}{2}\{\cos(A-B) - \cos(A+B)\}$$

$$I = \int \sin 4x \sin 8x\, dx = \int \frac{1}{2}\{\cos(4x-8x) - \cos(4x+8x)\}\, dx$$

$$= \frac{1}{2}\int [(\cos(-4x) - \cos 12x)]\, dx = \frac{1}{2}\int (\cos 4x - \cos 12x)dx = \frac{1}{2}\left[\frac{\sin 4x}{4} - \frac{\sin 12x}{12}\right] + C$$

**Question 8:**

$$\frac{1-\cos x}{1+\cos x}$$

**Answer 8:**

$$\frac{1-\cos x}{1+\cos x} = \frac{2sin^2\frac{x}{2}}{2cos^2\frac{x}{2}} = tan^2\frac{x}{2} = sec^2\frac{x}{2} - 1$$

$$I = \int \frac{1-\cos x}{1+\cos x}dx = \int \left\{sec^2\frac{x}{2} - 1\right\}dx = \left[\frac{tan\frac{x}{2}}{\frac{1}{2}} - 1\right] + C = 2\,tan\frac{x}{2} + C$$

**Question 9:**

$$\frac{\cos x}{1+\cos x}$$

**Answer 9:**

$$\frac{\cos x}{1+\cos x} = \frac{cos^2\frac{x}{2} - sin^2\frac{x}{2}}{2cos^2\frac{x}{2}} = \frac{1}{2}\left[1 - tan^2\frac{x}{2}\right]$$

$$I = \int \frac{\cos x}{1+\cos x}dx = \frac{1}{2}\int \left(1 - tan^2\frac{x}{2}\right)dx = \frac{1}{2}\left(1 - sec^2\frac{x}{2} + 1\right)dx$$

$$= \frac{1}{2}\left(2 - sec^2\frac{x}{2}\right)dx = \frac{1}{2}\left[2x - \frac{tan\frac{x}{2}}{\frac{1}{2}}\right] + C = x - tan\frac{x}{2} + C$$

**Question 10:**

$$sin^4x$$

**Answer 10:**

$$sin^4x = sin^2x.sin^2x = \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-\cos 2x}{2}\right) = \frac{1}{4}(1-\cos 2x)^2 = \frac{1}{4}\left[1 + cos^2 2x - 2\cos 2x\right]$$

$$= \frac{1}{4}\left[1 + \left(\frac{1+\cos 4x}{2}\right) - 2\cos 2x\right] = \frac{1}{4}\left[1 + \frac{1}{2} + \frac{1}{2}\cos 4x - 2\cos 2x\right] = \frac{1}{4}\left[\frac{3}{2} + \frac{1}{2}\cos 4x - 2\cos 2x\right]$$

$$I = \int sin^4x dx = \frac{1}{4}\int \left[\frac{3}{2} + \frac{1}{2}\cos 4x - 2\cos 2x\right]dx = \frac{1}{4}\left[\frac{3}{2} + \frac{1}{2}\left(\frac{\sin 4x}{4}\right) - 2\frac{2\sin 2x}{2}\right] + C$$

$$= \frac{1}{8}\left[3x + \frac{\sin 4x}{4} - 2\sin 2x\right] + C = \frac{3x}{8} - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$$

**Question 11:**

$$cos^4 2x$$

**Answer 11:**

$$cos^4 2x = \quad (cos^2 2x)^2 = \left(\frac{1+\cos 4x}{2}\right)^2 = \frac{1}{4}[1 + cos^2 4x + 2\cos 4x]$$

$$= \frac{1}{4}\left[1 + \left(\frac{1+\cos 8x}{2}\right) + 2\cos 4x\right] = \frac{1}{4}\left[1 + \frac{1}{2} + \frac{1}{2}\cos 8x + 2\cos 4x\right] = \frac{1}{4}\left[\frac{3}{2} + \frac{1}{2}\cos 8x + 2\cos 4x\right]$$

$$I = \int cos^4 2x dx = \int \left[\frac{3}{8} + \frac{\cos 8x}{8} + \frac{\cos 4x}{2}\right] dx = \frac{3}{8}x + \frac{\sin 8x}{64} + \frac{\sin 4x}{8} + C$$

**Question 12:**

$$\frac{sin^2 x}{1+\cos x}$$

**Answer 12:**

$$\frac{sin^2 x}{1+\cos x} = \frac{\left(2sin\frac{x}{2}.cos\frac{x}{2}\right)^2}{2cos^2\frac{x}{2}} = \frac{4sin^2\frac{x}{2}.cos^2\frac{x}{2}}{2cos^2\frac{x}{2}} = 2sin^2\frac{x}{2} = 1 - \cos x$$

$$I = \int \frac{sin^2 x}{1+\cos x} dx = \int (1-\cos x) dx = x - \sin x + C$$

**Question 13:**

$$\frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}$$

**Answer 13:**

$$\frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} = \frac{-2\sin\frac{2x+2\alpha}{2} sin\frac{2x-2\alpha}{2}}{-2\sin\frac{x+\alpha}{2} sin\frac{x-\alpha}{2}} \qquad \left[\cos C - \cos D = -2sin\frac{C+D}{2} sin\frac{C-D}{2}\right]$$

$$= \frac{\sin(x+\alpha)\sin(x-\alpha)}{\sin\left(\frac{x+\alpha}{2}\right).\sin\left(\frac{x-\alpha}{2}\right)}$$

$$= \frac{\left[2\sin\left(\frac{x+\alpha}{2}\right).\cos\left(\frac{x+\alpha}{2}\right)\right]\left[2\sin\left(\frac{x-\alpha}{2}\right).\cos\left(\frac{x-\alpha}{2}\right)\right]}{\sin\left(\frac{x+\alpha}{2}\right).\sin\left(\frac{x-\alpha}{2}\right)}$$

$$= 4\cos\left(\frac{x+\alpha}{2}\right) cos\left(\frac{x-\alpha}{2}\right)$$

$$= 2\left[\cos\left(\frac{x+\alpha}{2} + \frac{x-\alpha}{2}\right) + \cos\left(\frac{x+\alpha}{2} - \frac{x-\alpha}{2}\right)\right]$$

$$= 2[\cos(x) + \cos\alpha] = 2\cos x + 2\cos\alpha$$

$$I = \int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha} dx = \int (2\cos x + 2\cos\alpha) dx$$

$$= 2\ [\sin x + x\cos\alpha] + C$$

## Question 14:

$$\frac{\cos x - \sin x}{1 + \sin 2x}$$

### Answer 14:

$$\frac{\cos x - \sin x}{1 + \sin 2x} = \frac{\cos x - \sin x}{(sin^2 x + cos^2 x) + 2 \sin x \cos x} = \frac{\cos x - \sin x}{(\sin x + \cos x)^2}$$

[since $sin^2 x + cos^2 x = 1$ and $\sin 2x = 2 \sin x \cos x$]

$$I = \int \frac{\cos x - \sin x}{1 + \sin 2x} dx = \int \frac{\cos x - \sin x}{(\sin x + \cos x)^2} dx$$

$Let\ \sin x + \cos x = t \quad \Rightarrow (\cos x - \sin x)\, dx = dt$

$$I = \int \frac{dt}{t^2} = \int t^{-2} dt = -t^{-1} + C = -\frac{1}{t} + C = -\frac{1}{\sin x + \cos x} + C$$

## Question 15:

$$tan^3 2x . \sec 2x$$

### Answer 15:

$tan^3 2x \sec 2x = tan^2 2x \tan 2x \sec 2x = (sec^2 2x - 1) \tan 2x \sec 2x$

$= sec^2 2x . \tan 2x \sec 2x - \tan 2x \sec 2x$

$$I = \int tan^3 2x . \sec 2x\, dx = \int tan^2 2x \tan 2x \sec 2x\, dx - \int \tan 2x \sec 2x\, dx$$

$$= \int tan^2 2x \tan 2x \sec 2x\, dx - \frac{\sec 2x}{2} + C$$

Let $\sec 2x = t \quad \Rightarrow 2 \sec 2x tan\, 2x\, dx = dt$

$$I = \int tan^3 2x . \sec 2x\, dx = \frac{1}{2} \int t^2 dt - \frac{\sec 2x}{2} + C = \frac{t^3}{6} - \frac{\sec 2x}{2} + C = \frac{(\sec 2x)^3}{6} - \frac{\sec 2x}{2} + C$$

## Question 16:

$$tan^4 x$$

### Answer 16:

$tan^4 x = tan^2 x\, tan^2 x = (sec^2 x - 1)\, tan^2 x = sec^2 x . tan^2 x - tan^2 x$

$= sec^2 x . tan^2 x - (sec^2 x - 1) = sec^2 x . tan^2 x - sec^2 x + 1$

$$I = \int tan^4 x dx = \int sec^2 x . tan^2 x\, dx - \int sec^2 x\, dx + \int 1\, dx$$

$$= \int sec^2 x . tan^2 x\, dx - \tan x + x + C \qquad ...(1)$$

Let $I_1 = \int sec^2 x . tan^2 x\, dx$

Let $\tan x = t \quad \Rightarrow \quad sec^2 x\, dx = dt$

$$I_1 = \int sec^2 x . tan^2 x\, dx = \int t^2 dt = \frac{t^3}{3} = \frac{tan^3 x}{3}$$

From equation (1), we obtain

$$I = \int tan^4 x\, dx = \frac{1}{3} tan^3 x - \tan x + x + C$$

**Question 17:**

$$\frac{\sin^3 x+\cos^3 x}{\sin^2 x\cos^2 x}$$

**Answer 17:**

$$\frac{sin^3x+cos^3x}{sin^2x.cos^2x}=\frac{sin^3x}{sin^2x.cos^2x}+\frac{cos^3x}{sin^2x.cos^2x}=\frac{\sin x}{cos^2x}+\frac{\cos x}{sin^2x}=\tan x\sec x+\cot x\ cosec\ x$$

$$I=\int\frac{sin^3x+cos^3x}{sin^2x.cos^2x}dx=\int(\tan x\sec x+\cot x\ cosec\ x)dx\ =\sec x-cosec\ x+C$$

**Question 18:**

$$\frac{\cos 2x+2sin^2x}{cos^2x}$$

**Answer 18:**

$$\frac{\cos 2x+2sin^2x}{cos^2x}=\frac{\cos 2x+(1-\cos 2x)}{cos^2x}=\frac{1}{cos^2x}=sec^2x$$

$$I=\int\frac{\cos 2x+2sin^2x}{cos^2x}dx=\int sec^2x\,dx=\tan x+C$$

**Question 19:**

$$\frac{1}{\sin x\ cos^3x}$$

**Answer 19:**

$$\frac{1}{\sin x\ cos^3x}=\frac{sin^2x+\ cos^2x}{\sin x\ cos^3x}=\frac{\sin x}{\cos^3 x}+\frac{1}{\sin x\ cos\ x}\ =\tan x\ sec^2x+\frac{cos^2x}{\left(\frac{\sin x\ \cos x}{cos^2x}\right)}$$

$$=\tan x\ sec^2x+\ \frac{sec^2x}{\tan x}$$

$$I=\int\frac{1}{\sin x\ cos^3x}dx=\int\tan x\ sec^2x\,dx+\int\frac{sec^2x}{\tan x}dx$$

Let $\tan x=t \quad \Rightarrow \quad sec^2x\,dx=dt$

$$I=\int\frac{1}{\sin x\ cos^3x}dx=\int t\,dt+\int\frac{1}{t}dt\ =\frac{t^2}{2}+\log|t|+C=\frac{1}{2}tan^2x+\log|\tan x|+C$$

**Question 20:**

$$\frac{\cos 2x}{(\cos x+\sin x)^2}$$

**Answer 20:**

$$\frac{\cos 2x}{(\cos x+\sin x)^2}=\frac{\cos 2x}{cos^2x+\ sin^2x+2\sin x cos\ x}=\frac{\cos 2x}{1+\sin 2x}$$

$$I=\int\frac{\cos 2x}{(\cos x+\sin x)^2}dx\ \ =\ \ \int\frac{\cos 2x}{1+\sin 2x}dx$$

Let $(1+\sin 2x)=t \quad \Rightarrow \quad 2\cos 2x\,dx\,dt$

$$I=\int \frac{\cos 2x}{(\cos x+\sin x)^2}dx=\frac{1}{2}\int \frac{1}{t}dt\ =\frac{1}{2}\log|t|+C$$

$$=\frac{1}{2}\log|1+\sin 2x|+C\ =\frac{1}{2}\log|\ (\cos x+\sin x)^2|+C=\log|\cos x+\sin x|+C$$

**Question 21:**

$$sin^{-1}(\cos x)$$

**Answer 21:**

$$I=\int sin^{-1}(\cos x)\,dx=\int sin^{-1}(\sin\left\{\frac{\pi}{2}-x\right\})\,dx=\int\left\{\frac{\pi}{2}-x\right\}dx=\frac{\pi}{2}x-\frac{x^2}{2}+C$$

**Question 22:**

$$\frac{1}{\cos(x-a)\cos(x-b)}$$

**Answer 22:**

$$\frac{1}{\cos(x-a)\cos(x-b)}=\frac{1}{\sin(x-a)}\ \left[\frac{\sin(a-b)}{\cos(x-a)\cos(x-b)}\right]$$

$$=\frac{1}{\sin(x-a)}\left[\frac{\sin\{\ (x-b)-\ (x-a)\}}{\cos(x-a)\cos(x-b)}\right]$$

$$=\frac{1}{\sin(x-a)}\left[\frac{\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)}{\cos(x-a)\cos(x-b)}\right]$$

$$=\frac{1}{\sin(x-a)}\ \ [\tan(x-b)-\tan(x-a)]$$

$$I=\int\frac{1}{\cos(x-a)\cos(x-b)}dx=\frac{1}{\sin(x-a)}\int\ [\tan(x-b)-\tan(x-a)]dx$$

$$=\frac{1}{\sin(x-a)}[-log|\cos(x-b)|+\ log|\cos(x-a)|]=\frac{1}{\sin(x-a)}\left[log\left|\frac{\cos(x-a)}{\cos(x-b)}\right|\right]+C$$

Choose the correct answer in Exercises 23 and 24.

**Question 23:**

$$\int\frac{sin^2x-cos^2x}{sin^2cos^2x}dx\quad \text{is equal to}$$

(A) $\tan x+\cot x+C$
(B) $\tan x+\operatorname{cosec} x+C$
(C) $-\ \tan x+\cot x+C$
(D) $\tan x+\sec x+C$

**Answer 23:**

$$I=\int\frac{sin^2x-cos^2x}{sin^2cos^2x}dx$$

$$=\int\left(\frac{sin^2x}{sin^2cos^2x}-\frac{cos^2x}{sin^2cos^2x}\right)dx\ =\ \int(sec^2x-cosec^2x)dx\ =\tan x+\cot x+C$$

Hence, the correct answer is (A).

**Question 24:**

$$\int \frac{e^x(1+x)}{\cos^2(e^x x)}dx \text{ equals}$$

$(A)\ -\cot(ex^x)+C$ $\quad$ $(B)\ \tan(xe^x)+C$

$(C)\ \tan(e^x)+C$ $\quad$ $(D)\ \cot(e^x)+C$

**Answer 24:**

$$\int \frac{e^x(1+x)}{\cos^2(e^x x)}dx$$

Put, $e^x x = t$

$\Rightarrow (e^x x + e^x.1)dx = dt$

$e^x(x+1)dx = dt$

$\therefore \int \frac{e^x(1+x)}{\cos^2(e^x x)}dx = \int \frac{dt}{\cos^2 t}$

$= \int \sec^2 t dt$

$= \tan t + C$

$= \tan(e^x x) + C$

Thus, the correct answer is B.

# Mathematics

**(Chapter – 7) (Integrals) (Exercise 7.4)**
**(Class – XII)**

Integrate the functions in Exercises 1 to 23.

**Question 1:**

$$\frac{3x^2}{x^6+1}$$

**Answer 1:**

Let $x^3 = t$

$\Rightarrow \; 3x^2 dx = dt$

$$I = \int \frac{3x^2}{x^6+1}dx = \int \frac{dt}{t^2+1} = tan^{-1}t + C = tan^{-1}(x^3) + C$$

**Question2:**

$$\frac{1}{\sqrt{1+4x^2}}$$

**Answer 2:**

Let $2x = t \quad \Rightarrow 2dx = dt$

$$I = \int \frac{1}{\sqrt{1+4x^2}}dx = \frac{1}{2}\int \frac{1}{\sqrt{1+t^2}}dt = \frac{1}{2}\left[\log\left|t+\sqrt{t^2+1}\right|\right] + C$$

$$= \frac{1}{2}\left[\log\left|2x+\sqrt{4x^2+1}\right|\right] + C \qquad \left[\text{Since} \int \frac{1}{\sqrt{x^2+a^2}}dt = \log\left|x+\sqrt{x^2+a^2}\right|\right]$$

**Question 3:**

$$\frac{1}{\sqrt{(2-x)^2+1}}$$

**Answer 3:**

Let $2 - x = t \quad \Rightarrow -dx = dt$

$$I = \int \frac{1}{\sqrt{(2-x)^2+1}}dx = -\int \frac{1}{\sqrt{t^2+1}}dt = -log\left|t+\sqrt{t^2+1}\right| + C$$

$$= -\log\left|2-x+\sqrt{(2-x)^2+1}\right| + C = \log\left|\frac{1}{(2-x)+\sqrt{x^2-4x+5}}\right| + C$$

**Question 4:**

$$\frac{1}{\sqrt{9-25x^2}}$$

**Answer 4:**

Let $5x = t$

$\Rightarrow 5dx = dt$

$$I = \int \frac{1}{\sqrt{9-25x^2}}dx = \frac{1}{5}\int \frac{1}{\sqrt{9-t^2}}dt = \frac{1}{5}\int \frac{1}{\sqrt{3^2-t^2}}dt$$

$$= \frac{1}{5}sin^{-1}\left(\frac{t}{3}\right) + C = \frac{1}{5}sin^{-1}\left(\frac{5x}{3}\right) + C$$

**Question 5:**

$$\frac{3x}{1+2x^4}$$

**Answer 5:**

Let $\sqrt{2}\,x^2 = t \quad \Rightarrow \quad 2\sqrt{2}\,xdx = dt$

$$I = \int \frac{3x}{1+2x^4}dx = \frac{2}{2\sqrt{2}}\int \frac{dt}{1+t^2} = \frac{2}{2\sqrt{2}}[tan^{-1}t] + C = \frac{2}{2\sqrt{2}}\left[tan^{-1}\left(\sqrt{2}\,x^2\right)\right] + C$$

**Question 6:**

$$\frac{x^2}{1-x^6}$$

**Answer 6:**

Let $x^3 = t \quad \Rightarrow 3t^2dx = dt$

$$I = \int \frac{x^2}{1-x^6}dx = \frac{1}{3}\int \frac{dt}{1-t^2} = \frac{1}{3}\left[\frac{1}{2}\log\left|\frac{1+t}{1-t}\right|\right] + C = \frac{1}{6}\log\left|\frac{1+x^3}{1-x^3}\right| + C$$

**Question 7:**

$$\frac{x-1}{\sqrt{x^2-1}}$$

**Answer 7:**

$$I = \int \frac{x-1}{\sqrt{x^2-1}}dx = \int \frac{x}{\sqrt{x^2-1}}dx - \int \frac{1}{\sqrt{x^2-1}}dx \qquad \dots(1)$$

For $\int \frac{x}{\sqrt{x^2-1}}dx$, let $x^2 - 1 = t$

$\Rightarrow \quad 2xdx = dt$

$$\therefore \int \frac{x}{\sqrt{x^2-1}}dx = \frac{1}{2}\int \frac{dt}{\sqrt{t}} = \frac{1}{2}\int t^{-\frac{1}{2}}dt = \frac{1}{2}\left[2t^{\frac{1}{2}}\right] = \sqrt{t} = \sqrt{x^2-1}$$

From equation (1), we obtain

$$I = \int \frac{x-1}{\sqrt{x^2-1}}dx = \int \frac{x}{\sqrt{x^2-1}}dx - \int \frac{1}{\sqrt{x^2-1}}dx = \sqrt{x^2-1} - \log\left|x + \sqrt{x^2-1}\right| + C$$

**Question 8:**

$$\frac{x^2}{\sqrt{x^6+a^6}}$$

**Answer 8:**

Let $x^3 = t$

$\Rightarrow 3x^2dx = dt$

$$I = \int \frac{x^2}{\sqrt{x^6+a^6}}dx = \frac{1}{3}\int \frac{dt}{\sqrt{t^2+(a^3)^2}}$$

$$= \frac{1}{3}\log\left|t + \sqrt{t^2+(a^3)^2}\right| + C = \frac{1}{3}\log\left|x^3 + \sqrt{x^6+a^6}\right| + C$$

**Question 9:**

$$\frac{sec^2x}{\sqrt{tan^2x+4}}$$

**Answer 9:**

Let $\tan x = t \quad \Rightarrow \quad sec^2xdx = dt$

$$I = \int \frac{sec^2x}{\sqrt{tan^2x+4}}dx = \int \frac{dt}{\sqrt{t^2+2^2}} = \log\left|t+\sqrt{t^2+2^2}\right| + C = \log\left|tan+\sqrt{tan^2x+4}\right| + C$$

**Question 10:**

$$\frac{1}{\sqrt{x^2+2x+2}}$$

**Answer 10:**

$$\int \frac{1}{\sqrt{x^2+2x+2}}dx = \int \frac{1}{\sqrt{(x+1)^2+(1)^2}}dx$$

Let $x+1=t$

$\therefore dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{x^2+2x+2}}dx = \int \frac{1}{\sqrt{t^2+1}}dt$$

$$= \log\left|t+\sqrt{t^2+1}\right|C$$

$$= \log\left|(x+1)+\sqrt{(x+1)^2+1}\right|+C$$

$$= \log\left|(x+1)+\sqrt{x^2+2x+2}\right|+C$$

**Question 11:**

$$\frac{1}{9x^2+6x+5}$$

**Answer 11:**

$$I = \int \frac{1}{9x^2+6x+5}dx = \int \frac{1}{(3x+1)^2+2^2}$$

Let $(3x+1) = t \quad \Rightarrow \quad 3dx = dt$

$$I = \int \frac{1}{(3x+1)^2+2^2}dx = \frac{1}{3}\int \frac{1}{t^2+2^2}dt = \frac{1}{3}\left[\frac{1}{2}tan^{-1}\left(\frac{t}{2}\right)\right] + C = \frac{1}{6}tan^{-1}\left(\frac{t}{2}\right) + C$$

**Question 12:**

$$\frac{1}{\sqrt{7-6x-x^2}}$$

**Answer 12:**

Here, $\quad 7-6x-x^2 = 7-(x^2+6x+9-9) = 7-(x^2+6x+9-9)$

$= 16-(x^2+6x+9) = 16-(x+3)^2 = (4)^2-(x+3)^2$

$$\therefore \mathrm{I} = \int \frac{1}{\sqrt{7-6x-x^2}}dx = \int \frac{1}{\sqrt{(4)^2-(x+3)^2}}dx$$

Let $x+3=t$

$\Rightarrow \quad dx = dt$

$$I = \int \frac{1}{\sqrt{(4)^2-(x+3)^2}}dx = \int \frac{1}{\sqrt{(4)^2-(t)^2}}dt = sin^{-1}\left(\frac{t}{4}\right) + C = sin^{-1}\left(\frac{x+3}{4}\right) + C$$

**Question 13:**

$$\frac{1}{\sqrt{(x-1)(x-2)}}$$

**Answer 13:**

$(x-1)(x-2)$ can be written as $x^2-3x+2$

Therefore $x^2-3x+2=x^2-3x+\frac{9}{4}-\frac{9}{4}+2$

$=\left(x-\frac{3}{2}\right)^2-\frac{1}{4}=\left(x-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2$

$I=\int\frac{1}{\sqrt{\left(x-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}}dx$

Let $x-\frac{3}{2}=t$

$\Rightarrow \quad dx=dt$

$=\int\frac{1}{\sqrt{(t)^2-\left(\frac{1}{2}\right)^2}}dt=\log\left|t+\sqrt{(t)^2-\left(\frac{1}{2}\right)^2}\right|+C=\log\left|\left(x-\frac{3}{2}\right)+\sqrt{x^2-3x+2}\right|+C$

**Question 14:**

$$\frac{1}{\sqrt{8+3x-x^2}}$$

**Answer 14:**

$8+3x-x^2$ can be written as $8-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)$

Therefore $8-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)=\frac{41}{4}-\left(x-\frac{3}{2}\right)^2$

$I=\int\frac{1}{\sqrt{8+3x-x^2}}dx=\int\frac{1}{\sqrt{\frac{41}{4}-\left(x-\frac{3}{2}\right)^2}}dx$

Let $x-\frac{3}{2}=t \quad \Rightarrow \quad dx=dt$

$\Rightarrow \quad I=\int\frac{1}{\sqrt{\frac{41}{4}-\left(x-\frac{3}{2}\right)^2}}dx=\int\frac{1}{\sqrt{\left(\frac{\sqrt{41}}{2}\right)^2-t^2}}dt$

$=sin^{-1}\left(\frac{t}{\frac{\sqrt{41}}{2}}\right)+C=sin^{-1}\left(\frac{x-\frac{3}{2}}{\frac{\sqrt{41}}{2}}\right)+C=sin^{-1}\left(\frac{2x-3}{\sqrt{41}}\right)+C$

## Question 15:

$$\frac{1}{\sqrt{(x-a)(x-b)}}$$

## Answer 15:

$(x-a)(x-b)$ can be written as $x^2-(x-b)x+ab$

Therefore $x^2-(x-b)x+ab = x^2-(x-b)x+\frac{(a+b)^2}{4}-\frac{(a+b)^2}{4}+ab$

$$=\left[x-\left(\frac{a+b}{2}\right)\right]^2-\frac{(a+b)^2}{4}$$

$$I=\int\frac{1}{\sqrt{(x-a)(x-b)}}dx=\int\frac{1}{\sqrt{\left[x-\left(\frac{a+b}{2}\right)\right]^2-\frac{(a+b)^2}{4}}}dx$$

Let $x-\left(\frac{a+b}{2}\right)=t \Rightarrow dx=dt$

$$I=\int\frac{1}{\sqrt{\left[x-\left(\frac{a+b}{2}\right)\right]^2-\frac{(a+b)^2}{4}}}dx=\int\frac{1}{\sqrt{t^2-\left(\frac{a+b}{2}\right)^2}}dt$$

$$=\log\left|t+\sqrt{t^2-\left(\frac{a+b}{2}\right)^2}\right|+C=\log\left|\left\{x-\left(\frac{a+b}{2}\right)\right\}+\sqrt{(x-a)(x-b)}\right|+C$$

## Question 16:

$$\frac{4x+1}{\sqrt{2x^2+x-3}}$$

## Answer 16:

Let $4x+1 = A\frac{d}{dx}(2x^2+x-3)+B \quad \Rightarrow 4x+1 = A(4x+1)+B \quad \Rightarrow 4x+1 = 4Ax+A+B$

Equating the coefficients of $x$ and constant term on both sides, we obtain

4A = 4 ⇒ A = 1 and A + B = 1 ⇒ B = 0

Let $2x^2+x-3=t \quad \Rightarrow (4x+1)dx=dt$

$$I=\int\frac{4x+1}{\sqrt{2x^2+x-3}}dx=\int\frac{1}{\sqrt{t}}dt = 2\sqrt{t}+C=2\sqrt{2x^2+x-3}+C$$

## Question 17:

$$\frac{x+2}{\sqrt{x^2-1}}$$

## Answer 17:

Let $x+2=A\frac{d}{dx}(x^2-1)+B \qquad \ldots(1)$

$\Rightarrow x+2=A(2x)+B$

Equating the coefficients of x and constant term on both sides, we obtain

$2A = 1 \quad \Rightarrow \quad A = \frac{1}{2}$

From (1), we obtain: $(x+2) = \frac{1}{2}(2x) + 2$

Then, $\int \frac{x+2}{\sqrt{x^2-1}}dx = \int \frac{\frac{1}{2}(2x)+2}{\sqrt{x^2-1}}dx = \frac{1}{2}\int \frac{(2x)}{\sqrt{x^2-1}}dx + \int \frac{2}{\sqrt{x^2-1}}dx \quad \ldots(2)$

In $\frac{1}{2}\int \frac{(2x)}{\sqrt{x^2-1}}dx$, let $x^2 - 1 = t \quad \Rightarrow \quad 2x\,dx = dt$

So, $\frac{1}{2}\int \frac{(2x)}{\sqrt{x^2-1}}dx = \frac{1}{2}\int \frac{dt}{\sqrt{t}} = \frac{1}{2}[2\sqrt{t}] = \sqrt{t} = \sqrt{x^2-1}$

and $\int \frac{2}{\sqrt{x^2-1}}dx = 2\int \frac{1}{\sqrt{x^2-1}}dx = 2log\left|x + \sqrt{x^2-1}\right|$

From equation (2), we obtain

$$\text{I} = \int \frac{x+2}{\sqrt{x^2-1}}dx = \sqrt{x^2-1} + 2log\left|x+\sqrt{x^2-1}\right| + C$$

**Question 18:**

$$\frac{5x-2}{1+2x+3x^2}$$

**Answer 18:**

Let $5x - 2 = A\frac{d}{dx}(1+2x+3x^2) + B \quad \Rightarrow 5x - 2 = A(2+6x) + B$

Equating the coefficient of $x$ and constant term on both sides, we have

$5 = 6A \quad \Rightarrow A = \frac{5}{6} \qquad$ and $\qquad 2A + B = -2 \quad \Rightarrow B = -\frac{11}{3}$

Therefore, $5x - 2 = \frac{5}{6}(2+6x) + \left(-\frac{11}{3}\right)$

$\therefore \text{I} = \int \frac{5x-2}{1+2x+3x^2}dx = \int \frac{\frac{5}{6}(2+6x) + \left(-\frac{11}{3}\right)}{1+2x+3x^2}dx$

$= \frac{5}{6}\int \frac{(2+6x)}{1+2x+3x^2}dx - \frac{11}{3}\int \frac{1}{1+2x+3x^2}dx$

Let $I_1 = \int \frac{(2+6x)}{1+2x+3x^2}$ and $I_2 = \int \frac{1}{1+2x+3x^2}dx$

$\therefore \text{I} = \int \frac{5x-2}{1+2x+3x^2}dx = \frac{5}{6}I_1 - \frac{11}{3}I_2 \quad \ldots(1)$

Let $1 + 2x + 3x^2 = t \quad \Rightarrow \quad (2+6x)dx = dt$

$I_1 = \int \frac{dt}{t} = \log|t| \quad \Rightarrow I_1 = \log|1+2x+3x^2| \quad \ldots(2)$

$I_2 = \int \frac{1}{1+2x+3x^2}dx$

$$1+2x+3x^2 \text{ can be written as } 1+3\left(x^2+\frac{2}{3}x\right)$$

Therefore, $1+3\left(x^2+\frac{2}{3}x\right)=1+3\left(x^2+\frac{2}{3}x+\frac{1}{9}-\frac{1}{9}\right)$

$=1+3\left(x+\frac{1}{3}\right)^2-\frac{1}{3}=\frac{2}{3}+3\left(x+\frac{1}{3}\right)^2=3\left[\left(x+\frac{1}{3}\right)^2+\frac{2}{9}\right]=3\left[\left(x+\frac{1}{3}\right)^2+\left(\frac{\sqrt{2}}{3}\right)^2\right]$

$I_2=\frac{1}{3}\int\frac{1}{\left[\left(x+\frac{1}{3}\right)^2+\left(\frac{\sqrt{2}}{3}\right)^2\right]}=\frac{1}{3}\left[\frac{1}{\frac{\sqrt{2}}{3}}tan^{-1}\left(\frac{x+\frac{1}{3}}{\frac{\sqrt{2}}{3}}\right)\right]=\frac{1}{3}\left[\frac{3}{\sqrt{2}}tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)\right]\frac{1}{\sqrt{2}}tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)$

Substituting the values from (2) and (3) in equation (1), we have

$\therefore \mathrm{I}=\int\frac{5x-2}{1+2x+3x^2}dx=\frac{5}{6}[\log|1+2x+3x^2|]-\frac{11}{3}\left[\frac{1}{\sqrt{2}}tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)\right]+C$

$=\frac{5}{6}\log|1+2x+3x^2|-\frac{11}{3\sqrt{2}}tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)+C$

**Question 19:**

$$\frac{6x+7}{\sqrt{(x-5)(x-4)}}$$

**Answer 19:**

$$\frac{6x+7}{\sqrt{(x-5)(x-4)}}=\frac{6x+7}{\sqrt{x^2-9x+20}}$$

Let $6x+7=A\frac{d}{dx}(x^2-9x+20)+B \Rightarrow 6x+7=A(2x-9)+B$

Equating the coefficients of $x$ and constant term, we have

$2A=6\Rightarrow A=3$ and $-9A+B=7\Rightarrow B=34$

$\therefore 6x+7=3(2x-9)+34$

$\therefore \mathrm{I}=\int\frac{6x+7}{\sqrt{x^2-9x+20}}=\int\frac{3(2x-9)+34}{\sqrt{x^2-9x+20}}dx$

$=3\int\frac{2x-9}{\sqrt{x^2-9x+20}}dx+34\int\frac{1}{\sqrt{x^2-9x+20}}dx$

Let $I_1=\int\frac{2x-9}{\sqrt{x^2-9x+20}}dx$ and $I_2=\int\frac{1}{\sqrt{x^2-9x+20}}dx$

$I=\int\frac{6x+7}{\sqrt{x^2-9x+20}}=3I_1+34I_2$ ... (1)

Then, $I_1=\int\frac{2x-9}{\sqrt{x^2-9x+20}}dx$

Let $x^2-9x+20=t \Rightarrow (2x-9)dx=dt$

$$I_1 = \int \frac{1}{\sqrt{t}}\,dt = 2\sqrt{t} = 2\sqrt{x^2 - 9x + 20} \qquad \dots (2)$$

$$\text{and} \quad I_2 = \int \frac{1}{\sqrt{x^2 - 9x + 20}}\,dx$$

$x^2 - 9x + 20$ can be written as $x^2 - 9x + 20 + \frac{81}{4} - \frac{81}{4}$

Therefore, $\quad x^2 - 9x + 20 + \frac{81}{4} - \frac{81}{4} = \left(x - \frac{9}{2}\right)^2 - \frac{1}{4} = \left(x - \frac{9}{2}\right)^2 - \left(\frac{1}{2}\right)^2$

$$I_2 = \int \frac{1}{\sqrt{\left(x - \frac{9}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}\,dx = \log\left|\left(x - \frac{9}{2}\right) + \sqrt{x^2 - 9x + 20}\right| \qquad \dots (3)$$

Substituting equations (2) and (3) in (1), we have

$$\therefore \mathrm{I} = \frac{6x+7}{\sqrt{x^2 - 9x + 20}}\,dx = 3\left[2\sqrt{x^2 - 9x + 20}\right] + 34\log\left|\left(x - \frac{9}{2}\right) + \sqrt{x^2 - 9x + 20}\right| + C$$

$$= 6\left[\sqrt{x^2 - 9x + 20}\right] + 34\log\left|\left(x - \frac{9}{2}\right) + \sqrt{x^2 - 9x + 20}\right| + C$$

## Question 20:

$$\frac{x+2}{\sqrt{4x - x^2}}$$

## Answer 20:

Let $x + 2 = A\frac{d}{dx}(4x - x^2) + B \qquad \Rightarrow x + 2 = A(4 - 2x) + B$

Equating the coefficients of $x$ and constant term on both sides, we have

$$-2A = 1 \ \Rightarrow A = -\frac{1}{2} \qquad \text{and} \qquad 4A + B = 2 \ \Rightarrow B = 4$$

Therefore, $\quad (x + 2) = -\frac{1}{2}(4 - 2x) + 4$

$$I = \int \frac{x+2}{\sqrt{4x - x^2}}\,dx = \int \frac{-\frac{1}{2}(4 - 2x) + 4}{\sqrt{4x - x^2}}\,dx = -\frac{1}{2}\int \frac{(4 - 2x)}{\sqrt{4x - x^2}}\,dx + 4\int \frac{1}{\sqrt{4x - x^2}}\,dx$$

Let $I_1 = \int \frac{(4 - 2x)}{\sqrt{4x - x^2}}\,dx$ and $I_2 = \int \frac{1}{\sqrt{4x - x^2}}\,dx \Rightarrow I = \int \frac{x+2}{\sqrt{4x - x^2}}\,dx = -\frac{1}{2}\,I_1 + 4\,I_2 \qquad \dots (1)$

Then $\quad I_1 = \int \frac{(4 - 2x)}{\sqrt{4x - x^2}}\,dx$

Let $4x - x^2 = t \qquad \Rightarrow (4 - 2x)dx = dt$

$$I_1 = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} = 2\sqrt{4x - x^2} \qquad \dots (2)$$

$$I_2 = \int \frac{1}{\sqrt{(2)^2 - (x-2)^2}} dx = sin^{-1}\left(\frac{x-2}{2}\right) \quad \dots (3)$$

Using equations (2) and (3) in (1), we obtain

$$I = \int \frac{x+2}{\sqrt{4x-x^2}} = -\frac{1}{2}\left(2\sqrt{4x-x^2}\right) + 4sin^{-1}\left(\frac{x-2}{2}\right) + C = -\sqrt{4x-x^2} + 4sin^{-1}\left(\frac{x-2}{2}\right) + C$$

**Question 21:**

$$\frac{x+2}{\sqrt{x^2+2x+3}}$$

**Answer 21:**

$$I = \int \frac{x+2}{\sqrt{x^2+2x+3}} dx = \frac{1}{2}\int \frac{2(x+2)}{\sqrt{x^2+2x+3}} dx = \frac{1}{2}\int \frac{2x+4}{\sqrt{x^2+2x+3}} dx$$

$$= \frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x+3}} dx + \frac{1}{2}\int \frac{2}{\sqrt{x^2+2x+3}} dx = \frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x+3}} dx + \int \frac{1}{\sqrt{x^2+2x+3}} dx$$

$$Let\ I_1 = \int \frac{2x+2}{\sqrt{x^2+2x+3}} dx \ \ and \ \ I_2 = \int \frac{1}{\sqrt{x^2+2x+3}} dx$$

$$\Rightarrow I = \int \frac{x+2}{\sqrt{x^2+2x+3}} dx = \frac{1}{2} I_1 + I_2$$

Now, $I_1 = \int \frac{2x+2}{\sqrt{x^2+2x+3}} dx$

$Let \ \ x^2 + 2x + 3 = t$

$\Rightarrow (2x+2)dx = dt$

$$I_1 = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} = 2\sqrt{x^2+2x+3} \quad \dots (2)$$

$$I_2 = \int \frac{1}{\sqrt{x^2+2x+3}} dx$$

Here, $x^2 + 2x + 3 = x^2 + 2x + 1 + 2$

$= (x+1)^2 + \left(\sqrt{2}\right)^2$

So, $I_2 = \int \frac{1}{\sqrt{(x+1)^2 + \left(\sqrt{2}\right)^2}} dx = log\left|(x+1) + \sqrt{x^2+2x+3}\right| \quad \dots (3)$

Using equations (2) and (3) in (1), we have

$$I = \int \frac{x+2}{\sqrt{x^2+2x+3}} dx$$

$$= \frac{1}{2}\left[2\sqrt{x^2+2x+3}\right] + \log\left[(x+1) + \sqrt{x^2+2x+3}\right] + C$$

$$= \sqrt{x^2+2x+3} + \log\left[(x+1) + \sqrt{x^2+2x+3}\right] + C$$

**Question 22:**

$$\frac{x+3}{x^2-2x-5}$$

**Answer 22:**

$$\text{Let} \quad (x+3) = A\frac{d}{dx}(x^2-2x-5)+B \qquad \Rightarrow (x+3) = A(2x-2)+B$$

Equating the coefficients of $x$ and constant term on both sides, we obtain

$$2A = 1 \;\Rightarrow A = \frac{1}{2} \qquad \text{and} \qquad -2A+B = 3 \;\Rightarrow B = 4$$

$$\text{Therefore,} \qquad (x+3) = \frac{1}{2}(2x-2)+4$$

$$I = \int \frac{x+3}{x^2-2x-5}dx = \int \frac{\frac{1}{2}(2x-2)+4}{x^2-2x-5}dx = \frac{1}{2}\int \frac{2x-2}{x^2-2x-5}dx + \int \frac{1}{x^2-2x-5}dx$$

$$\text{Let} \;\; I_1 = \int \frac{2x-2}{x^2-2x-5}dx \;\text{ and }\; I_2 = \int \frac{1}{x^2-2x-5}dx$$

$$\Rightarrow I = \int \frac{x+3}{x^2-2x-5}dx = \frac{1}{2}I_1 + 4\,I_2 \qquad \ldots(1)$$

$$I_1 = \int \frac{2x-2}{x^2-2x-5}dx$$

Let $x^2-2x-5 = t \qquad \Rightarrow (2x-2)dx = dt$

$$I_1 = \int \frac{dt}{t} = \log|t| = \log|x^2-2x-5| \qquad \ldots(2)$$

$$I_2 = \int \frac{1}{x^2-2x-5}dx = \int \frac{1}{(x^2-2x+1)-6}dx = \int \frac{1}{(x-1)^2+\left(\sqrt{6}\right)^2}dx$$

$$= \frac{1}{2\sqrt{6}}\log\left(\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right) \qquad \ldots(3)$$

Substituting (2) and (3) in (1), we have

$$I = \int \frac{x+3}{x^2-2x-5}dx = \frac{1}{2}\log|x^2-2x-5| + \frac{4}{2\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right| + C$$

$$= \frac{1}{2}\log|x^2-2x-5| + \frac{2}{\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right| + C$$

**Question 23:**

$$\frac{5x+3}{\sqrt{x^2+4x+10}}$$

**Answer 23:**

$$\text{Let} \quad 5x+3 = A\frac{d}{dt}(x^2+4x+10)+B \qquad \Rightarrow 5x+3 = A(2x+4)+B$$

Equating the coefficients of $x$ and constant term, we have

$$2A = 5 \quad \Rightarrow A = \frac{5}{2} \qquad \text{and} \qquad 4A + B = 3 \quad \Rightarrow B = -7$$

Therefore, $\quad 5x + 3 = \frac{5}{2}(2x + 4) - 7$

$$I = \int \frac{5x+3}{\sqrt{x^2+4x+10}}dx = \int \frac{\frac{5}{2}(2x+4)-7}{\sqrt{x^2+4x+10}}dx = \frac{5}{2}\int \frac{2x+4}{\sqrt{x^2+4x+10}}dx - 7\int \frac{1}{\sqrt{x^2+4x+10}}dx$$

Let $I_1 = \int \frac{2x+4}{\sqrt{x^2+4x+10}}dx$ and $I_2 = \int \frac{1}{\sqrt{x^2+4x+10}}dx$

$$\Rightarrow I = \int \frac{5x+3}{\sqrt{x^2+4x+10}}dx = \frac{5}{2}I_1 - 7I_2 \qquad \dots(1)$$

$$\text{Now, } I_1 = \int \frac{2x+4}{\sqrt{x^2+4x+10}}dx$$

Let $x^2 + 4x + 10 = t$

$\Rightarrow (2x+4)dx = dt$

$$I_1 = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} = 2\sqrt{x^2+4x+10} \qquad \dots(2)$$

$$I_2 = \int \frac{1}{\sqrt{x^2+4x+10}}dx = \int \frac{1}{\sqrt{(x^2+4x+4)+6}}dx$$

$$= \int \frac{1}{\sqrt{(x+2)^2 + \left(\sqrt{6}\right)^2}}dx$$

$$= \int \frac{1}{\sqrt{(x+2)^2 + \left(\sqrt{6}\right)^2}}dx$$

$$= \log\left|(x+2) + \sqrt{(x^2+4x+10}\right| \qquad \dots(3)$$

Using equations (2) and (3) in (1), we have

$$\text{I} = \int \frac{5x+3}{\sqrt{x^2+4x+10}}dx$$

$$= \frac{5}{2}\left|2\sqrt{x^2+4x+10}\right| - 7\log\left|(x+2) + \sqrt{(x^2+4x+10}\right| + C$$

$$= 5\left|\sqrt{x^2+4x+10}\right| - 7\log\left|(x+2) + \sqrt{(x^2+4x+10}\right| + C$$

Choose the correct answer in Exercises 24 and 25.

**Question 24:**

$$\int \frac{dx}{x^2 + 2x + 2} \text{ equals}$$

(A) $x\ tan^{-1}(x+1) + C$

(B) $tan^{-1}(x+1) + C$

(C) $(x+1)\ tan^{-1}x + C$

(D) $tan^{-1}x + C$

**Answer 24:**

$$I = \int \frac{dx}{x^2 + 2x + 2} = \int \frac{dx}{(x^2 + 2x + 1) + 1}$$

$$= \int \frac{dx}{(x+1)^2 + 1} dx = [tan^{-1}(x+1)] + C$$

Hence, the correct Answer is (B).

**Question 25:**

$$\int \frac{dx}{\sqrt{9x - 4x^2}} \text{ equals}$$

(A) $\frac{1}{9} sin^{-1}\left(\frac{9x-8}{8}\right) + C$

(B) $\frac{1}{2} sin^{-1}\left(\frac{8x-9}{8}\right) + C$

(C) $\frac{1}{3} sin^{-1}\left(\frac{9x-8}{8}\right) + C$

(D) $\frac{1}{2} sin^{-1}\left(\frac{9x-8}{9}\right) + C$

**Answer 25:**

$$\int \frac{dx}{\sqrt{9x - 4x^2}}$$

$$= \int \frac{1}{\sqrt{-4\left(x^2 - \frac{9}{4}x\right)}} dx$$

$$= \int \frac{1}{\sqrt{-4\left(x^2 - \frac{9}{4}x + \frac{81}{64} - \frac{81}{64}\right)}} dx$$

$$= \int \frac{1}{\sqrt{-4\left[\left(x - \frac{9}{8}\right)^2 - \left(\frac{9}{8}\right)^2\right]}} dx$$

$$= \frac{1}{2} \int \frac{1}{\left(\frac{9}{8}\right)^2 - \left(x - \frac{9}{8}\right)^2} dx$$

$$= \frac{1}{2}\left[\sin^{-1}\left(\frac{x - \frac{9}{8}}{\frac{9}{8}}\right)\right] + C \qquad \left(\int \frac{dy}{\sqrt{a^2 - y^2}} = \sin^{-1}\frac{y}{a} + C\right)$$

$$= \frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right) + C$$

Hence, the correct option is B.

# Mathematics

**(Chapter – 7) (Integrals) (Exercise 7.5)**
**(Class – XII)**

Integrate the rational functions in Exercises 1 to 21.

**Question 1:**

$$\frac{x}{(x+1)(x+2)}$$

**Answer 1:**

$$\text{Let } \frac{x}{(x+1)(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)} = \frac{A(x+2)+B(x+1)}{(x+1)(x+2)}$$

$\Rightarrow x = A(x+2) + B(x+1)$

Equating the coefficients of $x$ and constant term, we have

A + B = 1 and 2A + B = 0

On solving, we have:

A = −1 and B = 2

$$\Rightarrow \frac{x}{(x+1)(x+2)} = \frac{-1}{(x+1)} + \frac{2}{(x+2)}$$

$$I = \int \frac{x}{(x+1)(x+2)}dx = \int \frac{-1}{(x+1)} + \frac{2}{(x+2)}dx$$

$$= -\log|x+1| + 2\log|x+2| + C$$

$$= \log(x+2)^2 - \log|x+1| + C = \log\frac{(x+2)^2}{(x+1)} + C$$

**Question 2:**

$$\frac{1}{x^2-9}$$

**Answer 2:**

$$\text{Let } \frac{1}{(x+3)(x-3)} = \frac{A}{(x+3)} + \frac{B}{(x-3)} = \frac{A(x-3)+B(x+3)}{(x+3)(x-3)}$$

$\Rightarrow 1 = A(x-3) + B(x+3)$

Equating the coefficients of $x$ and constant term, we have

A + B = 0 and −3A + 3B = 1

On solving, we have

$$A = -\frac{1}{6} \text{ and } B = \frac{1}{6}$$

$$\Rightarrow \frac{1}{(x+3)(x-3)} = \frac{-1}{6(x+3)} + \frac{1}{6(x-3)}$$

$$I = \int \frac{1}{x^2-9}dx = \int \left(\frac{-1}{6(x+3)} + \frac{1}{6(x-3)}\right)dx$$

$$= -\frac{1}{6}\log|x+3| + \frac{1}{6}\log|x-3| + C$$

$$= -\frac{1}{6}\log\left|\frac{(x-3)}{(x+3)}\right| + C$$

**Question 3:**

$$\frac{3x-1}{(x-1)(x-2)(x-3)}$$

**Answer 3:**

Let $\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$

$\Rightarrow \frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2)}{(x-1)(x-2)(x-3)}$

$\Rightarrow 3x-1 = A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2) \qquad \ldots (1)$

Equating the coefficients of $x^2$, $x$ and constant term, we have

A + B + C = 0, – 5A – 4B – 3C = 3 and 6A + 3B + 2C = – 1

Solving these equations, we have

A = 1, B = –5 and C = 4

$\Rightarrow \frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{1}{(x-1)} - \frac{5}{(x-2)} + \frac{4}{(x-3)}$

$I = \int \frac{3x-1}{(x-1)(x-2)(x-3)} dx = \int \left[\frac{1}{(x-1)} - \frac{5}{(x-2)} + \frac{4}{(x-3)}\right] dx$

$= \log|x-1| - 5\log|x-2| + 4\log|x-3| + C$

**Question 4:**

$$\frac{x}{(x-1)(x-2)(x-3)}$$

**Answer 4:**

Let $\frac{x}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$

$\Rightarrow \frac{x}{(x-1)(x-2)(x-3)} = \frac{A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2)}{(x-1)(x-2)(x-3)}$

$x = A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2) \qquad \ldots (1)$

Equating the coefficients of $x^2$, $x$ and constant term, we have

A + B + C = 0, – 5A – 4B – 3C = 1 and 6A + 4B + 2C = 0

Solving these equations, we have

$$A = \frac{1}{2}, \quad B = -2 \quad \text{and} \quad C = \frac{3}{2}$$

$\Rightarrow \frac{x}{(x-1)(x-2)(x-3)} = \frac{1}{2(x-1)} - \frac{2}{(x-2)} + \frac{3}{2(x-3)}$

$I = \int \frac{x}{(x-1)(x-2)(x-3)} dx = \int \left[\frac{1}{2(x-1)} - \frac{2}{(x-2)} + \frac{3}{2(x-3)}\right] dx$

$= \frac{1}{2}\log|x-1| - 2\log|x-2| + \frac{3}{2}\log|x-3| + C$

**Question 5:**

$$\frac{2x}{x^2+3x+2}$$

**Answer 5:**

$$\frac{2x}{x^2+3x+2}=\frac{A}{(x+1)}+\frac{B}{(x+2)}=\frac{A(x+2)+B(x+1)}{x^2+3x+2}$$

$\Rightarrow 2x = A(x+2)+B(x+1) \quad \dots(1)$

Equating the coefficients of $x$ and constant term, we have

A + B = 2 and 2A + B = 0

Solving these equations, we have

A = −2 and B = 4

$$\Rightarrow \frac{2x}{x^2+3x+2}=\frac{-2}{(x+1)}+\frac{4}{(x+2)}$$

$$I=\int \frac{2x}{(x-1)(x-2)(x-3)}dx = \int\left[\frac{4}{(x+2)}-\frac{2}{(x+1)}\right]dx$$

$= 4\log|x+2| - 2\log|x+1| + C$

**Question 6:**

$$\frac{1-x^2}{x(1-2x)}$$

**Answer 6:**

It can be seen that the given integrand is not a proper fraction.

Therefore, on dividing $(1-x^2)$ by $x(1-2x)$, we have

$$\frac{1-x^2}{x(1-2x)}=\frac{1}{2}+\frac{1}{2}\left(\frac{2-x}{x(1-2x)}\right)$$

Let $\frac{2-x}{x(1-2x)}=\frac{A}{x}+\frac{B}{(1-2x)} \quad \Rightarrow (2-x)=A(1-2x)+Bx \quad \dots(1)$

Equating the coefficients of $x^2$, $x$ and constant term, we have

− 2A + B = − 1 and A = 2

Solving these equations, we have

A = 2 and B = 3

Therefore, $\frac{2-x}{x(1-2x)}=\frac{2}{x}+\frac{3}{(1-2x)}$

Substituting in equation (1), we have

$$\frac{1-x^2}{x(1-2x)}=\frac{1}{2}+\frac{1}{2}\left(\frac{2}{x}+\frac{3}{(1-2x)}\right)$$

$$I=\int\frac{1-x^2}{x(1-2x)}dx=\int\left\{\frac{1}{2}+\frac{1}{2}\left(\frac{2}{x}+\frac{3}{(1-2x)}\right)dx\right\}$$

$=\frac{x}{2}+\log|x|+\frac{3}{2(-2)}\log|1-2x|+C$

$=\frac{x}{2}+\log|x|-\frac{3}{4}\log|1-2x|+C$

**Question 7:**

$$\frac{x}{(x^2+1)(x-1)}$$

**Answer 7:**

Let $\dfrac{x}{(x^2+1)(x-1)} = \dfrac{Ax+B}{(x^2+1)} + \dfrac{C}{(x-1)}$

$\Rightarrow x = (Ax+B)(x-1) + C(x^2+1) \quad \Rightarrow x = Ax^2 - Ax + Bx - B + Cx^2 + C$

Equating the coefficients of $x^2$, $x$ and constant term, we have

A + C = 0, −A + B = 1 and −B + C = 0

On solving these equations, we have

$$A = -\frac{1}{2}, \qquad B = \frac{1}{2} \quad \text{and} \quad C = \frac{1}{2}$$

From equation (1), we have

$$\frac{x}{(x^2+1)(x-1)} = \frac{\left(-\frac{1}{2}x + \frac{1}{2}\right)}{(x^2+1)} + \frac{\frac{1}{2}}{(x-1)}$$

$$I = \int \frac{x}{(x^2+1)(x-1)} = -\frac{1}{2}\int \frac{x}{(x^2+1)}dx + \frac{1}{2}\int \frac{1}{(x^2+1)}dx + \frac{1}{2}\int \frac{1}{(x-1)}dx$$

$$= -\frac{1}{4}\int \frac{2x}{(x^2+1)}dx + \frac{1}{2}tan^{-1}x + \frac{1}{2}\log|x-1| + C$$

Consider $\displaystyle\int \frac{2x}{(x^2+1)}dx$, $Let\ (x^2+1) = t \qquad \Rightarrow 2xdx = dt$

Therefore, $\displaystyle\int \frac{2x}{(x^2+1)}dx = \int \frac{dt}{t} = \log|t| = \log|x^2+1|$

$$I = \int \frac{x}{(x^2+1)(x-1)}dx = -\frac{1}{4}\log|x^2+1| + \frac{1}{2}tan^{-1}x + \frac{1}{2}\log|x-1| + C$$

$$= \frac{1}{2}\log|x-1| - \frac{1}{4}\log|x^2+1| + \frac{1}{2}tan^{-1}x + C$$

**Question 8:**

$$\frac{x}{(x-1)^2(x+2)}$$

**Answer 8:**

Let $\dfrac{x}{(x-1)^2(x+2)} = \dfrac{A}{(x-1)} + \dfrac{B}{(x-1)^2} + \dfrac{C}{(x+2)}$

$\Rightarrow x = A(x-1)(x+2) + B(x+2) + C\,(x-1)^2$

Equating the coefficients of $x^2$, $x$ and constant term, we have

A + C = 0, A + B – 2C = 1 and −2A + 2B + C = 0

On solving, we have

$$A = \frac{2}{9}, \qquad B = \frac{1}{3} \quad \text{and} \quad C = -\frac{2}{9}$$

Therefore, $\dfrac{x}{(x-1)^2\,(x+2)} = \dfrac{2}{9(x-1)} + \dfrac{1}{3(x-1)^2} + \dfrac{-2}{9(x+2)}$

$$I = \int \frac{x}{(x-1)^2\,(x+2)}dx = \frac{2}{9}\int \frac{1}{(x-1)}dx + \frac{1}{3}\int \frac{1}{(x-1)^2}dx - \frac{2}{9}\int \frac{1}{(x+2)}dx$$

$$= \frac{2}{9}\log|x-1| + \frac{1}{3}\left(\frac{-1}{x-1}\right) - \frac{2}{9}\log|x+2| + C = \frac{2}{9}\log\left|\frac{x-1}{x+2}\right| - \frac{1}{3(x-1)} + C$$

**Question 9:**

$$\frac{3x+5}{x^3-x^2-x+1}$$

**Answer 9:**

$$\frac{3x+5}{x^3-x^2-x+1} = \frac{3x+5}{(x-1)^2(x+1)}$$

Let $\dfrac{3x+5}{(x-1)^2(x+1)} = \dfrac{A}{(x-1)} + \dfrac{B}{(x-1)^2} + \dfrac{C}{(x+1)}$

$\Rightarrow 3x+5 = A(x-1)(x+1) + B(x+1) + C\,(x-1)^2$

$\Rightarrow 3x+5 = A(x^2-1) + B(x+1) + C(x^2+1-2x)$ ... (1)

Equating the coefficients of $x^2$, $x$ and constant term, we have

A + C = 0, B – 2C = 3 and – A + B + C = 5

On solving, we have

$$A = -\frac{1}{2}, \quad B = 4 \quad \text{and} \quad C = \frac{1}{2}$$

$$\frac{3x+5}{(x-1)^2(x+1)} = \frac{-1}{2(x-1)} + \frac{4}{(x-1)^2} + \frac{1}{2(x+1)}$$

$$I = \int \frac{3x+5}{(x-1)^2(x+1)}dx$$

$$= -\frac{1}{2}\int \frac{1}{(x-1)}dx + 4\int \frac{4}{(x-1)^2}dx + \frac{1}{2}\int \frac{1}{(x+1)}dx$$

$$= -\frac{1}{2}\log|x-1| + 4\left(\frac{-1}{x-1}\right) + \frac{1}{2}\log|x+1| + C = \frac{1}{2}\log\left|\frac{x+1}{x-1}\right| - \frac{4}{(x-1)} + C$$

**Question 10:**

$$\frac{2x-3}{(x^2-1)(2x+3)}$$

**Answer 10:**

$$\frac{2x-3}{(x^2-1)(2x+3)} = \frac{2x-3}{(x+1)(x-1)(2x+3)}$$

Let $\dfrac{2x-3}{(x+1)(x-1)(2x+3)} = \dfrac{A}{(x+1)} + \dfrac{B}{(x-1)} + \dfrac{C}{(2x+3)}$

$\Rightarrow (2x-3) = A(x-1)(2x+3) + B(x+1)(2x+3) + C(x+1)(x-1)$

$\Rightarrow (2x-3) = (2A+2B+C)x^2 + (A+5B)x + (-3A+3B-C)$

Equating the coefficients of $x^2$, $x$ and constant, we have

2A + 2B + C = 0, A + 5B = 2 and – 3A + 3B – C = – 3

On solving, we have

$$A = \frac{5}{2}, \quad B = \frac{1}{10} \quad \text{and} \quad C = -\frac{24}{5}$$

$$\frac{2x-3}{(x+1)(x-1)(2x+3)} = \frac{5}{2(x+1)} - \frac{1}{10(x-1)} - \frac{24}{5(2x+3)}$$

$$I = \int \frac{2x-3}{(x^2-1)(2x+3)} dx$$

$$= \frac{5}{2}\int \frac{1}{(x+1)} dx - \frac{1}{10}\int \frac{1}{(x-1)} dx - \frac{24}{5}\int \frac{1}{(2x+3)} dx$$

$$= \frac{5}{2}\log|x+1| - \frac{1}{10}\log|x-1| - \frac{24}{5\times 2}\log|2x+3| + C$$

$$= \frac{5}{2}\log|x+1| - \frac{1}{10}\log|x-1| - \frac{12}{5}\log|2x+3| + C$$

**Question 11:**

$$\frac{5x}{(x+1)(x^2-4)}$$

**Answer 11:**

$$\frac{5x}{(x+1)(x^2-4)} = \frac{5x}{(x+1)(x+2)(x-2)}$$

Let $\dfrac{5x}{(x+1)(x+2)(x-2)} = \dfrac{A}{(x+1)} + \dfrac{B}{(x+2)} + \dfrac{C}{(x-2)}$

$\Rightarrow 5x = A(x+2)(x-2) + B(x+1)(x-2) + C(x+1)(x+2) \qquad \ldots (1)$

Equating the coefficients of $x^2$, $x$ and constant, we have

A + B + C = 0, – B + 3C = 5 and – 4A – 2B + 2C = 0

On solving, we have

$$A = \frac{5}{3}, \quad B = -\frac{5}{2} \quad \text{and} \quad C = -\frac{5}{6}$$

$$\frac{5x}{(x+1)(x+2)(x-2)} = \frac{5}{3(x+1)} - \frac{5}{2(x+2)} - \frac{5}{6(x-2)}$$

$$I = \int \frac{5x}{(x+1)(x+2)(x-2)} dx$$

$$= \frac{5}{3}\int \frac{1}{(x+1)} dx - \frac{5}{2}\int \frac{1}{(x+2)} dx + \frac{5}{6}\int \frac{1}{(x-2)} dx$$

$$= \frac{5}{3}\log|x+1| - \frac{5}{2}\log|x+2| + \frac{5}{6}\log|x-2| + C$$

**Question 12:**

$$\frac{x^3+x+1}{x^2-1}$$

**Answer 12:**

It can be seen that the given integrand is not a proper fraction.

Therefore, on dividing $(x^3 + x + 1)$ by $x^2 - 1$, we have

$$\frac{x^3+x+1}{x^2-1} = x + \frac{2x+1}{x^2-1}$$

Let $\frac{2x+1}{x^2-1} = \frac{A}{(x-1)} + \frac{B}{(x+1)} \quad \Rightarrow 2x+1 = A(x+1) + B(x-1) \quad \ldots (1)$

Equating the coefficients of $x$ and constant, we have

$A + B = 2$ and $-A + B = 1$

On solving, we have

$$A = \frac{1}{2} \quad \text{and} \quad B = \frac{3}{2}$$

Therefore, $\frac{x^3+x+1}{x^2-1} = x + \frac{1}{2(x-1)} + \frac{3}{2(x+1)}$

$$I = \int \frac{x^3+x+1}{x^2-1}dx = \int x\,dx + \frac{1}{2}\int \frac{1}{(x-1)}dx + \frac{3}{2}\int \frac{1}{(x+1)}dx$$

$$= \frac{x^2}{2} + \frac{1}{2}\log|x+1| + \frac{3}{2}\log|x-1| + C$$

**Question 13:**

$$\frac{2}{(1-x)(1+x^2)}$$

**Answer 13:**

Let $\frac{2}{(1-x)(1+x^2)} = \frac{A}{(1-x)} + \frac{Bx+C}{(1+x^2)}$

$\Rightarrow 2 = A(1+x^2) + (Bx+C)(1-x)$

$\Rightarrow 2 = A + Ax^2 + Bx - Bx^2 + C - Cx$

Equating the coefficient of $x^2$, $x$ and constant term, we have

$A - B = 0,$ $B - C = 0$ and $A + C = 2$

On solving these equations, we have

$A = 1,$ $B = 1$ and $C = 1$

Therefore, $\frac{2}{(1-x)(1+x^2)} = \frac{1}{(1-x)} + \frac{x+1}{(1+x^2)}$

$$I = \int \frac{2}{(1-x)(1+x^2)}dx = \int \frac{1}{(1-x)}dx + \int \frac{x}{(1+x^2)}dx + \int \frac{1}{(1+x^2)}dx$$

$$= -\int \frac{1}{(x-1)}dx + \frac{1}{2}\int \frac{2x}{(1+x^2)}dx + \int \frac{1}{(1+x^2)}dx$$

$$= -\log|x-1| + \frac{1}{2}\log|1+x^2| + tan^{-1}x + C$$

**Question 14:**

$$\frac{3x-1}{(x+2)^2}$$

**Answer 14:**

Let $\dfrac{3x-1}{(x+2)^2}=\dfrac{A}{(x+2)}+\dfrac{B}{(x+2)^2}$

$\Rightarrow 3x-1=A(x+2)+B$

Equating the coefficient of $x$ and constant term, we have

A = 3 and 2A + B = −1 ⇒ B = −7

Therefore, $\dfrac{3x-1}{(x+2)^2}=\dfrac{3}{(x+2)}-\dfrac{7}{(x+2)^2}$

$$I=\int\frac{3x-1}{(x+2)^2}dx=3\int\frac{1}{(x+2)}dx-7\int\frac{1}{(x+2)^2}dx$$

$$=3\log|x+2|-7\left(\frac{-1}{(x+2)}\right)+C\ =3\log|x+2|+\frac{7}{(x+2)}+C$$

**Question 15:**

$$\frac{1}{x^4-1}$$

**Answer 15:**

$$\frac{1}{x^4-1}=\frac{1}{(x^2-1)(x^2+1)}=\frac{1}{(x+1)(x-1)(x^2+1)}$$

Let $\dfrac{1}{(x-1)(x+1)(x^2+1)}=\dfrac{A}{(x+1)}+\dfrac{B}{(x-1)}+\dfrac{Cx+D}{(x^2+1)}$

$\Rightarrow 1=A(x-1)(x^2+1)+B(x+1)(x^2+1)+(Cx+D)(x-1)(x+1)$

$\Rightarrow 1=A(x^3+x-x^2-1)+B(x^3+x+x^2+1)+Cx^3+Dx^2-Cx-D$

$\Rightarrow 1=(A+B+C)x^3+(-A+B+D)x^2+(A+B-C)x+(-A+B-D)$

Equating the coefficient of $x^3, x^2, x$ and constant term, we have

$A+B+C=0$, $-A+B+D=0$, $A+B-C=0$ and $-A+B-D=1$

On solving these equations, we have

$$A=-\frac{1}{4},\quad B=\frac{1}{4},\quad C=0\quad \text{and}\quad D=-\frac{1}{2}$$

Therefore, $\dfrac{1}{x^4-1}=\dfrac{-1}{4(x+1)}+\dfrac{1}{4(x-1)}-\dfrac{1}{2(x^2+1)}$

$$I=\int\frac{1}{x^4-1}dx=-\frac{1}{4}\int\frac{1}{(x+1)}dx+\frac{1}{4}\int\frac{1}{(x-1)}dx+-\frac{1}{2}\int\frac{1}{(x^2+1)}dx$$

$$=\int\frac{1}{x^4-1}dx=-\frac{1}{4}\log|x+1|+\frac{1}{4}\log|x-1|-\frac{1}{2}tan^{-1}x+C$$

$$=\frac{1}{4}\log\left|\frac{x-1}{x+1}\right|-\frac{1}{2}tan^{-1}x+C$$

**Question 16:**

$$\frac{1}{x(x^n+1)}$$

[Hint: multiply numerator and denominator by $x^{n-1}$ and put $x^n = t$]

**Answer 16:**

$$\frac{1}{x(x^n+1)} = \frac{x^{n-1}}{x^{n-1}x(x^n+1)} = \frac{x^{n-1}}{x^n(x^n+1)}$$

Let $x^n \ = t \ \ i.e. \ nx^{n-1}dx = dt$

$$I = \int \frac{1}{x(x^n+1)}dx = \int \frac{x^{n-1}}{x^n(x^n+1)}dx = \frac{1}{n}\int \frac{1}{t(t+1)}dt$$

Let $\frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{(t+1)} \quad \Rightarrow 1 = A(t+1) + Bt \quad \ldots(1)$

Equating the coefficients of $t$ and constant, we have

A = 1 and B = −1

Therefore, $\frac{1}{t(t+1)} = \frac{1}{t} - \frac{1}{(1+t)}$

$$I = \int \frac{1}{x(x^n+1)}dx = \frac{1}{n}\int \left\{\frac{1}{t} - \frac{1}{(1+t)}\right\}dt = \frac{1}{n}[\log|t| - \log|t+1|] + C$$

$$= -\frac{1}{n}[\log|x^n| - \log|x^n+1|] + C = \frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right| + C$$

**Question 17:**

$$\frac{\cos x}{(1-\sin x)(2-\sin x)} \qquad [\text{Hint: Put } sin\, x \ = \ t]$$

**Answer 17:**

$$\frac{\cos x}{(1-\sin x)(2-\sin x)}$$

Let $\sin x = t \ \Rightarrow \cos x\, dx = dt$

$$I = \int \frac{\cos x}{(1-\sin x)(2-\sin x)}dx = \int \frac{dt}{(1-t)(2-t)}$$

Let $\frac{1}{(1-t)(2-t)} = \frac{A}{(1-t)} + \frac{B}{(2-t)}$

$\Rightarrow 1 = A(2-t) + B(1-t) \qquad \ldots(1)$

Equating the coefficients of $t$ and constant, we have

– 2A – B = 0 and 2A + B = 1

On solving, we have

A = 1 and B = −1

Therefore, $\frac{1}{(1-t)(2-t)} = \frac{1}{(1-t)} + \frac{-1}{(2-t)}$

$$I = \int \frac{\cos x}{(1-\sin x)dx(2-\sin x)} = \int \left\{\frac{1}{(1-t)} + \frac{-1}{(2-t)}\right\}dt$$

$$= -\log|1-t| + \log|2-t| + C = \log\left|\frac{2-t}{1-t}\right| + C = \log\left|\frac{2-\sin x}{1-\sin x}\right| + C$$

**Question 18:**

$$\frac{(x^2+1)\ (x^2+2)}{(x^2+3)\ (x^2+4)}$$

**Answer 18:**

$$\frac{(x^2+1)\ (x^2+2)}{(x^2+3)\ (x^2+4)} = 1 - \frac{(4x^2+10)}{(x^2+3)\ (x^2+4)}$$

Let $\dfrac{(4x^2+10)}{(x^2+3)\ (x^2+4)} = -\dfrac{Ax+B}{(x^2+3)} + \dfrac{Cx+D}{(x^2+4)}$

$\Rightarrow 4x^2+10 = (Ax+B)\ (x^2+4) + (Cx+D)(x^2+3)$

$\Rightarrow 4x^2+10 = Ax^3 + 4Ax + Bx^2 + 4B + Cx^3 + 3cx + Dx^2 + 3D$

$\Rightarrow 4x^2+10 = (A+C)x^3 + (B+D)x^2 + (4A+3C)x + (4B+3D)$

Equating the coefficients of $x^3$, $x^2$, $x$ and constant term, we have

A + C = 0, B + D = 4, 4A + 3C = 0 and 4B + 3D = 10

On solving these equations, we have

A = 0, B = −2, C = 0 and D = 6

Therefore, $\dfrac{(4x^2+10)}{(x^2+3)\ (x^2+4)} = \dfrac{-2}{(x^2+3)} + \dfrac{6}{(x^2+4)}$

$$\Rightarrow \frac{(x^2+1)\ (x^2+2)}{(x^2+3)\ (x^2+4)} = 1 - \left(\frac{-2}{(x^2+3)} + \frac{6}{(x^2+4)}\right)$$

$$I = \int \frac{(x^2+1)\ (x^2+2)}{(x^2+3)\ (x^2+4)} dx = \int \left\{1 + \left(\frac{2}{(x^2+3)} - \frac{6}{(x^2+4)}\right)\right\} dx$$

$$= \left\{1 + \frac{2}{x^2+\left(\sqrt{3}\right)^2} + \frac{6}{x^2+2^2}\right\} = x + 2\left(\frac{1}{\sqrt{3}} tan^{-1}\frac{x}{\sqrt{3}}\right) - 6\left(\frac{1}{2} tan^{-1}\frac{x}{2}\right) + C$$

$$= x + \frac{2}{\sqrt{3}} tan^{-1}\frac{x}{\sqrt{3}} - 3tan^{-1}\frac{x}{2} + C$$

**Question 19:**

$$\frac{2x}{(x^2+1)\ (x^2+3)}$$

**Answer 19:**

$$\frac{2x}{(x^2+1)\ (x^2+3)}$$

Let $x^2 = t \quad \Rightarrow 2x\,dx = dt$

$$I = \int \frac{2x}{(x^2+1)\ (x^2+3)} dx = \int \frac{dt}{(t+1)\ (t+3)} \qquad \text{... (1)}$$

Let $\dfrac{1}{(t+1)\ (t+3)} = \dfrac{A}{(t+1)} + \dfrac{B}{(t+3)} \Rightarrow 1 = A(t+3) + B(t+1)$ ... (2)

Equating the coefficients of $t$ and constant, we have

A + B = 0 and 3A + B = 1

On solving, we have

$$A = \frac{1}{2} \qquad \text{and} \qquad b = -\frac{1}{2}$$

Therefore, $$\frac{1}{(t+1)\ (t+3)} = \frac{1}{2(t+1)} - \frac{2}{2\,(t+3)}$$

$$I = \int \frac{2x}{(x^2+1)\ (x^2+3)} dx$$

$$= \int \left\{\frac{1}{2(t+1)} - \frac{2}{2\,(t+3)}\right\} dt$$

$$= \frac{1}{2}\log|t+1| - \frac{1}{2}\log|t+3| + C = \frac{1}{2}\log\left|\frac{t+1}{t+3}\right| + C = \frac{1}{2}\log\left|\frac{x^2+1}{x^2+3}\right| + C$$

**Question 20**:

$$\frac{1}{x(x^4-1)}$$

**Answer 20:**

$$\frac{1}{x(x^4-1)}$$

Multiplying numerator and denominator by $x^3$, we have

$$\frac{1}{x(x^4-1)} = \frac{x^3}{x^4(x^4-1)}$$

$$I = \int \frac{1}{x(x^4-1)} dx = \int \frac{x^3}{x^4(x^4-1)} dx$$

Let $x^4 = t \quad \Rightarrow \quad 4x^3 dx = dt$

$$I = \int \frac{1}{x(x^4-1)} dx = \frac{1}{4}\int \frac{dt}{t(t-1)}$$

Let $\frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{(t-1)}$

$\Rightarrow 1 = A(t-1) + Bt \quad \ldots..(1)$

Equating the coefficients of $t$ and constant, we have

A = −1 and B = 1

Therefore, $$\frac{1}{t(t-1)} = \frac{-1}{t} + \frac{1}{(t-1)}$$

$$I = \int \frac{1}{x(x^4-1)} dx$$

$$= \frac{1}{4}\int \left\{\frac{-1}{t} + \frac{1}{(t-1)}\right\} dt$$

$$= \frac{1}{4}\ [-\log|t| + \log|t-1|] + C$$

$$= \frac{1}{4}\log\left|\frac{t-1}{t}\right| + C = \frac{1}{4}\log\left|\frac{x^4-1}{x^4}\right| + C$$

**Question 21:**

$$\frac{1}{(e^x-1)} \quad \text{Hint: Put } e^x = t$$

**Answer 21:**

$$\frac{1}{(e^x-1)}$$

$Let \quad e^x = t \quad \Rightarrow e^x dx = dt$

$$I = \int \frac{1}{(e^x-1)}dx = \int \frac{1}{(t-1)} \times \frac{dt}{t} = \int \frac{dt}{t(t-1)}$$

Let $\dfrac{1}{t(t-1)} = \dfrac{A}{t} + \dfrac{B}{(t-1)} \quad \Rightarrow 1 = A(t-1) + Bt \qquad \text{... (1)}$

Equating the coefficients of $t$ and constant, we have

A = −1 and B = 1

Therefore, $\dfrac{1}{t(t-1)} = \dfrac{-1}{t} + \dfrac{1}{(t-1)}$

$$I = \int \frac{1}{t(t-1)}dt = \log\left|\frac{t-1}{t}\right| + C = \log\left|\frac{e^x-1}{e^x}\right| + C$$

Choose the correct answer in each of the Exercises 22 and 23.

**Question 22:**

$$\int \frac{x\,dx}{(x-1)(x-2)} \text{ equals}$$

(A) $\log\left|\frac{(x-1)^2}{x-2}\right| + C$

(B) $\log\left|\frac{(x-2)^2}{x-2}\right| + C$

(C) $\log\left|\left(\frac{x-1}{x-2}\right)^2\right| + C$

(D) $\log|(x-1)(x-2)| + C$

**Answer 22:**

Let $\dfrac{x}{(x-1)(x-2)} = \dfrac{A}{(x-1)} + \dfrac{B}{(x-2)} \quad \Rightarrow x = A(x-2) + B(x-1)$

Equating the coefficients of $x$ and constant, we have

A + B = 1 and $-2A - B = 0$

Solving the equations, we have

A = −1 and B = 2

Therefore, $\dfrac{x}{(x-1)(x-2)} = -\dfrac{1}{(x-1)} + \dfrac{1}{(x-2)}$

$$I = \int \frac{x}{(x-1)(x-2)}dx = \int \left\{-\frac{1}{(x-1)} + \frac{1}{(x-2)}\right\}dx$$

$= -\log|x-1| + 2\log|x-2| + C$

$= \log\left|\dfrac{(x-2)^2}{x-1}\right| + C$

Hence, the correct Answer is (B).

**Question 23:**

$$\int \frac{dx}{x(x^2+1)} \text{ equals}$$

(A) $\log|x| - \frac{1}{2}\log(x^2+1) + C$

(B) $\log|x| + \frac{1}{2}\log(x^2+1) + C$

(C) $-\log|x| + \frac{1}{2}\log(x^2+1) + C$

(D) $\frac{1}{2}\log|x| + \log(x^2+1) + C$

**Answer 23:**

$$\text{Let } \frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{(x^2+1)}$$

$\Rightarrow 1 = A(x^2+1) + (Bx+C)x$

Equating the coefficients of $x^2$, $x$ and constant term, we have

A + B = 0, C = 0 and A = 1

On solving these equations, we have

A = 1, B = −1 and C = 0

$$\text{Therefore, } \frac{1}{x(x^2+1)} = \frac{1}{x} + \frac{-x}{(x^2+1)}$$

$I = \int \frac{1}{x(x^2+1)} dx$

$= \int \left\{\frac{1}{x} + \frac{-x}{(x^2+1)}\right\} dt$

$= \log|x| - \frac{1}{2}\log|x^2+1| + C$

Hence, the correct Answer is (A).

# Mathematics

**(Chapter – 7) (Integrals) (Exercise 7.6)**
**(Class – XII)**

Integrate the functions in Exercises 1 to 22

**Question 1:**

$$x \sin x$$

**Answer 1:**

$$I = \int x \sin x \, dx$$

Taking x as first function and sin x as second function and integrating by parts, we have

$$I = x\int \sin x \, dx - \int \left\{\left(\frac{d}{dx}x\right)\int \sin x\right\} dx$$

$$= x(-\cos x) - \int 1.(-\cos x)dx = -x\cos x + \sin x + C$$

**Question 2:**

$$x \sin 3x$$

**Answer 2:**

$$I = \int x \sin 3x \, dx$$

Taking x as first function and sin 3x as second function and integrating by parts, we have

$$I = x\int \sin 3x \, dx - \int \left\{\left(\frac{d}{dx}x\right)\int \sin 3x dx\right\} dx$$

$$= x\left(\frac{-\cos 3x}{3}\right) - \int 1.\left(\frac{-\cos 3x}{3}\right)dx = -\frac{x\cos 3x}{3} + \frac{1}{3}\int \cos 3x \, dx$$

$$= -\frac{x\cos 3x}{3} + \frac{1}{9}\sin 3x + C$$

**Question 3:**

$$x^2 e^x$$

**Answer 3:**

$$I = \int x^2 e^x \, dx$$

Taking $x^2$ as first function and $e^x$ as second function and integrating by parts, we have

$$I = x^2\int e^x \, dx - \int \left\{\left(\frac{d}{dx}x^2\right)\int e^x dx\right\} dx$$

$$= x^2 e^x - \int 2x.e^x dx = x^2 e^x - 2\int x.e^x dx$$

Again integrating by parts, we have

$$I = x^2 e^x - 2\left[x.\int e^x dx - \int \left\{\left(\frac{d}{dx}x^2\right)\int e^x dx\right\} dx\right]$$

$$= x^2 e^x - 2\left[xe^x - \int e^x dx\right] = x^2 e^x - 2[xe^x - e^x]$$

$$= x^2 e^x - 2xe^x + 2e^x + C$$

$$= e^x(x^2 - 2x + 2) + C$$

**Question 4:**

$$x \log x$$

**Answer 4:**

$$I = \int x \log x \, dx$$

Taking $\log x$ as first function and x as second function and integrating by parts, we have

$I = \log x \int x \, dx - \int \left\{ \left( \frac{d}{dx} \log x \right) \int x dx \right\} dx$

$= \log x . \frac{x^2}{2} - \int \frac{1}{x} . \frac{x^2}{2} dx \ = \frac{x^2 \log 2x}{2} - \int \frac{x}{2} dx = \frac{x^2 \log 2x}{2} - \frac{x^4}{4} + C$

**Question 5:**

$$x \log 2x$$

**Answer 5:**

$$I = \int x \log 2x \, dx$$

Taking $\log 2x$ as first function and $x$ as second function and integrating by parts, we have

$I = \log 2x \int x \, dx - \int \left\{ \left( \frac{d}{dx} \log 2x \right) \int x dx \right\} dx$

$= \log 2x . \frac{x^2}{2} - \int \frac{2}{2x} . \frac{x^2}{2} dx = \frac{x^2 \log 2x}{2} - \int \frac{x}{2} dx = \frac{x^2 \log 2x}{2} - \frac{x^4}{4} + C$

**Question 6:**

$$x^2 \log x$$

**Answer 6:**

$$I = \int x^2 \log x \, dx$$

Taking $\log x$ as first function and $x^2$ as second function and integrating by parts, we have

$I = \log x \int x^2 \, dx - \int \left\{ \left( \frac{d}{dx} \log x \right) \int x^2 dx \right\} dx$

$= \log x . \left( \frac{x^3}{3} \right) - \int \frac{1}{x} . \frac{x^3}{3} dx = \frac{x^3 \log x}{3} - \frac{1}{3} \int x^2 dx = \frac{x^3 \log x}{3} - \frac{x^3}{9} + C$

**Question 7:**

$$x \, sin^{-1} x$$

**Answer 7:**

$$I = \int x \, sin^{-1} x \, dx$$

Taking $sin^{-1} x$ as first function and $x$ as second function and integrating by parts, we have

$I = sin^{-1} x \int x \, dx - \int \left\{ \left( \frac{d}{dx} sin^{-1} x \right) \int x \, dx \right\} dx$

$= sin^{-1} x \left( \frac{x^2}{2} \right) - \int \frac{1}{\sqrt{1 - x^2}} . \frac{x^2}{2} dx = \frac{x^2 sin^{-1} x}{2} + \frac{1}{2} \int \frac{-x^2}{\sqrt{1 - x^2}} dx$

$$= \frac{x^2 sin^{-1}x}{2} + \frac{1}{2}\int \left\{\frac{1-x^2}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}}\right\} dx = \frac{x^2 sin^{-1}x}{2} + \frac{1}{2}\int \left\{\sqrt{1-x^2} - \frac{1}{\sqrt{1-x^2}}\right\} dx$$

$$= \frac{x^2 sin^{-1}x}{2} + \frac{1}{2}\left\{\int \sqrt{1-x^2}dx - \int \frac{1}{\sqrt{1-x^2}}\ dx\right\}$$

$$= \frac{x^2 sin^{-1}x}{2} + \frac{1}{2}\left\{\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2} sin^{-1}x - sin^{-1}x\right\} + C$$

$$= \frac{x^2 sin^{-1}x}{2} + \frac{x}{4}\sqrt{1-x^2} + \frac{1}{2} sin^{-1}x - sin^{-1}x + C$$

$$= \frac{1}{4}(2x^2-1) sin^{-1}x + \frac{x}{4}\sqrt{1-x^2} + C$$

**Question 8:**

$$x\ tan^{-1}x$$

**Answer 8:**

$$I = \int x\ tan^{-1}x\ dx$$

Taking $tan^{-1}x$ as first function and $x$ as second function and integrating by parts, we have

$$I = tan^{-1}x \int x\ dx - \int \left\{\left(\frac{d}{dx} tan^{-1}x\right)\int x\ dx\right\} dx$$

$$= tan^{-1}x\left(\frac{x^2}{2}\right) - \int \frac{1}{1+x^2}.\frac{x^2}{2} dx = \frac{x^2 tan^{-1}x}{2} - \frac{1}{2}\int \frac{x^2}{1+x^2} dx$$

$$= \frac{x^2 tan^{-1}x}{2} - \frac{1}{2}\int \left\{\frac{x^2+1}{1+x^2} - \frac{1}{1+x^2}\right\} dx = \frac{x^2 tan^{-1}x}{2} - \frac{1}{2}\left(1 - \frac{1}{1+x^2}\right) dx$$

$$= \frac{x^2 tan^{-1}x}{2} - \frac{1}{2}(x - tan^{-1}x) = \frac{x^2}{2} tan^{-1}x - \frac{x}{2} + \frac{1}{2} tan^{-1}x + C$$

**Question 9:**

$$x\ cos^{-1}x$$

**Answer 9:**

$$I = \int x\ cos^{-1}x\ dx$$

Taking $cos^{-1}x$ as first function and $x$ as second function and integrating by parts, we have

$$I = cos^{-1}x \int x\ dx - \int \left\{\left(\frac{d}{dx} cos^{-1}x\right)\int x\ dx\right\} dx$$

$$= cos^{-1}x\left(\frac{x^2}{2}\right) - \int \frac{-1}{\sqrt{1-x^2}}.\frac{x^2}{2} dx = \frac{x^2 cos^{-1}x}{2} - \frac{1}{2}\int \frac{1-x^2-1}{\sqrt{1-x^2}} dx$$

$$= \frac{x^2 cos^{-1}x}{2} - \frac{1}{2}\int \left\{\frac{1-x^2}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}}\right\} dx$$

$$= \frac{x^2 cos^{-1}x}{2} - \frac{1}{2}\int \left\{\sqrt{1-x^2} + \left(\frac{-1}{\sqrt{1-x^2}}\right)\right\} dx$$

$$= \frac{x^2 cos^{-1}x}{2} - \frac{1}{2}\int \sqrt{1-x^2}\, dx - \frac{1}{2}\int \left(\frac{-1}{\sqrt{1-x^2}}\right) dx$$

$$= \frac{x^2 cos^{-1}x}{2} - \frac{1}{2} I_1 - \frac{1}{2} cos^{-1}x \qquad \dots(1)$$

Where $I_1 = \int \sqrt{1-x^2} . 1\, dx$

$$I_1 = x\sqrt{1-x^2} - \int \frac{d}{dx}\sqrt{1-x^2} \int x\, dx \ = x\sqrt{1-x^2} - \int \frac{-2x}{\sqrt{1-x^2}}\, x\, dx$$

$$= x\sqrt{1-x^2} - \int \frac{1-x^2-1}{\sqrt{1-x^2}}\, x\, dx \ = x\sqrt{1-x^2} - \left\{\int \sqrt{1-x^2}\, dx + \int \frac{-dx}{\sqrt{1-x^2}}\right\}$$

$$I_1 = x\sqrt{1-x^2} - \{I_1 + cos^{-1}x\}$$

$$\Rightarrow 2I_1 = x\sqrt{1-x^2} - cos^{-1}x \quad \Rightarrow I_1 = \frac{x}{2}\sqrt{1-x^2} - \frac{1}{2} cos^{-1}x$$

Substituting in (1), we have

$$I = \frac{x^2 cos^{-1}x}{2} - \frac{1}{2}\left(\frac{x}{2}\sqrt{1-x^2} - \frac{1}{2} cos^{-1}x\right) - \frac{1}{2} cos^{-1}x$$

$$= \frac{(2x^2-1)}{4} cos^{-1}x - \frac{x}{4}\sqrt{1-x^2} + C$$

**Question 10:**

$$(sin^{-1}x)^2$$

**Answer 10:**

$$I = \int (sin^{-1}x)^2 \ .1\, dx$$

Taking $(sin^{-1}x)^2$ as first function and 1 as second function and integrating by parts, we have

$$I = (sin^{-1}x)^2 \int 1\, dx - \int \left\{\frac{d}{dx}(sin^{-1}x)^2 . \int 1\, dx\right\} dx$$

$$= (sin^{-1}x)^2 . x - \int \frac{2 sin^{-1}x}{\sqrt{1-x^2}} . x\, dx = x(sin^{-1}x)^2 + \int sin^{-1}x \left(\frac{-2x}{\sqrt{1-x^2}}\right) dx$$

$$= x(sin^{-1}x)^2 + \left[sin^{-1}x \int \frac{-2x}{\sqrt{1-x^2}} dx - \int \left\{\left(\frac{d}{dx} sin^{-1}x\right) \int \frac{-2x}{\sqrt{1-x^2}} dx\right\} dx\right]$$

$$= x(sin^{-1}x)^2 + \left[sin^{-1}x 2\sqrt{1-x^2} - \int \frac{1}{\sqrt{1-x^2}} . 2\sqrt{1-x^2} dx\right]$$

$$= x(sin^{-1}x)^2 + 2\sqrt{1-x^2}\, sin^{-1}x - \int 2dx \ = x(sin^{-1}x)^2 + 2\sqrt{1-x^2}\, sin^{-1}x - 2x + C$$

**Question 11:**

$$\frac{x\cos^{-1}x}{\sqrt{1-x^2}}$$

**Answer 11:**

$$I=\int \frac{x\cos^{-1}x}{\sqrt{1-x^2}}dx=\int \frac{-1}{2}\frac{-2x}{\sqrt{1-x^2}}.\cos^{-1}x\,dx$$

Taking $\cos^{-1}x$ as first function and $\left(\frac{-2x}{\sqrt{1-x^2}}\right)$ as second function and integrating by parts, we have

$$I=\frac{-1}{2}\left[\cos^{-1}x\int \frac{-2x}{\sqrt{1-x^2}}dx-\int\left\{\left(\frac{d}{dx}\cos^{-1}x\right)\int \frac{-2x}{\sqrt{1-x^2}}dx\right\}dx\right]$$

$$=\frac{-1}{2}\left[\cos^{-1}x.2\sqrt{1-x^2}-\int \frac{-1}{\sqrt{1-x^2}}.2\sqrt{1-x^2}dx\right]$$

$$=\frac{-1}{2}\left[2\sqrt{1-x^2}\cos^{-1}x+\int 2\,dx\right]$$

$$=\frac{-1}{2}\left[2\sqrt{1-x^2}\cos^{-1}x+2x\right]+C$$

$$=-\left[\sqrt{1-x^2}\cos^{-1}x+x\right]+C$$

**Question 12:**

$$x\sec^2x$$

**Answer 12:**

$$I=\int x\sec^2xdx$$

Taking $x$ as first function and sec $2x$ as second function and integrating by parts, we have

$$I=x\int \sec^2xdx-\int\left\{\left\{\frac{d}{dx}x\right\}\int \sec^2x\,dx\right\}dx$$

$$=x\tan x-\int 1.\tan x\,dx=x\tan x+\log|\cos x|+C$$

**Question 13:**

$$\tan^{-1}x$$

**Answer 13:**

$$I=\int \tan^{-1}x$$

Taking $\tan^{-1}x$ as first function and 1 as second function and integrating by parts, we have

$$I=\tan^{-1}x\int 1dx-\int\left\{\left\{\frac{d}{dx}\tan^{-1}x\right\}\int 1.dx\right\}dx$$

$$=\tan^{-1}x.x-\int \frac{1}{1+x^2}x\,dx=x\tan^{-1}x-\frac{1}{2}\int \frac{2x}{1+x^2}dx$$

$$=x\tan^{-1}x-\frac{1}{2}\log[1+x^2]+C$$

## Question 14:

$$x(\log x)^2$$

## Answer 14:

$$I = \int x(\log x)^2 dx$$

Taking$(\log x)^2$ as first function and $x$ as second function and integrating by parts, we have

$$I = (\log x)^2 \int xdx - \int \left\{ \left\{ \frac{d}{dx}(\log x)^2 \right\} \int xdx \right\} dx$$

$$= \frac{x^2}{2}(\log x)^2 - \left[ \int 2\log x \frac{1}{x} . \frac{x^2}{2} dx \right] = \frac{x^2}{2}(\log x)^2 - \int x \log x \, dx$$

Again integrating by parts, we have

$$= \frac{x^2}{2}(\log x)^2 - \left[ \log x \int xdx - \int \left\{ \left\{ \frac{d}{dx} \log x \right\} \int x \, dx \right\} dx \right]$$

$$= \frac{x^2}{2}(\log x)^2 - \left[ \frac{x^2}{2} - \log x - \int \frac{1}{2} . \frac{x^2}{2} dx \right]$$

$$= \frac{x^2}{2}(\log x)^2 - \frac{x^2}{2} \log x + \frac{x^2}{4} + C$$

## Question 15:

$$(x^2 + 1) \log x$$

## Answer 15:

$$I = \int (x^2 + 1) \log x = \int x^2 \log x \, dx + \int \log x \, dx$$

Let $I = I_1 + I_2$ ...(1)

Let $I_1 = \int x^2 \log x \, dx$ and $I_2 = \int \log x \, dx$

Taking $\log x$ as first function and $x^2$ as second function and integrating by parts, we have

$$I_1 = \log x \int x^2 \, dx - \int \left\{ \left\{ \frac{d}{dx} \log x \right\} \int x^2 \, dx \right\} dx$$

$$= \log x . \frac{x^3}{3} - \int \frac{1}{x} . \frac{x^3}{3} dx = \frac{x^3}{3} \log x - \frac{1}{3} \left( \int x^2 \, dx \right) = \frac{x^3}{3} \log x - \frac{x^3}{9} + C_1 \quad ...(2)$$

Taking $\log x$ as first function and 1 as second function and integrating by parts, we have

$$I_2 = \log x \int 1 \, dx - \int \left\{ \left\{ \frac{d}{dx} \log x \right\} \int 1 \, dx \right\} dx$$

$$= \log x . x - \int \frac{1}{x} . x \, dx = x \log x - \int 1 \, dx = x \log x - x + C_2 \quad ...(3)$$

Using equations (2) and (3) in (1), we have

$$I = \frac{x^3}{3} \log x - \frac{x^3}{9} + C_1 + x \log x - x + C_2$$

$$= \frac{x^3}{3} \log x - \frac{x^3}{9} + x \log x - x + C) \quad [C = C_1 + C_2]$$

**Question 16:**

$$e^x \ (\sin x + \cos x)$$

**Answer 16:**

$$I = \int e^x \ (\sin x + \cos x) dx$$

Let $f(x) = \sin x \quad \Rightarrow f'(x) = \cos x$

Here, $\quad I = \int e^x \ (\sin x + \cos x) dx = \int e^x \{f(x) + f'(x)\} dx$

It is known that, $\quad \int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$

Therefore, $\quad I = e^x \, sinx + C$

**Question 17:**

$$\frac{xe^x}{(1+x)^2}$$

**Answer 17:**

$$I = \int \frac{xe^x}{(1+x)^2} dx = \int e^x \left\{\frac{x}{(1+x)^2}\right\} dx$$

$$= \int e^x \left\{\frac{1+x-1}{(1+x)^2}\right\} dx = \int e^x \left\{\frac{1}{1+x} - \frac{1}{(1+x)^2}\right\} dx$$

Let $f(x) = \frac{1}{1+x} \quad \Rightarrow f'(x) = \frac{-1}{(1+x)^2}$

$$I = \int \frac{xe^x}{(1+x)^2} dx = \int e^x \left\{\frac{1}{1+x} - \frac{1}{(1+x)^2}\right\} dx = \int e^x \{f(x) + f'(x)\} dx$$

It is known that, $\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$

$$I = \int \frac{xe^x}{(1+x)^2} dx = \frac{e^x}{1+x} + C$$

**Question 18:**

$$e^x \left(\frac{1+\sin x}{1+\cos x}\right)$$

**Answer 18:**

$$e^x \left(\frac{1+\sin x}{1+\cos x}\right) = e^x \left(\frac{sin^2\frac{x}{2} + \ cos^2\frac{x}{2} + 2sin\frac{x}{2}\ cos\frac{x}{2}}{2cos^2\frac{x}{2}}\right)$$

$$= e^x \frac{\left(sin\frac{x}{2} + cos\frac{x}{2}\right)^2}{2cos^2\frac{x}{2}} = \frac{1}{2} e^x \left(\frac{sin\frac{x}{2} + cos\frac{x}{2}}{cos\frac{x}{2}}\right)^2$$

$$= \frac{1}{2} e^x \ \left[tan\frac{x}{2} + 1\right]^2$$

$$=\frac{1}{2}e^x\left[1+tan\frac{x}{2}\right]^2=\frac{1}{2}e^x\left[1+tan^2\frac{x}{2}+2tan\frac{x}{2}\right]$$

$$=\frac{1}{2}e^x\left[sec^2\frac{x}{2}+2tan\frac{x}{2}\right]$$

$$I=\int\frac{e^x(1+sinx)}{(1+\cos x)}dx=\int e^x\left[sec^2\frac{x}{2}+2tan\frac{x}{2}\right]dx$$

Let $2tan\frac{x}{2}=f(x) \quad \Rightarrow \quad f'(x)=\sec^2\frac{x}{2}$

$$I=\int e^x\{f'(x)+f(x)\}\,dx$$

It is known that, $\int e^x\{f(x)+f'(x)\}\,dx=e^x f(x)+C$

$$I=\int\frac{e^x(1+sinx)}{(1+\cos x)}dx=e^x\,tan\frac{x}{2}+C$$

**Question 19:**

$$e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)$$

**Answer 19:**

$$I=\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx$$

Let $\frac{1}{x}=f(x) \quad \Rightarrow \quad f'(x)=\frac{-1}{x^2}$

It is known that $\int e^x\{f(x)+f'(x)\}\,dx=e^x f(x)+C$

$$I=\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx=\frac{e^x}{x}+C$$

**Question 20:**

$$\frac{(x-3)\,e^x}{(x-1)^3}$$

**Answer 20:**

$$\int e^x\left\{\frac{x-3}{(x-1)^3}\right\}dx=\int e^x\left\{\frac{x-1-2}{(x-1)^3}\right\}dx$$

$$=\int e^x\left\{\frac{1}{(x-1)^2}-\frac{2}{(x-1)^3}\right\}dx$$

Let $f(x)=\frac{1}{(x-1)^2} \quad f'(x)=\frac{-2}{(x-1)^3}$

It is known that,

$$\int e^x\{f(x)+f'(x)\}dx=e^x f(x)+C$$

$$\therefore\int e^x\left\{\frac{(x-3)}{(x-1)^2}\right\}dx=\frac{e^x}{(x-1)^2}+C$$

**Question 21:**

$$e^{2x}\sin x$$

**Answer 21:**

$$I = \int e^{2x}\sin x\,dx \qquad \ldots(1)$$

Integrating by parts, we have

$$I = \sin x\int e^{2x}\,dx - \int\left\{\left(\frac{d}{dx}\sin x\right)\int e^{2x}\,dx\right\}dx$$

$$I = \sin x\,.\frac{e^{2x}}{2} - \int \cos x\frac{e^{2x}}{2}dx = \frac{e^{2x}\sin x}{2} - \frac{1}{2}\int e^{2x}\cos x\,dx$$

Again integrating by parts, we have

$$I = \frac{e^{2x}\sin x}{2} - \frac{1}{2}\left[\cos x\int e^{2x}\,dx - \int\left\{\left(\frac{d}{dx}\cos x\right)\int e^{2x}\,dx\right\}dx\right]$$

$$I = \frac{e^{2x}\sin x}{2} - \frac{1}{2}\left[\cos x.\frac{e^{2x}}{2} - \int(-\sin x)\frac{e^{2x}}{2}dx\right] = \frac{e^{2x}\sin x}{2} - \frac{1}{2}\left[\frac{e^{2x}\cos x}{2} + \frac{1}{2}\int e^{2x}\sin x\,dx\right]$$

$$I = \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4} - \frac{1}{4}I \qquad \text{[From (1)]}$$

$$I + \frac{1}{4}I = \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4} \qquad \Rightarrow \frac{5}{4}I = \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4}$$

$$I = \frac{4}{5}\left[\frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4}\right] + C = \frac{e^{2x}}{5}[2\sin x - \cos x] + C$$

**Question 22:**

$$\sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

**Answer 22:**

Let $x = \tan\theta \qquad \Rightarrow dx = \sec^2\theta\,d\theta$

Therefore, $\sin^{-1}\left(\frac{2x}{1+x^2}\right) = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \sin^{-1}(\sin 2\theta) = 2\theta$

$$I = \int \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx = \int 2\theta\sec^2\theta\,d\theta = 2\int\theta\sec^2\theta\,d\theta$$

Integrating by parts, we have

$$I = 2\left[\theta\int\sec^2\theta\,d\theta - \int\left\{\left(\frac{d}{d\theta}\theta\right)\int\sec^2\theta\,d\theta\right\}d\theta\right] = 2\left[\theta\tan\theta - \int\tan\theta\,d\theta\right]$$

$$= 2[\theta\tan\theta - \log|\cos\theta|] + C = 2\left[x\tan^{-1}x + \log\left|\frac{1}{\sqrt{1+x^2}}\right|\right] + C$$

$$= 2x\tan^{-1}x + 2\log(1+x^2)^{-\frac{1}{2}} + C = 2x\tan^{-1}x + 2\left[-\frac{1}{2}\log(1+x^2)\right] + C$$

$$= 2x\tan^{-1}x - \log(1+x^2) + C$$

**Choose the correct answer in Exercises 23 and 24.**

## Question 23:

$$\int x^2 e^{x^3} dx \quad \text{equals}$$

(A) $\frac{1}{3}e^{x^3} + C$ (B) $\frac{1}{3}e^{x^2} + C$ (C) $\frac{1}{2}e^{x^3} + C$ (D) $\frac{1}{2}e^{x^2} + C$

## Answer 23:

$$I = \int x^2 e^{x^3} dx$$

Let $x^3 = t$

$\Rightarrow 3x^2 dx = dt$

$$I = \frac{1}{3}\int (e^t)dt = \frac{1}{3}(e^t) + C = \frac{1}{3}e^{x^3} + C$$

Hence, the correct answer is (A).

## Question 24:

$$\int e^x \sec x(1 + \tan x)\, dx \text{ equals}$$

(A) $e^x \cos x + C$ (B) $e^x \sec x + C$ (C) $e^x \sin x + C$ (D) $e^x \tan x + C$

## Answer 24:

$$I = \int e^x \sec x(1 + \tan x)\, dx = \int e^x (\sec x + esc\, x \tan x) dx$$

Let $\sec x = f(x)$

$\Rightarrow \sec x\; tan\; x = f'(x)$

It is known that $\int e^x \{f(x) + f'(x)\}\, dx = e^x f(x) + C$

$I = \int e^x (\sec x + esc\, x \tan x) dx$

$= e^x \sec x + C$

Hence, the correct answer is (B).

(Chapter – 7) (Integrals) (Exercise 7.7)
(Class – XII)

**Integrate the functions in Exercises 1 to 9.**

## Question 1:

$$\sqrt{4-x^2}$$

### Answer 1:

$$I=\int \sqrt{4-x^2}\,dx=\int \sqrt{(2)^2-(x)^2}\,dx$$

Using, $\int \sqrt{a^2-x^2}=\frac{x}{2}\int \sqrt{a^2-x^2}+\frac{a^2}{2}sin^{-1}\frac{x}{a}+C$

$$I=\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}sin^{-1}\frac{x}{a}+C=\frac{x}{2}\sqrt{4-x^2}+2sin^{-1}\frac{x}{a}+C$$

## Question 2:

$$\sqrt{1-4x^2}$$

### Answer 2:

$$I=\int \sqrt{1-4x^2}dx=\int \sqrt{(1)^2-(2x)^2}\,dx$$

Let $2x=t\ \ so\ \ 2dx=dt\quad \Rightarrow I=\frac{1}{2}\int \sqrt{(1)^2-(t)^2}\ dt$

Using, $\int \sqrt{a^2-x^2}=\frac{x}{2}\int \sqrt{a^2-x^2}+\frac{a^2}{2}sin^{-1}\frac{x}{a}+C$

$$I=\frac{1}{2}\left[\frac{1}{2}\sqrt{1-t^2}+\frac{1}{2}sin^{-1}t\right]+C=\frac{t}{4}\sqrt{1-t^2}+\frac{1}{4}sin^{-1}2x+C$$

$$=\frac{x}{2}\sqrt{1-4x^2}+\frac{1}{4}sin^{-1}2x+C$$

## Question 3:

$$\sqrt{x^2+4x+6}$$

### Answer 3:

$$I=\int \sqrt{x^2+4x+6}\,dx=\int \sqrt{x^2+4x+4+2}\ dx$$

$$=\int \sqrt{(x^2+4x+4)+2}\ dx$$

$$=\int \sqrt{(x+2)^2+\left(\sqrt{2}\right)^2}\,dx$$

Using, $\int \sqrt{x^2+a^2}=\frac{x}{2}\int \sqrt{x^2+a^2}+\frac{a^2}{2}log\left|x+\sqrt{x^2+a^2}\right|+C$

$$I=\frac{(x+2)}{2}\sqrt{x^2+4x+6}+\frac{2}{2}log\left|(x+2)+\sqrt{x^2+4x+6}\right|+C$$

$$=\frac{(x+2)}{2}\sqrt{x^2+4x+6}+2log\left|(x+2)+\sqrt{x^2+4x+6}\right|+C$$

**Question4:**

$$\sqrt{x^2+4x+1}$$

**Answer 4:**

$$I=\int\sqrt{x^2+4x+1}\,dx=\int\sqrt{(x^2+4x+4)-3}\,dx=\int\sqrt{(x+2)^2-\left(\sqrt{3}\right)^2}\,dx$$

Using, $\int\sqrt{x^2-a^2}=\frac{x}{2}\int\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$

$$I=\frac{(x+2)}{2}\sqrt{x^2+4x+1}-\frac{3}{2}\log\left|(x+2)+\sqrt{x^2+4x+1}\right|+C$$

**Question 5:**

$$\sqrt{1-4x-x^2}$$

**Answer 5:**

$$I=\int\sqrt{1-4x-x^2}dx=\int\sqrt{1-(x^2+4x+4-4)}\,dx=\int\sqrt{\left(\sqrt{5}\right)^2-(x+2)^2}\,dx$$

Using, $\int\sqrt{a^2-x^2}=\frac{x}{2}\int\sqrt{a^2-x^2}+\frac{a^2}{2}sin^{-1}\frac{x}{a}+C$

$$I=\frac{(x+2)}{2}\sqrt{1-4x-x^2}+\frac{5}{2}sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right)+C$$

**Question 6:**

$$\sqrt{x^2+4x-5}$$

**Answer 6:**

$$I=\int\sqrt{x^2+4x-5}\,dx=\int\sqrt{(x^2+4x+4)-9}\,dx=\int\sqrt{(x+2)^2-(3)^2}\,dx$$

Using, $\int\sqrt{x^2-a^2}\,dx=\frac{x}{2}\int\sqrt{x^2-a^2}-\frac{a^2}{2}log\left|x+\sqrt{x^2-a^2}\right|+C$

$$I=\frac{(x+2)}{2}\sqrt{x^2+4x-5}-\frac{9}{2}\log\left|(x+2)+\sqrt{x^2+4x-5}\right|+C$$

**Question 7:**

$$\sqrt{1+3x-x^2}$$

**Answer 7:**

$$I=\int\sqrt{1+3x-x^2}\,dx=\int\sqrt{1-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)}\,dx$$

$$=\int\sqrt{\left(1+\frac{9}{4}\right)-\left(x-\frac{3}{2}\right)^2}\,dx=\int\sqrt{\left(\frac{\sqrt{13}}{2}\right)^2-\left(x-\frac{3}{2}\right)^2}\,dx$$

Using, $\int\sqrt{a^2-x^2}=\frac{x}{2}\int\sqrt{a^2-x^2}+\frac{a^2}{2}sin^{-1}\frac{x}{a}+C$

$$I = \frac{(x-\frac{3}{2})}{2}\sqrt{1+3x-x^2} + \frac{13}{4\times 2} sin^{-1}\left(\frac{x-\frac{3}{2}}{\frac{\sqrt{13}}{2}}\right) + C$$

$$= \frac{(2x-3)}{4}\sqrt{1+3x-x^2} + \frac{13}{8} sin^{-1}\left(\frac{2x-3}{\sqrt{13}}\right) + C$$

**Question 8:**

$$\sqrt{x^2+3x}$$

**Answer 8:**

$$I = \int \sqrt{x^2+3x}\, dx = \int \sqrt{x^2+3x+\frac{9}{4}-\frac{9}{4}} dx = \sqrt{\left(x+\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 dx}$$

Using, $\int \sqrt{x^2-a^2}\, dx = \frac{x}{2}\int \sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + C$

$$I = \frac{\left(x+\frac{3}{2}\right)}{2}\sqrt{x^2+3x} - \frac{\frac{9}{4}}{2}\log\left|\left(x+\frac{3}{2}\right)+\sqrt{x^2+3x}\right| + C$$

$$= \frac{(2x+3)}{4}\sqrt{x^2+3x} - \frac{9}{8}\log\left|\left(x+\frac{3}{2}\right)+\sqrt{x^2+3x}\right| + C$$

**Question 9:**

$$\sqrt{1+\frac{x^2}{9}}$$

**Answer 9:**

$$I = \int \sqrt{1+\frac{x^2}{9}} dx = \frac{1}{3}\int \sqrt{9+x^2}\, dx = \frac{1}{3}\int \sqrt{(3)^2+x^2}\, dx$$

Using, $\int \sqrt{x^2+a^2}\, dx = \frac{x}{2}\int \sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + C$

$$I = \frac{1}{3}\left[\frac{x}{2}\sqrt{x^2+9} - \frac{9}{2}\log\left|x+\sqrt{x^2+9}\right|\right] + C = \frac{x}{6}\sqrt{x^2+9} + \frac{3}{2}\log\left|x+\sqrt{x^2+9}\right| + C$$

**Choose the correct answer in Exercises 10 to 11.**

**Question 10:**

$\int \sqrt{1+x^2}\, dx$ is equal to

(A) $\frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}\log\left|x+\sqrt{1+x^2}\right| + C$

(B) $\frac{2}{3}(1+x^2)^{\frac{3}{2}} + C$

(C) $\frac{2}{3}x(1+x^2)^{\frac{1}{2}} + C$

(D) $\frac{x^2}{2}\sqrt{1+x^2} + \frac{1}{2}x^2\log\left|x+\sqrt{1+x^2}\right| + C$

**Answer 10:**

Using, $\int \sqrt{x^2+a^2}\, dx = \frac{x}{2}\int \sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + C$

$$I = \int \sqrt{1+x^2}\, dx = \frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}\log\left|x+\sqrt{1+x^2}\right| + C$$

Hence, the correct answer is (A).

**Question 11:**

$$\int \sqrt{x^2 - 8x + 7}\, dx \quad \text{is equals to}$$

(A) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7} + 9\log\left|x-4+\sqrt{x^2-8x+7}\right| + C$

(B) $\frac{1}{2}(x+4)\sqrt{x^2-8x+7} + 9\log\left|x+4+\sqrt{x^2-8x+7}\right| + C$

(C) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7} - 3\sqrt{2}\log\left|x-4+\sqrt{x^2-8x+7}\right| + C$

(D) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7} - \frac{9}{2}\log\left|x-4+\sqrt{x^2-8x+7}\right| + C$

**Answer 11:**

$$I = \int \sqrt{x^2 - 8x + 7}\, dx$$

$$= \int \sqrt{(x^2 - 8x + 16) - 9}\ dx$$

$$= \int \sqrt{(x-4)^2 - (3)^2}\, dx$$

Using, $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\int \sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + C$

$$I = \frac{(x-4)}{2}\sqrt{x^2 - 8x + 7} - \frac{9}{2}\log\left|(x-4) + \sqrt{x^2 - 8x + 7}\right| + C$$

Hence, the correct answer is (D).

**Evaluate the definite integrals in Exercises 1 to 20.**

## Question 1:

$$\int_{-1}^{1}(x+1)dx$$

### Answer 1:

$$I=\int_{-1}^{1}(x+1)dx=\left[\frac{x^2}{2}+x\right]_{-1}^{1}=\left(\frac{1}{2}+1\right)-\left(\frac{1}{2}-1\right)=\frac{1}{2}+1-\frac{1}{2}+1=2$$

## Question 2:

$$\int_{2}^{3}\frac{1}{x}dx$$

### Answer 2:

$$I=\int\frac{1}{x}dx=\log x\Big|_2^3=\log|3|-\log|2|=\log\left|\frac{3}{2}\right|$$

## Question 3:

$$\int_{1}^{2}(4x^3-5x^2+6x+9)dx$$

### Answer 3:

$$I=\int_{1}^{2}(4x^3-5x^2+6x+9)dx=\left[4\left(\frac{x^4}{4}\right)-5\left(\frac{x^3}{3}\right)+6\left(\frac{x^2}{2}\right)+9(x)\right]_1^2$$

$$=\left[x^4-\frac{5x^3}{3}+3x^2+9x\right]_1^2=\left\{2^4-\frac{5(2)^3}{3}+3(2)^2+9(2)\right\}-\left\{(1)^4-\frac{5(1)^3}{3}+3(1)^2+9(1)\right\}$$

$$=\left\{16-\frac{40}{3}+12+18\right\}-\left\{1-\frac{5}{3}+3+9\right\}=16-\frac{40}{3}+12+18-1+\frac{5}{3}-3-9$$

$$=33-\frac{35}{3}=\frac{64}{3}$$

## Question 4:

$$\int_{0}^{\frac{\pi}{4}}\sin 2x\,dx$$

### Answer 4:

$$I=\int_{0}^{\frac{\pi}{4}}\sin 2x\,dx=\left[\frac{-\cos 2x}{2}\right]_0^{\frac{\pi}{4}}$$

$$=-\frac{1}{2}\left[\cos 2\left(\frac{\pi}{4}\right)-\cos 0\right]$$

$$=-\frac{1}{2}\left[\cos\left(\frac{\pi}{2}\right)-\cos 0\right]$$

$$=-\frac{1}{2}(0-1)=\frac{1}{2}$$

## Question 5:

$$\int_0^{\frac{\pi}{2}} \cos 2x\, dx$$

### Answer 5:

Let $I = \int_0^{\frac{\pi}{2}} \cos 2x dx$

$$\int \cos 2x dx = \left(\frac{\sin 2x}{2}\right) = F(x)$$

Using second fundamental theorem of calculus, we get

$$I = F\left(\frac{\pi}{2}\right) - F(0)$$

$$= \frac{1}{2}\left[\sin 2\left(\frac{\pi}{2}\right) - \sin 0\right] = \frac{1}{2}[\sin \pi - \sin 0]$$

$$= \frac{1}{2}[0 - 0] = 0$$

## Question 6:

$$\int_4^5 e^x dx$$

### Answer 6:

$$I = \int_4^5 e^x dx = [e^x]_4^5 = e^5 - e^4 = e^4(e-1)$$

## Question 7:

$$\int_0^{\frac{\pi}{4}} \tan x\, dx$$

### Answer 7:

$$I = \int_0^{\frac{\pi}{4}} \tan x\, dx = [-\log|\cos x|]_0^{\frac{\pi}{4}}$$

$$= -\log\left|\cos\frac{\pi}{4}\right| + \log|\cos 0| = -\log\left|\frac{1}{\sqrt{2}}\right| + \log|1| = -\log(2)^{-\frac{1}{2}} = \frac{1}{2}\log 2$$

## Question 8:

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x\, dx$$

### Answer 8:

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x\, dx = [\log|\operatorname{cosec} x - \cot x|]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$$

$$= \log\left|\operatorname{cosec}\frac{\pi}{4} - \cot\frac{\pi}{4}\right| - \log\left|\operatorname{cosec}\frac{\pi}{6} - \cot\frac{\pi}{6}\right|$$

$$= \log|\sqrt{2} - 1| - \log|2 - \sqrt{3}|$$

$$= \log\left(\frac{\sqrt{2} - 1}{2 - \sqrt{3}}\right)$$

**Question 9:**

$$\int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

**Answer 9:**

$$I = \int_0^1 \frac{dx}{\sqrt{1-x^2}} = [sin^{-1}x]_0^1$$

$$= sin^{-1}(1) - sin^{-1}(0) = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

**Question 10:**

$$\int_0^1 \frac{dx}{1+x^2}$$

**Answer 10:**

$$I = \int_0^1 \frac{dx}{1+x^2} = [tan^{-1}x]_0^1 = tan^{-1}(1) - tan^{-1}(0) = \frac{\pi}{4}$$

**Question 11:**

$$\int_2^3 \frac{dx}{x^2-1}$$

**Answer 11:**

Let $I = \int_2^3 \frac{dx}{x^2-1}$

$$\int \frac{dx}{x^2-1} = \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| = F(x)$$

Using second fundamental theorem of calculus, we get

$$I = F(3) - F(2)$$

$$= \frac{1}{2}\left[\log\left|\frac{3-1}{3+1}\right| - \log\left|\frac{2-1}{2+1}\right|\right] = \frac{1}{2}\left[\log\left|\frac{2}{4}\right| - \log\left|\frac{1}{3}\right|\right]$$

$$= \frac{1}{2}\left[\log\frac{1}{2} - \log\frac{1}{3}\right]$$

$$= \frac{1}{2}\left[\log\frac{3}{2}\right]$$

**Question 12:**

$$\int_0^{\frac{\pi}{2}} cos^2x\, dx$$

**Answer 12:**

$$I = \int_0^{\frac{\pi}{2}} cos^2x\, dx = \int_0^{\frac{\pi}{2}} \left(\frac{1+\cos 2x}{2}\right) dx$$

$$= \left[\frac{x}{2} + \frac{\sin 2x}{4}\right]_0^{\frac{\pi}{2}} = \left[\frac{1}{2}\left(x + \frac{\sin 2x}{2}\right)\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2}\left[\left(\frac{\pi}{2} - \frac{\sin \pi}{2}\right) - \left(0 + \frac{\sin \pi}{2}\right)\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{2} + 0 - 0 - 0\right] = \frac{\pi}{4}$$

**Question 13:**

$$\int_2^3 \frac{x\,dx}{x^2+1}$$

**Answer 13:**

$$I=\int_2^3 \frac{x}{x^2+1}dx=\frac{1}{2}\int_2^3 \frac{2x}{x^2+1}dx=\left[\frac{1}{2}\log(1+x^2)\right]_2^3$$

$$=\frac{1}{2}[\log\{1+(3)^2\}-\log\{1+(2)^2\}]=\frac{1}{2}\ [\log(10)-\log(5)]\ =\frac{1}{2}\left[\log\frac{10}{5}\right]=\frac{1}{2}\log 2$$

**Question14:**

$$\int_0^1 \frac{2x+3}{5x^2+1}dx$$

**Answer 14:**

$$I=\int_0^1 \frac{2x+3}{5x^2+1}dx$$

Here, $\int \frac{2x+3}{5x^2+1}dx=\frac{1}{5}\int \frac{5(2x+3)}{5x^2+1}dx=\frac{1}{5}\int \frac{10x+15}{5x^2+1}dx=\frac{1}{5}\int \frac{10x}{5x^2+1}dx+3\int \frac{1}{5x^2+1}dx$

$$=\frac{1}{5}\int \frac{10x}{5x^2+1}dx+3\int \frac{1}{5\left(x^2+\frac{1}{5}\right)}dx=\frac{1}{5}\log(5x^2+1)+\frac{3}{5}.\frac{1}{\frac{1}{\sqrt{5}}}tan^{-1}\frac{x}{\frac{1}{\sqrt{5}}}$$

$$=\frac{1}{5}\log(5x^2+1)+\frac{3}{\sqrt{5}}tan^{-1}(\sqrt{5}x)$$

Therefore, $I=\int_0^1 \frac{2x+3}{5x^2+1}dx=\left\{\frac{1}{5}\log(5+1)+\frac{3}{\sqrt{5}}tan^{-1}(\sqrt{5})\right\}-\left\{\frac{1}{5}\log(1)+\frac{3}{\sqrt{5}}tan^{-1}(0)\right\}$

$$=\frac{1}{5}\log(6)+\frac{3}{\sqrt{5}}tan^{-1}(\sqrt{5})$$

**Question 15:**

$$\int_0^1 xe^{x^2}dx$$

**Answer 15:**

$$I=\int_0^1 xe^{x^2}dx$$

Put $x^2=t$

$\Rightarrow\ 2xdx=dt$

If $x\to 0, t\to 0$ and if $x\to 1, t\to 1$

Therefore, $I=\frac{1}{2}\int_0^1 e^t dt=\frac{1}{2}[e^t]_0^1$

$$=\frac{1}{2}e-\frac{1}{2}e^0=\frac{1}{2}(e-1)$$

**Question 16:**

$$\int_1^2 \frac{5x^2}{x^2+4x+3}\,dx$$

**Answer 16:**

$$I = \int_1^2 \frac{5x^2}{x^2+4x+3}dx$$

Dividing $5x^2$ by $x^2+4x+3$, we have

$$I = \int_1^2 \left\{5 - \frac{20x+15}{x^2+4x+3}\right\}dx = \int_1^2 5\,dx - \int_1^2 \frac{20x+15}{x^2+4x+3}dx = [5x]_1^2 - \int_1^2 \frac{20x+15}{x^2+4x+3}dx$$

$I = 5 - I_1$ where $I_1 = \int_1^2 \frac{20x+15}{x^2+4x+3}dx \quad \ldots(1)$

$$\text{Now } I_1 = \int_1^2 \frac{20x+15}{x^2+4x+3}dx$$

Let $20x+15 = A\frac{d}{dx}(x^2+4x+3)+B \quad \Rightarrow 20x+15 = 2Ax + (4A+B)$

Equating the coefficients of $x$ and constant term, we have

A = 10 and B = −25

$$I_1 = 10\int_1^2 \frac{2x+4}{x^2+4x+3}dx - 25\int_1^2 \frac{dx}{x^2+4x+3}$$

Let $x^2+4x+3 = t \quad \Rightarrow (2x+4)dx = dt$

$$I_1 = 10\int \frac{dt}{t} - 25\int \frac{dx}{(x+2)^2-1^2} = 10\log t - 25\left[\frac{1}{2}\log\left(\frac{x+2-1}{x+2+1}\right)\right]$$

$$= [10\log(x^2+4x+3)]_1^2 - 25\left[\frac{1}{2}\log\left(\frac{x+1}{x+3}\right)\right]_1^2 = [10\log 15 - 10\log 8] - 25\left[\frac{1}{2}\log\frac{3}{5} - \frac{1}{2}\log\frac{2}{4}\right]$$

$$= [10\log(5\times 3) - 10\log(4\times 2)] - \frac{25}{2}[\log 3 - \log 5 - \log 2 + \log 4]$$

$$= [10\log 5 + 10\log 3) - 10\log 4 + 10\log 2] - \frac{25}{2}[\log 3 - \log 5 - \log 2 + \log 4]$$

$$= \left[10 + \frac{25}{2}\right]\log 5 + \left[-10 - \frac{25}{2}\right]\log 4 + \left[10 - \frac{25}{2}\right]\log 3 + \left[-10 + \frac{25}{2}\right]\log 2$$

$$= \frac{45}{2}\log 5 - \frac{45}{2}\log 4 - \frac{5}{2}\log 3 + \frac{5}{2}\log 2 = \frac{45}{2}log\frac{5}{4} - \frac{5}{2}\log\frac{3}{2}$$

Substituting the value of $I_1$ in (1), we have

$$I = 5 - I_1 = 5 - \left[\frac{45}{2}log\frac{5}{4} - \frac{5}{2}\log\frac{3}{2}\right]$$

$$= 5 - \frac{5}{2}\left[9\,log\frac{5}{4} - \log\frac{3}{2}\right]$$

## Question 17:

$$\int_0^{\frac{\pi}{4}} (2sec^2 x + x^3 + 2)\, dx$$

## Answer 17:

$$I = \int_0^{\frac{\pi}{4}} (2sec^2 x + x^3 + 2)\, dx = \left[2\tan x + \frac{x^4}{4} + 2x\right]_0^{\frac{\pi}{4}}$$

$$= \left[\left\{2\tan\frac{\pi}{4} + \frac{1}{4}\left(\frac{\pi}{4}\right)^4 + 2\left(\frac{\pi}{4}\right)\right\} - \{2\tan 0 + 0 + 0\}\right]$$

$$= 2\tan\frac{\pi}{4} + \frac{\pi^4}{4^5} + \frac{\pi}{2} = 2 + \frac{\pi}{2} + \frac{\pi^4}{1024}$$

## Question 18:

$$\int_0^{\pi} (\sin^2\frac{x}{2} - \cos^2\frac{x}{2})\, dx$$

## Answer 18:

Let $I = \int_0^{\pi}\left(\sin^2\frac{x}{2} - \cos^2\frac{x}{2}\right)dx = -\int_0^{\pi}\left(\cos^2\frac{x}{2} - \sin^2\frac{x}{2}\right)dx$

$= -\int_0^{\pi}\cos x dx$

$\int \cos x dx = \sin x = F(x)$

Using second fundamental theorem of calculus, we get

$I = F(\pi) - F(0)$

$= \sin\pi - \sin 0$

$= 0$

## Question 19:

$$\int_0^2 \frac{6x+3}{x^2+4}\, dx$$

## Answer 19:

$$I = \int_0^2 \frac{6x+3}{x^2+4}\,dx = 3\int_0^2 \frac{2x+1}{x^2+4}\,dx = 3\int_0^2 \frac{2x}{x^2+4}\,dx + 3\int_0^2 \frac{1}{x^2+4}$$

$$= \left[3\log(x^2+4) + \frac{3}{2}tan^{-1}\frac{x}{2}\right]_0^2$$

$$= \left\{3\log(2^2+4) + \frac{3}{2}tan^{-1}\left(\frac{2}{2}\right)\right\} - \left\{3\log(0+4) + \frac{3}{2}tan^{-1}\left(\frac{0}{2}\right)\right\}$$

$$= 3\log 8 + \frac{3}{2}tan^{-1}1 - 3\log 4 - \frac{3}{2}tan^{-1}0$$

$$= 3\log 8 + \frac{3}{2}tan^{-1}\left(\frac{\pi}{4}\right) - 3\log 4 - 0$$

$$= 3\log\left(\frac{8}{4}\right) + \frac{3\pi}{4} = 3\log 2 + \frac{3\pi}{4}$$

**Question 20:**

$$\int_0^1 \left(xe^x + sin\frac{\pi x}{4}\right) dx$$

**Answer 20:**

$$I = \int_0^1 \left(xe^x + sin\frac{\pi x}{4}\right) dx$$

Here,

$$\int \left(xe^x + sin\frac{\pi x}{4}\right) dx = x\int e^x dx - \int \left\{\left(\frac{d}{dx}x\right)\int e^x dx\right\} dx + \left\{\frac{-\cos\frac{\pi x}{4}}{\frac{\pi}{4}}\right\}$$

$$= xe^x - \int e^x dx - \frac{4}{\pi}\cos\frac{\pi x}{4}$$

$$= xe^x - e^x - \frac{4}{\pi}cos\frac{\pi x}{4}$$

Therefore,

$$I = \left[xe^x - e^x - \frac{4}{\pi}cos\frac{\pi x}{4}\right]_0^1$$

$$= \left(1.e^1 - e^1 - \frac{4}{\pi}cos\frac{\pi}{4}\right) - \left(0.e^0 - e^0 - \frac{4}{\pi}cos\,0\right)$$

$$= e - e - \frac{4}{\pi}\left(\frac{1}{\sqrt{2}}\right) + 1 + \frac{4}{\pi}$$

$$= 1 + \frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}$$

**Choose the correct answer in Exercises 21 and 22.**

**Question 21:**

$$\int_1^{\sqrt{3}} \frac{dx}{1+x^2} \quad \text{equals}$$

(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{12}$

**Answer 21:**

$$\int_1^{\sqrt{3}} \frac{dx}{1+x^2} = [\tan^{-1} x]_1^{\sqrt{3}}$$

$$= tan^{-1}\sqrt{3} - tan^{-1}1$$

$$= \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$$

Hence, the correct answer is (D).

**Question 22:**

$$\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2} \text{ equals}$$

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{12}$ (C) $\frac{\pi}{24}$ (D) $\frac{\pi}{4}$

**Answer 22:**

$$\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2} = \int_0^{\frac{2}{3}} \frac{dx}{2^2+(3x)^2}$$

Let $3x = t$

$\Rightarrow \ 3dx = dt$

Therefore,

$$\int \frac{dx}{(2)^2+(3x)^2} = \frac{1}{3}\int \frac{dx}{(2)^2+(t)^2}$$

$$= \frac{1}{3}\left[\frac{1}{2} tan^{-1} \frac{t}{2}\right]$$

$$= \frac{1}{6} tan^{-1}\left(\frac{3x}{2}\right)$$

Now,

$$I = \left[\frac{1}{6} tan^{-1}\left(\frac{3x}{2}\right)\right]_0^{\frac{2}{3}}$$

$$= \frac{1}{6} tan^{-1}\left(\frac{3}{2}.\frac{2}{3}\right) - \frac{1}{6} tan^{-1} 0$$

$$= \frac{1}{6} \times \frac{\pi}{4} = \frac{\pi}{24}$$

Hence, the correct answer is (C).

**Evaluate the integrals in Exercises 1 to 8 using substitution.**

## Question 1:

$$\int_0^1 \frac{x}{x^2+1}dx$$

### Answer 1:

$$I = \int_0^1 \frac{x}{x^2+1}dx$$

Let $x^2+1=t$
$\Rightarrow$ $2x\,dx = dt$
When $x = 0, t = 1$ and when $x = 1, t = 2$

$$I = \int_0^1 \frac{x}{x^2+1}dx = \frac{1}{2}\int_1^2 \frac{dt}{t} = \frac{1}{2}[\log|t|]_1^2$$

$$= \frac{1}{2}[\log 2 - \log 1] = \frac{1}{2}\log 2$$

## Question 2:

$$\int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\, cos^5\phi\; d\phi$$

### Answer 2:

$$I = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\, cos^5\phi\; d\phi = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\, cos^4\phi \cos\phi\; d\phi$$

Let $\sin\phi = t \quad \Rightarrow \quad \cos\phi\; d\phi = dt$

When $\phi = 0, t = 0$ and when $\phi = \frac{\pi}{2}, t = 1$

$$I = \int_0^1 \sqrt{t}\,(1-t^2)^2\,dt = \int_0^1 t^{\frac{1}{2}}(1+t^4-2t^2)dt = \int_0^1 \left[t^{\frac{1}{2}} + t^{\frac{9}{2}} - 2t^{\frac{5}{2}}\right]dt$$

$$= \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{t^{\frac{11}{2}}}{\frac{11}{2}} - \frac{t^{\frac{7}{2}}}{\frac{7}{2}}\right]_0^1 = \frac{2}{3}+\frac{2}{11}-\frac{4}{7} = \frac{154+42-132}{231} = \frac{64}{231}$$

## Question 3:

$$\int_0^1 sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$$

### Answer 3:

$$I = \int_0^1 sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$$

Let $x = \tan\theta \quad \Rightarrow dx = sec^2\theta\; d\theta$

When $x = 0, \theta = 0$ and when $x = 1, \theta = \frac{\pi}{4}$

$$I = \int_0^{\frac{\pi}{4}} sin^{-1}\left(\frac{2\tan\theta}{1+tan^2\theta}\right) sec^2\theta\; d\theta = \int_0^{\frac{\pi}{4}} sin^{-1}(\sin 2\theta)\, sec^2\theta\; d\theta$$

$$= \int_0^{\frac{\pi}{4}} 2\,\theta.sec^2\theta\; d\theta = 2\int_0^{\frac{\pi}{4}} \theta\,.sec^2\theta\; d\theta$$

Taking θ as first function and $sec^2\theta$ as second function and integrating by parts, we have

$$I = 2\left[\theta\, \int sec^2\theta\; d\theta\;\; \int\left\{\left(\frac{d}{dx}\theta\right)\int sec^2\theta\; d\theta\right\}d\theta\right]_0^{\frac{\pi}{4}} = 2\left[\theta\tan\theta - \int \tan\theta\; d\theta\,\right]_0^{\frac{\pi}{4}}$$

$$= 2[\theta\tan\theta - \log|\cos\theta|]_0^{\frac{\pi}{4}} = 2\left[\frac{\pi}{4}\tan\frac{\pi}{4} - \log\left|\cos\frac{\pi}{4}\right| - \log|\cos 0|\,\right]$$

$$= 2\left[\frac{\pi}{4} + \log\left(\frac{1}{\sqrt{2}}\right) - \log 1\right] = 2\left[\frac{\pi}{4} - \frac{1}{2}\log 2\right] = \frac{\pi}{2} - \log 2$$

**Question 4:**

$$\int_0^2 x\sqrt{x+2}\;\;(Put\; x + 2 = t^2)$$

**Answer 4:**

$$\int_0^2 x\sqrt{x+2}\; dx$$

Let $x + 2 = t^2 \quad \Rightarrow dx = 2t\, dt$

When $x = 0, t = \sqrt{2}$ and when $x = 2, t = 2$

$$I = \int_0^2 x\sqrt{x+2}\; dx = \int_{\sqrt{2}}^{t}(t^2 - 2)\sqrt{t^2}\; 2t\, dt \;=\; 2\int_{\sqrt{2}}^{2}(t^2 - 2)t^2\, dt$$

$$=\; 2\int_{\sqrt{2}}^{2}(t^4 - 2t^2)\, dt \;=\; 2\left[\frac{t^5}{5} - \frac{2t^3}{3}\right]_{\sqrt{2}}^{2} \;=\; 2\left[\frac{32}{5} - \frac{16}{3} - \frac{4\sqrt{2}}{5} + \frac{4\sqrt{2}}{3}\right]$$

$$=\; 2\left[\frac{96 - 80 - 12\sqrt{2} + 20\sqrt{2}}{15}\right] = 2\left[\frac{16 + 8\sqrt{2}}{15}\right] = \frac{16(2+\sqrt{2})}{15} = \frac{16\sqrt{2}(\sqrt{2}+1)}{15}$$

**Question 5:**

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1 + cos^2 x}dx$$

**Answer 5:**

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{1 + cos^2 x}dx$$

Let $\cos x = t \quad \Rightarrow -\sin x\, dx = dt$

When $x = 0, t = 1$ and when $x = \dfrac{\pi}{2}, t = 0$

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{1 + cos^2 x}dx = -\int_1^0 \frac{dt}{1 + t^2}$$

$$= -[tan^{-1}t]_1^0 = -[tan^{-1}0 - tan^{-1}1] = -\left[-\frac{\pi}{4}\right] \;=\frac{\pi}{4}$$

**Question 6:**

$$\int_0^2 \frac{dx}{x+4-x^2}$$

**Answer 6:**

$$I=\int_0^2 \frac{dt}{x+4-x^2}=\int_0^2 \frac{dx}{-(x^2-x-4)}=\int_0^2 \frac{dx}{-(x^2-x+\frac{1}{4}-\frac{1}{4}-4)}$$

$$=\int_0^2 \frac{dx}{\left[-\left(x-\frac{1}{2}\right)^2-\frac{17}{4}\right]}=\int_0^2 \frac{dx}{\left[\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2\right]}$$

Let $x-\frac{1}{2}=t \quad \Rightarrow \quad dx=dt$

When $x=0, t=-\frac{1}{2}$ and when $x=2, t=\frac{3}{2}$

Therefore, $\quad I=\int_4^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2}=\int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{dt}{\left(\frac{\sqrt{17}}{2}\right)^2-t^2}$

$$=\left[\frac{1}{2\left(\frac{\sqrt{17}}{2}\right)} log\left(\frac{\frac{\sqrt{17}}{2}+t}{\frac{\sqrt{17}}{2}-t}\right)\right]_{-\frac{1}{2}}^{\frac{3}{2}}=\frac{1}{\sqrt{17}}\left[\log\frac{\frac{\sqrt{17}}{2}+\frac{3}{2}}{\frac{\sqrt{17}}{2}-\frac{3}{2}}-\frac{\log\frac{\sqrt{17}}{2}-\frac{1}{2}}{\log\frac{\sqrt{17}}{2}+\frac{1}{2}}\right]$$

$$=\frac{1}{\sqrt{17}}\left[\log\frac{\sqrt{17}+3}{\sqrt{17}-3}-\log\frac{\sqrt{17}-1}{\sqrt{17}+1}\right]=\frac{1}{\sqrt{17}}\log\frac{\sqrt{17}+3}{\sqrt{17}-3}\times\frac{\sqrt{17}+1}{\sqrt{17}-1}=\frac{1}{\sqrt{17}}\log\left[\frac{17+3+4\sqrt{17}}{17+3-4\sqrt{17}}\right]$$

$$=\frac{1}{\sqrt{17}}\log\left[\frac{20+4\sqrt{17}}{20-4\sqrt{17}}\right]=\frac{1}{\sqrt{17}}\log\left(\frac{5+\sqrt{17}}{5-\sqrt{17}}\right)=\frac{1}{\sqrt{17}}\log\left[\frac{(5+\sqrt{17})(5-\sqrt{17})}{25-17}\right]$$

$$=\frac{1}{\sqrt{17}}\log\left[\frac{25+17+10\sqrt{17}}{8}\right]=\frac{1}{\sqrt{17}}\log\left[\frac{42+10\sqrt{17}}{8}\right]=\frac{1}{\sqrt{17}}\log\left[\frac{21+5\sqrt{17}}{4}\right]$$

**Question 7:**

$$\int_{-1}^1 \frac{dx}{x^2+2x+5}$$

**Answer 7:**

$$I=\int_{-1}^1 \frac{dx}{x^2+2x+5}=\int_{-1}^1 \frac{dx}{(x^2+2x+1)+4}=\int_{-1}^1 \frac{dx}{(x+1)^2+(2)^2}$$

Let $x+1=t \quad \Rightarrow dx=dt$

When $x=-1, t=0$ and when $x=1, t=2$

$$I=\int_{-1}^1 \frac{dx}{(x+1)^2+(2)^2}=\int_0^2 \frac{dt}{(t)^2+(2)^2}=\left[\frac{1}{2}tan^{-1}\frac{t}{2}\right]_0^2=\frac{1}{2}tan^{-1}1-\frac{1}{2}tan^{-1}0=\frac{1}{2}\left(\frac{\pi}{4}\right)=\frac{\pi}{8}$$

**Question 8:**

$$\int_1^2 \left(\frac{1}{x}-\frac{1}{2x^2}\right)e^{2x}dx$$

**Answer 8:**

$$I=\int_1^2 \left(\frac{1}{x}-\frac{1}{(2x)^2}\right)e^{2x}dx$$

Let $2x = t \quad \Rightarrow 2dx = dt$

When $x = 1, t = 2$ and when $x = 2, t = 4$

$$I=\int_2^1 \left(\frac{1}{x}-\frac{1}{(t)^2}\right)e^{t}\ dx=\frac{1}{2}\int_2^1 \left(\frac{2}{t}-\frac{2}{t^2}\right)e^{t}dt=\int_2^1 \left(\frac{1}{t}-\frac{1}{t^2}\right)e^{t}dt$$

If $\frac{1}{t}=f(t)$ then, $f'(t)=-\frac{1}{t^2}$

$$I=\int_2^4 \left(\frac{1}{t}-\frac{1}{t^2}\right)e^{t}dt=\int_2^4 e^t[f(t)+f'(t)]\,dt=[e^tf(t)]_2^4=\left[e^t.\frac{2}{t}\right]_2^4=\left[\frac{e^t}{t}\right]_2^4=\frac{e^4}{4}-\frac{e^2}{2}=\frac{e^2(e^2-2)}{4}$$

**Choose the correct answer in Exercises 9 and 10.**

**Question 9:**

The value of the integral $\int_{\frac{1}{3}}^{1}\frac{(x-x^3)^{\frac{1}{3}}}{x^4}dx$ is

(A) 6 (B) 0 (C) 3 (D) 4

**Answer 9:**

$$I=\int_{\frac{1}{3}}^{1}\frac{(x-x^3)^{\frac{1}{3}}}{x^4}$$

Let $x=\sin\theta \quad \Rightarrow dx=\cos\theta\, d\theta$

When $x=\frac{1}{3}, \quad \theta=sin^{-1}\left(\frac{1}{3}\right)$ and when $x=1, \quad \theta=\frac{\pi}{2}$

$$I=\int_{sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\frac{(\sin\theta-sin^3\theta)^{\frac{1}{3}}}{sin^4\theta}\cos\theta d\theta=\int_{sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\frac{(\sin\theta)^{\frac{1}{3}}(1-sin^2\theta)^{\frac{1}{3}}}{sin^4\theta}\cos\theta d\theta$$

$$=\int_{sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\frac{(\sin\theta)^{\frac{1}{3}}(\cos\theta)^{\frac{2}{3}}}{sin^2\theta sin^2\theta}\cos\theta d\theta=\int_{sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\frac{(\cos\theta)^{\frac{5}{3}}}{(\sin\theta)^{\frac{5}{3}}}cosec^2\,\theta d\theta=\int_{sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}(\cot\theta)^{\frac{5}{3}}cosec^2\,\theta d\theta$$

Let $\cot\theta=t \quad \Rightarrow -cosec^2\,\theta d\theta = dt$

When $\theta=sin^{-1}\left(\frac{1}{3}\right), t=2\sqrt{2}$ and when $\theta=\frac{\pi}{2}$, $t=0$

$$I=\int_{2\sqrt{2}}^{0}(t)^{\frac{5}{3}}\,dt=-\left[\frac{3}{8}(t)^{\frac{8}{3}}\right]_{2\sqrt{2}}^{0}=-\frac{3}{8}\left[(t)^{\frac{8}{3}}\right]_{2\sqrt{2}}^{0}=-\frac{3}{8}\left[(-2\sqrt{2})^{\frac{8}{3}}\right]$$

$$=\frac{3}{8}\left[(\sqrt{8})^{\frac{8}{3}}\right]=\frac{3}{8}\left[(8)^{\frac{4}{3}}\right]=\frac{3}{8}[16]=3\times 2=6$$

Hence, the correct answer is (A).

**Question 10:**

If $f(x)=\int_0^x t\sin t\,dt$ , then $f'(x)$ is

(A) $\cos x+x\ \sin x$ (B) $x\sin x$ (C) $x\cos x$ (D) $\sin x+x\ \cos x$

**Answer 10:**

$f(x)=\int_0^x t\sin t\,dt$ , Integrating by parts, we have

$$f(x)=t\int_0^x \sin t\,dt-\int_0^x\left\{\left(\frac{d}{dt}t\right)\int \sin t\,dt\right\}dt$$

$$=[t(-\cos t)]_0^x-\int_0^x(-\cos t)\,dt$$

$$\Rightarrow f(x)=[-t\cos t+\sin t)]_0^x=-x\cos x+\sin x$$

$$\Rightarrow f'(x)=-[\{x(-\sin x)\}+\cos x\ ]+\cos x$$

$$=x\sin x-\cos x+\cos x$$

$$=x\sin x$$

Hence, the correct answer is (B).

**By using the properties of definite integrals, evaluate the integrals in Exercises 1 to 19.**

## Question 1:

$$\int_0^{\frac{\pi}{2}} cos^2x \; dx$$

### Answer 1:

$$I = \int_0^{\frac{\pi}{2}} cos^2x \; dx \qquad \dots(1)$$

$$I = \int_0^{\frac{\pi}{2}} cos^2\left(\frac{\pi}{2} - x\right) dx \qquad \left[\text{Using property: } \int_0^a f(x)dx = \int_0^a f(a-x)\,dx\right]$$

$$I = \int_0^{\frac{\pi}{2}} sin^2x \; dx \qquad \dots(2)$$

Adding (1) and (2), we have

$$2I = \int_0^{\frac{\pi}{2}} (sin^2x + cos^2x) \; dx$$

$$2I = \int_0^{\frac{\pi}{2}} 1 \; dx \quad \Rightarrow 2I = [x]_0^{\frac{\pi}{2}} \quad \Rightarrow 2I = \frac{\pi}{2} \quad \Rightarrow I = \frac{\pi}{4}$$

## Question 2:

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

### Answer 2:

$$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \qquad \dots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\sqrt{\sin\left(\frac{\pi}{2} - x\right)} + \sqrt{\cos\left(\frac{\pi}{2} - x\right)}} dx \qquad \left[\text{Using property: } \int_0^a f(x)dx = \int_0^a f(a-x)\,dx\right]$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \qquad \dots(2)$$

Adding (1) and (2), we have

$$2I = \int_0^{\frac{\pi}{2}} \left(\frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}}\right) dx$$

$$2I = \int_0^{\frac{\pi}{2}} \left(\frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}\right) dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \; dx \quad 2I = [x]_0^{\frac{\pi}{2}} \quad \Rightarrow 2I = \frac{\pi}{2} \quad \Rightarrow I = \frac{\pi}{4}$$

**Question 3:**

$$\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x\, dx}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}\, dx$$

**Answer 3:**

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}\, dx \qquad \dots(1)$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)}{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right) + \cos^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)}\, dx \qquad \left[\text{Using property: } \int_0^a f(x)dx = \int_0^a f(a-x)\, dx\right]$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos^{\frac{3}{2}} x}{\cos^{\frac{3}{2}} x + \sin^{\frac{3}{2}} x}\, dx \qquad \dots(2)$$

Adding (1) and (2), we have

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}\, dx \quad \Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1\, dx$$

$$\Rightarrow \quad I = [x]_0^{\frac{\pi}{2}} \quad \Rightarrow 2I = \frac{\pi}{2} \quad \Rightarrow I = \frac{\pi}{4}$$

**Question4:**

$$\int_0^{\frac{\pi}{2}} \frac{\cos^5 x\, dx}{\sin^5 x + \cos^5 x}\, dx$$

**Answer 4:**

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos^5 x\, dx}{\sin^5 x + \cos^5 x}\, dx \qquad \dots(1)$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos^5\left(\frac{\pi}{2} - x\right) dx}{\sin^5\left(\frac{\pi}{2} - x\right) + \cos^5\left(\frac{\pi}{2} - x\right)}\, dx \qquad \left[\text{Using property: } \int_0^a f(x)dx = \int_0^a f(a-x)\, dx\right]$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x\, dx}{\cos^5 x + \sin^5 x}\, dx \qquad \dots(2)$$

Adding (1) and (2), we have

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x + \cos^5 x}{\cos^5 x + \sin^5 x}\, dx \quad \Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1\, dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2} \quad \Rightarrow I = \frac{\pi}{4}$$

**Question 5:**

$$\int_{-5}^{5} |x+2|\, dx$$

**Answer 5:**

$$I = \int_{-5}^{5} |x+2|\, dx$$

Here, $(x+2) < 0$ if $x < -2$ and $(x+2) > 0$ if $x > -2$, therefore

$$I = \int_{-5}^{-2} -(x+2)dx + \int_{-2}^{5} (x+2)dx \qquad \left[\text{Using property: } \int_a^b f(x) = \int_a^e f(x) + \int_e^b f(x)\right]$$

$$I = -\left[\frac{x^2}{2} + 2x\right]_{-5}^{-2} + \left[\frac{x^2}{2} + 2x\right]_{-2}^{5}$$

$$= -\left[\frac{(-2)^2}{2} + 2(-2) - \frac{(-5)^2}{2} - 2(-5)\right] + \left[\frac{(5)^2}{2} + 2(5) - \frac{(2)^2}{2} + 2(2)\right]$$

$$I = -\left[2 - 4 - \frac{25}{2} + 10\right] + \left[\frac{25}{2} + 10 - 2 + 4\right] = -2 + 4 + \frac{25}{2} - 10 + \frac{25}{2} + 10 - 2 + 4 = 29$$

**Question 6:**

$$\int_{2}^{8} |x-5| dx$$

**Answer 6:**

$$I = \int_{2}^{8} |x-5| dx$$

Here, $(x-5) < 0$ if $x < 5$ and $(x-5) > 0$ if $x > 5$, therefore

$$I = \int_{2}^{5} -(x-5)dx + \int_{2}^{8} (x-5)dx \qquad \left[\text{Using property: } \int_a^b f(x) = \int_a^e f(x) + \int_e^b f(x)\right]$$

$$I = -\left[\frac{x^2}{2} - 5x\right]_{2}^{5} + \left[\frac{x^2}{2} - 5x\right]_{5}^{8} I = -\left[\frac{25}{2} - 25 - 2 + 10\right] + \left[32 - 40 - \frac{25}{2} + 25\right] = 9$$

**Question 7:**

$$\int_{0}^{1} x(1-x)^n dx$$

**Answer 7:**

$$I = \int_0^1 x(1-x)^n dx = \int_0^1 (1-x)(1-(1-x))^n dx \quad \left[\text{Using property: } \int_0^a f(x)dx = \int_0^a f(a-x)\, dx\right]$$

$$= \int_0^1 (1-x)(x)^n dx = \int_0^1 (x^n - x^{n+1}) dx = \left[\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\right]_0^1$$

$$= \left[\frac{1}{n+1} - \frac{1}{n+2}\right] = \frac{(n+2)-(n+1)}{(n+1)(n+2)} = \frac{1}{(n+1)(n+2)}$$

**Question 8:**

$$\int_0^{\frac{\pi}{4}} \log(1+\tan x)\,dx$$

**Answer 8:**

$$I = \int_0^{\frac{\pi}{4}} \log(1+\tan x)\,dx \qquad \ldots(1)$$

$$= \int_0^{\frac{\pi}{4}} \log\left[1+\tan\left(\frac{\pi}{4}-x\right)\right]dx \qquad \left[\text{Using property: } \int_0^a f(x)dx = \int_0^a f(a-x)\,dx\right]$$

$$= \int_0^{\frac{\pi}{4}} \log\left[1+\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right]dxI$$

$$= \int_0^{\frac{\pi}{4}} \log\left[1+\frac{1-\tan x}{1+\tan x}\right]dxI = \int_0^{\frac{\pi}{4}} \log\frac{2}{(1+\tan x)}\,dx$$

$$= \int_0^{\frac{\pi}{4}} \log 2\,dx - \int_0^{\frac{\pi}{4}} \log(1+\tan x)\,dx = \int_0^{\frac{\pi}{4}} \log 2\,dx - I \qquad [\text{From } (1)]$$

$$2I = [x\log 2]_0^{\frac{\pi}{4}} \qquad \Rightarrow 2I = \frac{\pi}{4}\log 2 \qquad \Rightarrow I = \frac{\pi}{8}\log 2$$

**Question 9:**

$$\int_0^2 x\sqrt{2-x}\,dx$$

**Answer 9:**

$$I = \int_0^2 x\sqrt{2-x}\,dx$$

$$I = \int_0^2 (2-x)\sqrt{x}\,dx \qquad \left[\text{Using property: } \int_0^a f(x)dx = \int_0^a f(a-x)\,dx\right]$$

$$= \int_0^2 \left\{2\,x^{\frac{1}{2}} - x^{\frac{3}{2}}\right\}dx = \left[2\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right) - \left(\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right)\right]_0^1$$

$$= \left[\frac{4}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}\right]_0^1 = \frac{4}{3}(2)^{\frac{3}{2}} - \frac{2}{5}(2)^{\frac{5}{2}}$$

$$= \frac{4\times 2\sqrt{2}}{3} - \frac{2}{5}\times 4\sqrt{2}$$

$$= \frac{8\sqrt{2}}{3} - \frac{8\sqrt{2}}{5} = \frac{40\sqrt{2}-24\sqrt{2}}{15} = \frac{16\sqrt{2}}{15}$$

**Question 10:**

$$\int_0^{\frac{\pi}{2}}(2\log\sin x-\log\sin 2x)dx$$

**Answer 10:**

$$I=\int_0^{\frac{\pi}{2}}(2\log\sin x-\log\sin 2x)dx=\int_0^{\frac{\pi}{2}}\{(2\log\sin x-\log(2\sin x\cos x)\}dx$$

$$=\int_0^{\frac{\pi}{2}}\{(2\log\sin x-\log\sin x-log\cos x-\log 2)\}dx$$

$$I=\int_0^{\frac{\pi}{2}}\{(\log\sin x-\log\cos x-\log 2)\}dx \quad \dots(1)$$

Using property, $\left\{\int_0^a f(x)dx=\int_0^a f(a-x)dx\right\}$, we have

$$I=\int_0^{\frac{\pi}{2}}\{(\log\cos x-\log\sin x-\log 2)\}dx \quad \dots(2)$$

Adding (1) and (2), we have

$$2I=\int_0^{\frac{\pi}{2}}(-\log 2-\log 2)dx \quad \Rightarrow 2I=-2\log 2\int_0^{\frac{\pi}{2}}1\,dx$$

$$\Rightarrow\ I=-\log 2\left[\frac{\pi}{2}\right] \quad \Rightarrow I=\frac{\pi}{2}(-\log 2)$$

$$\Rightarrow I=-\frac{\pi}{2}\log 2$$

**Question 11:**

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}sin^2x\ \ dx$$

**Answer 11:**

$$I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}sin^2x\ \ dx$$

Here, $sin^2(-x)=\ (\sin(-x))^2=sin^2x$, therefore, $sin^2x$ is an even function.

We know that if $f(x)$ is an even function, then $\int_{-a}^{a}f(x)dx=2\int_0^a f(x)dx$

$$I=2\int_0^{\frac{\pi}{2}}sin^2x\ \ dx=2\int_0^{\frac{\pi}{2}}\frac{1-\cos 2x}{2}dx$$

$$=\int_0^{\frac{\pi}{2}}(1-\cos 2x)\ \ dx=\left[x-\frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}}=\frac{\pi}{2}-0=\frac{\pi}{2}$$

**Question 12:**

$$\int_0^{\pi} \frac{x\,dx}{1+\sin x}$$

**Answer 12:**

$I = \int_0^{\pi} \frac{x\,dx}{1+\sin x}$ ... (1)

$I = \int_0^{\pi} \frac{(\pi - x)\,dx}{1+\sin(\pi - x)}$ $\left[\text{Using property: } \int_0^a f(x)dx = \int_0^a f(a-x)\,dx\right]$

$I = \int_0^{\pi} \frac{(\pi - x)\,dx}{1+\sin x}$ ... (2)

Adding (1) and (2), we have

$2I = \int_0^{\pi} \frac{\pi\,dx}{1+\sin x}$

$\Rightarrow 2I = \pi \int_0^{\pi} \frac{(1-\sin x)\,dx}{(1+\sin x)(1-\sin x)}$

$\Rightarrow 2I = \pi \int_0^{\pi} \frac{1-\sin x}{cos^2 x}dx$

$2I = \pi \int_0^{\pi} \{sec^2 x - \tan x \sec x\}\,dx$ $\Rightarrow 2I = \pi\,[\tan x \sec x]_0^{\pi}$

$\Rightarrow 2I = \pi[2]$ $\Rightarrow I = \pi$

**Question 13:**

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin^7 x\,dx$$

**Answer 13:**

$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin^7 x\,dx$ ... (1)

Here, $sin^7(-x) =$

$[(\sin(-x))]^7 = (-\sin x)^2$

$= -sin^7 x,$

Therefore , $sin^7 x$ is an odd function.

We know that if $f(x)$ is an odd function, then $\int_{-a}^{a} f(x)dx = 0$

Therefore, $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin^7 x\,dx = 0$

**Question 14:**

$$\int_0^{2\pi} cos^5 x\, dx$$

**Answer 14:**

$$I = \int_0^{2\pi} cos^5 x\, dx \quad \dots(1)$$

We know that,

$$\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx & \text{if } f(2a-x) = f(x) \\ 0 & \text{if } f(2a-x) = -f(x) \end{cases}$$

Here, $\quad cos^5(2\pi - x) = cos^5 x$

$$\therefore I = 2\int_0^{\pi} cos^5 x\, dx$$

Now, $\quad cos^5(\pi - x) = -cos^5 x$

$$\therefore I = 2(0) = 0$$

**Question 15:**

$$\int_0^{\frac{\pi}{2}} \frac{sin\, x - cos\, x}{1 + \sin x \cos x}\, dx$$

**Answer 15:**

$$I = \int_0^{\frac{\pi}{2}} \frac{sin\, x - cos\, x}{1 + \sin x \cos x}\, dx \quad \dots(1)$$

$$= \int_0^{\frac{\pi}{2}} \frac{sin\left(\frac{\pi}{2} - x\right) - cos\left(\frac{\pi}{2} - x\right)}{1 + \sin\left(\frac{\pi}{2} - x\right)\cos\left(\frac{\pi}{2} - x\right)}\, dx \qquad \left[\text{Using property: } \int_0^a f(x)dx = \int_0^a f(a-x)\,dx\right]$$

$$= \int_0^{\frac{\pi}{2}} \frac{cos\, x - sin\, x}{1 + \sin x \cos x}\, dx \quad \dots(2)$$

Adding (1) and (2), we have

$$2I = \int_0^{\frac{\pi}{2}} \frac{0}{1 + \sin x \cos x}\, dx = 0 \quad \Rightarrow I = 0$$

**Question 16:**

$$\int_0^{\pi} \log(1 + \cos x)\, dx$$

**Answer 16:**

$$I = \int_0^{\pi} \log(1 + \cos x)\, dx \quad \dots(1)$$

$$I = \int_0^{\pi} \log(1 + \cos(\pi - x))\, dx \qquad \left[\text{Using property: } \int_0^a f(x)dx = \int_0^a f(a-x)\,dx\right]$$

$I = \int_0^\pi \log(1 - \cos x)\, dx \qquad \dots (2)$

Adding (1) and (2), we have

$$2I = \int_0^\pi \{\log(1 + \cos x) + \log(1 - \cos x)\}\, dx = \int_0^\pi \log(1 - cos^2 x)\, dx$$

$$2I = \int_0^\pi \log(sin^2 x)\, dx = 2\int_0^\pi \log(sinx)\, dx$$

$\Rightarrow I = \int_0^\pi \log(sinx)\, dx \qquad \dots (3)$

We know that,

$$\int_0^{2a} f(x)\, dx = \begin{cases} 2\int_0^a f(x)\, dx & \text{if } f(2a - x) = f(x) \\ 0 & \text{if } f(2a - x) = -f(x) \end{cases}$$

Here, $\quad \sin(\pi - x) = \sin x$

$\Rightarrow I = 2\int_0^{\frac{\pi}{2}} \log \sin x\, dx \quad \dots (4)$

$I = \int_0^{\frac{\pi}{2}} \log \sin\left(\frac{\pi}{2} - x\right) dx = 2\int_0^{\frac{\pi}{2}} \log \cos x\, dx \qquad \dots (5)$

Adding (4) and (5), we have

$$2I = 2\int_0^{\frac{\pi}{2}} (\log \sin x + \log \cos x\,)dx \quad \Rightarrow I = \int_0^{\frac{\pi}{2}} (\log \sin x + \log \cos x + \log 2 - \log 2\,)dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} (\log 2 \sin x \cos x - \log 2)\, dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log 2 \sin x \cos x\, dx - \int_0^{\frac{\pi}{2}} \log 2\, dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log \sin 2x\, dx - \log 2\, [x]_0^{\frac{\pi}{2}} = \int_0^{\frac{\pi}{2}} \log \sin 2x\, dx - \frac{\pi}{2} \log 2$$

Let $\quad 2x = t \quad \Rightarrow 2\, dx = dt$

When $x = 0, t = 0$ and when $x = \frac{\pi}{2}, t = \pi$

$$I = \frac{1}{2}\int_0^\pi log\, sin\, t\, dt - \frac{\pi}{2} log\, 2 = \frac{1}{2} I - \frac{1}{2} log\, 2$$

$$\Rightarrow \frac{1}{2} I = -\frac{\pi}{2} log\, 2 \qquad \Rightarrow I = -\pi \log 2$$

**Question 17:**

$$\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}\,dx$$

**Answer 17:**

$$I = \int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}\,dx \quad \dots(1)$$

$\left[\text{Using property: } \int_0^a f(x)dx = \int_0^a f(a-x)\,dx\right]$, we have

$$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}\,dx \quad \dots(2)$$

Adding (1) and (2), we have

$$2I = \int_0^a \frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}\,dx \quad \Rightarrow 2I = \int_0^a 1\,dx \Rightarrow 2I = [x]_0^a \quad \Rightarrow 2I = a \quad \Rightarrow I = \frac{a}{2}$$

**Question 18:**

$$\int_0^4 |x-1|\,dx$$

**Answer 18:**

$$I = \int_0^4 |x-1|\,dx$$

Here, $(x-1) < 0$ if $x < 1$ and $(x-1) > 0$ if $x > 1$, therefore

$$I = \int_0^1 -(x-1)\,dx + \int_1^4 (x-1)\,dx \qquad \left[\text{Using property: } \int_a^b f(x) = \int_a^e f(x) + \int_e^b f(x)\right]$$

$$= \left[x - \frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2} - x\right]_1^4 = 1 - \frac{1}{2} + \frac{(4)^2}{2} - 4 - \frac{1}{2} + 1 = 1 - \frac{1}{2} + 8 - 4 - \frac{1}{2} + 1 = 5$$

**Question 19:**

Show that $\int_0^a f(x)g(x)dx = 2\int_0^a f(x)dx$, if $f$ and $g$ are defined as $f(x) = f(a-x)$ and $g(x) + g(a-x) = 4$.

**Answer 19:**

$$I = \int_0^a f(x)g(x)dx \qquad \dots(1)$$

$$I = \int_0^a f(a-x)g(a-x)dx \qquad \left[\text{Using property: } \int_0^a f(x)dx = \int_0^a f(a-x)\,dx\right]$$

$$I = \int_0^a f(x)g(a-x)dx \quad \dots(2)$$

Adding (1) and (2), we have

$2I = \int_0^a \{f(x)g(x) + f(x)g(a-x)\}dx \quad \Rightarrow 2I = \int_0^a \{f(x)\{g(x) + g(a-x)\}\}dx$

$\Rightarrow 2I = \int_0^a f(x) \times 4\, dx \qquad$ [Given that: $g(x) + g(a-x) = 4$]

$\Rightarrow I = 2\int_0^a f(x)dx$

**Choose the correct answer in Exercises 20 and 21.**

## Question 20:

The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^3 dx + x\cos x + tan^5 x + 1)\, dx$ is

(A) 0 (B) 2 (C) $\pi$ (D) 1

## Answer 20:

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^3 + x\cos x + tan^5 x + 1)\, dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^3 dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\cos x\, dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} tan^5 x\, dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1\, dx$$

We know that,

$$\int_{-a}^{a} f(x)\, dx = \begin{cases} 2\int_0^a f(x)\, dx & \text{if } f(x) \text{ is an even function} \\ 0 & \text{if } f(x) \text{ is an odd function} \end{cases}$$

Therefore, $\quad I = 0 + 0 + 0 + 2\int_0^{\frac{\pi}{2}} 1\, dx = 2[x]_0^{\frac{\pi}{2}} = \frac{2\pi}{2} = \pi$

Hence, the correct answer is (C).

## Question 21:

The value of $\int_0^{\frac{\pi}{2}} \log\left(\frac{4 + 3\sin x}{4 + 3\cos x}\right) dx$ is

(A) 2 (B) $\frac{3}{4}$ (C) 0 (D) $-2$

## Answer 21:

$$I = \int_0^{\frac{\pi}{2}} \log\left(\frac{4 + 3\sin x}{4 + 3\cos x}\right) dx \quad \ldots (1)$$

$$I = \int_0^{\frac{\pi}{2}} \log\left(\frac{4 + 3\sin\left(\frac{\pi}{2} - x\right)}{4 + 3\cos\left(\frac{\pi}{2} - x\right)}\right) dx \qquad \left[\text{Using property: } \int_0^a f(x)dx = \int_0^a f(a-x)\, dx\right]$$

$$I = \int_0^{\frac{\pi}{2}} \log\left(\frac{4 + 3\cos x}{4 + 3\sin x}\right) dx \quad \ldots (2)$$

Adding (1) and (2), we have,

$$2I = \int_0^{\frac{\pi}{2}} \left\{\log\left(\frac{4 + 3\sin x}{4 + 3\cos x}\right) + \log\left(\frac{4 + 3\cos x}{4 + 3\sin x}\right)\right\} dx \Rightarrow 2I = \int_0^{\frac{\pi}{2}} \left\{\log \frac{(4 + 3\sin x)}{(4 + 3\cos x)} \times \frac{(4 + 3\cos x)}{(4 + 3\sin x)}\right\} dx$$

$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \log 1\, dx \quad \Rightarrow I = 0$

Hence, the correct answer is (C).

**Integrate the functions in Exercises 1 to 23.**

## Question 1:

$$\frac{1}{x-x^3}$$

### Answer 1:

$$\frac{1}{x-x^3}=\frac{1}{x(1-x^2)}=\frac{1}{x(1-x)(1+x)}$$

Let $\frac{1}{x(1-x)(1+x)}=\frac{A}{x}+\frac{B}{(1-x)}+\frac{C}{(1+x)} \Rightarrow 1=A(1-x^2)+Bx(1+x)+Cx(1-x)$

$\Rightarrow 1=A-Ax^2+Bx+Bx^2+Cx-Cx^2$

Equating the coefficients of $x^2$, x, and constant term, we have

$-A+B-C=0$, $B+C=0$ and $A=1$

On solving these equations, we have

$$A=1,\quad B=\frac{1}{2}\quad \text{and}\quad C=-\frac{1}{2}$$

From equation (1), we have

$$\frac{1}{x(1-x)(1+x)}=\frac{1}{x}+\frac{1}{2(1-x)}-\frac{1}{2(1+x)}$$

Now, $I=\int\frac{1}{x(1-x)(1+x)}dx=\int\frac{1}{x}dx+\frac{1}{2}\int\frac{1}{(1-x)}dx-\frac{1}{2}\int\frac{1}{(1+x)}dx$

$=\log|x|-\frac{1}{2}\log|(1-x)|-\frac{1}{2}\log|(1+x)|=\log|x|-\log\left|(1-x)^{\frac{1}{2}}\right|-\log\left|(1+x)^{\frac{1}{2}}\right|$

$=\log\left|\frac{x}{(1-x)^{\frac{1}{2}}(1+x)^{\frac{1}{2}}}\right|+C=\log\left|\left(\frac{x^2}{1-x^2}\right)^{\frac{1}{2}}\right|+C=\frac{1}{2}\log\left|\frac{x^2}{1-x^2}\right|+C$

## Question 2:

$$\frac{1}{\sqrt{x+a}+\sqrt{x+b}}$$

### Answer 2:

$$\frac{1}{\sqrt{x+a}+\sqrt{x+b}}=\frac{1}{\sqrt{x+a}+\sqrt{x+b}}\times\frac{\sqrt{x+a}-\sqrt{x+b}}{\sqrt{x+a}-\sqrt{x+b}}=\frac{\sqrt{x+a}-\sqrt{x+b}}{(x+a)-(x+b)}=\frac{\sqrt{x+a}-\sqrt{x+b}}{a-b}$$

$I=\int\frac{1}{\sqrt{x+a}+\sqrt{x+b}}dx$

$=\frac{1}{a-b}\int\left(\sqrt{x+a}-\sqrt{x+b}\right)dx$

$=\frac{1}{a-b}\left[\frac{(x+a)^{\frac{3}{2}}}{\frac{3}{2}}-\frac{(x+b)^{\frac{3}{2}}}{\frac{3}{2}}\right]$

$=\frac{2}{3(a-b)}\left[(x+a)^{\frac{3}{2}}-(x+b)^{\frac{3}{2}}\right]+C$

**Question 3:**

$$\frac{1}{x\sqrt{ax-x^2}} \text{ [Hint:Put } x=\frac{a}{t}\text{]}$$

**Answer 3:**

$$I=\int\frac{1}{x\sqrt{ax-x^2}}dx$$

Let $x=\frac{a}{t} \quad \Rightarrow \; dx=-\frac{a}{t^2}\,dt$

$$I=\int\frac{1}{x\sqrt{ax-x^2}}dx=\int\frac{1}{\frac{a}{t}\sqrt{a.\frac{a}{t}-\left(\frac{a}{t}\right)^2}}\left(-\frac{a}{t^2}\,dt\right)=-\int\frac{1}{at}.\frac{1}{\sqrt{\frac{1}{t}-\frac{1}{t^2}}}dt$$

$$=-\frac{1}{a}\int\frac{1}{\sqrt{\frac{t^2}{t}-\frac{t^2}{t^2}}}dt=-\frac{1}{a}\int\frac{1}{\sqrt{t-1}}dt=-\frac{1}{a}\left[2\sqrt{t-1}\right]+C$$

$$=-\frac{1}{a}\left[2\sqrt{\frac{a}{x}-1}\right]+C=-\frac{2}{a}\left[\frac{\sqrt{a-x}}{\sqrt{x}}\right]+C=-\frac{2}{a}\left[\sqrt{\frac{a-x}{x}}\right]+C$$

**Question 4:**

$$\frac{1}{x^2(x^4+1)^{\frac{3}{4}}}$$

**Answer 4:**

$$I=\int\frac{1}{x^2(x^4+1)^{\frac{3}{4}}}dx$$

Multiplying numerator and denominator by $x^{-3}$, we have

$$\frac{x^{-3}}{x^{-3}.x^2(x^4+1)^{\frac{3}{4}}}=\frac{x^{-3}(x^4+1)^{\frac{3}{4}}}{x^{-3}.x^2}=\frac{(x^4+1)^{\frac{3}{4}}}{x^5.(x^4)^{-\frac{3}{4}}}=\frac{1}{x^5}\left(\frac{x^4+1}{x^4}\right)^{-\frac{3}{4}}=\frac{1}{x^5}\left(1+\frac{1}{x^4}\right)^{-\frac{3}{4}}$$

Let $\frac{1}{x^4}=t \quad \Rightarrow \; -\frac{4}{x^5}dx=dt$

$\Rightarrow \frac{1}{x^5}\,dx=-\frac{dt}{4}$

$$I=\int\frac{1}{x^2(x^4+1)^{\frac{3}{4}}}dx=\int\frac{1}{x^5}\left(1+\frac{1}{x^4}\right)^{-\frac{3}{4}}dx$$

$$=-\frac{1}{4}\int(1+t)^{-\frac{3}{4}}\,dt$$

$$=-\frac{1}{4}\left[\frac{(1+t)^{\frac{1}{4}}}{\frac{1}{4}}\right]+C=-\frac{1}{4}\frac{\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}}{\frac{1}{4}}+C$$

$$=-\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}+C$$

## Question 5:

$$\frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}} \qquad \left[Hint \quad \frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}}=\frac{1}{x^{\frac{1}{3}}\left(1+x^{\frac{1}{6}}\right)} \quad \text{Put } x=t^{6}\right]$$

## Answer 5:

$$\frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}}=\frac{1}{x^{\frac{1}{3}}\left(1+x^{\frac{1}{6}}\right)}$$

Let $x=t^{6} \quad \Rightarrow dx=6t^{5}dt$

$$I=\int\frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}}dx=\int\frac{1}{x^{\frac{1}{3}}\left(1+x^{\frac{1}{6}}\right)}dx=\int\frac{6t^{5}}{t^{2}(1+t)}dt=6\int\frac{6t^{3}}{(1+t)}dt$$

Dividing $t^{3}$ by denominator $1+t$, we have

$$I=\int\frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}}dx=6\int\left\{(t^{2}-t+1)-\frac{1}{1+t}\right\}dt=6\left[\left(\frac{t^{3}}{3}\right)-\left(\frac{t^{2}}{2}\right)+t-\log|1+t|\right]$$

$$=2\,x^{\frac{1}{2}}-3\,x^{\frac{1}{3}}+6\,x^{\frac{1}{6}}-6\log\left(1+x^{\frac{1}{6}}\right)+C=2\sqrt{2}-3\,x^{\frac{1}{3}}+6\,x^{\frac{1}{6}}-6\log\left(1+x^{\frac{1}{6}}\right)+C$$

## Question 6:

$$\frac{5x}{(x+1)(x^{2}+9)}$$

## Answer 6:

Let $\frac{5x}{(x+1)(x^{2}+9)}=\frac{A}{(x+1)}+\frac{Bx+C}{(x^{2}+9)} \qquad \ldots(1)$

$\Rightarrow 5x=A(x^{2}+9)+(Bx+C)(x+1) \Rightarrow 5x=Ax^{2}+9A+Bx^{2}+Bx+Cx+C$

Equating the coefficients of $x^{2}$, x, and constant term, we have

A + B = 0, B + C = 5 and 9A + C = 0

On solving these equations, we have

$$A=-\frac{1}{2}, \quad B=\frac{1}{2} \quad \text{and} \quad C=\frac{9}{2}$$

From equation (1), we have

$$\frac{5x}{(x+1)(x^{2}+9)}=\frac{-1}{2(x+1)}+\frac{\frac{x}{2}+\frac{9}{2}}{(x^{2}+9)}$$

Now, $I=\int\frac{5x}{(x+1)(x^{2}+9)}dx=\int\left\{\frac{-1}{2(x+1)}+\frac{(x+9)}{2(x^{2}+9)}\right\}dx$

$$=-\frac{1}{2}\log|x+1|+\frac{1}{2}\int\frac{x}{(x^{2}+9)}dx+\frac{9}{2}\int\frac{1}{(x^{2}+9)}dx$$

$$=-\frac{1}{2}\log|x+1|+\frac{1}{4}\int\frac{2x}{(x^{2}+9)}dx+\frac{9}{2}\int\frac{1}{(x^{2}+9)}dx$$

$$=-\frac{1}{2}\log|x+1|+\frac{1}{4}\log|x^{2}+9|+\frac{9}{2}.\frac{1}{3}tan^{-1}\frac{x}{3}+C=-\frac{1}{2}\log|x+1|+\frac{1}{4}\log(x^{2}+9)+\frac{3}{2}tan^{-1}\frac{x}{3}+C$$

**Question 7:**

$$\frac{\sin x}{\sin(x-a)}$$

**Answer 7:**

$$I=\int \frac{\sin x}{\sin(x-a)}dx$$

Let $(x-a)=t \quad \Rightarrow \; dx=dt$

$$I=\int \frac{\sin x}{\sin(x-a)}dx=\int \frac{\sin(t+a)}{\sin t}dt=\int \frac{\sin t\cos a+\cos t\sin a}{\sin t}\;dt$$

$$=\int (\cos a+\cot t\sin a)\;dt=t\cos a+\sin a\log|\sin t|+C_1$$

$$=(x-a)\cos a+\sin a\log|\sin(x-a)|+C_1=x\cos a+\sin a\log|\sin(x-a)|-a\cos a+C_1$$

$$=\sin a\log|\sin(x-a)|+x\cos a+C \qquad [\text{Where } C=C_1-a\cos a]$$

**Question 8:**

$$\frac{e^{5\log x}-e^{4\log x}}{e^{3\log x}-e^{2\log x}}$$

**Answer 8:**

$$\frac{e^{5\log x}-e^{4\log x}}{e^{3\log x}-e^{2\log x}}=\frac{e^{4\log x}\left(e^{\log x}-1\right)}{e^{2\log x}\left(e^{\log x}-1\right)}=\frac{e^{4\log x}}{e^{2\log x}}=\frac{x^4}{x^2}=x^2 \qquad \left[\text{Since } e^{m\log x}=e^{\log x^m}=x^m\right]$$

Therefore, $\quad I=\int \dfrac{e^{5\log x}-e^{4\log x}}{e^{3\log x}-e^{2\log x}}dx=\int x^2dx=\dfrac{x^3}{3}+C$

**Question 9:**

$$\frac{\cos x}{\sqrt{4-sin^2x}}$$

**Answer 9:**

$$\frac{\cos x}{\sqrt{4-sin^2x}}$$

Let $\sin x=t \quad \Rightarrow \quad \cos x dx=dt$

$$I=\int \frac{\cos x}{\sqrt{4-sin^2x}}dx=\int \frac{dt}{\sqrt{2^2-(t)^2}}=sin^{-1}\left(\frac{t}{2}\right)+C=sin^{-1}\left(\frac{\sin x}{2}\right)+C$$

**Question 10:**

$$\frac{sin^8x-cos^8x}{1-2\,sin^2x\,cos^2x}$$

**Answer 10:**

$$\frac{sin^8x-cos^8x}{1-2\,sin^2x cos^2x}=\frac{(sin^4x+cos^4x)(sin^4x-cos^4x)}{sin^2x+cos^2x-\;sin^2x\,cos^2x-\;sin^2x\,cos^2x}$$

$$=\frac{(sin^4x+cos^4x)(sin^2x-cos^2x)(sin^2x+cos^2x)}{(sin^2x-\;sin^2x\,cos^2x)+(cos^2x-\;sin^2x\,cos^2x)}$$

$$= \frac{(sin^4x + cos^4x)(sin^2x - cos^2x)}{sin^2x(1 - cos^2x) + cos^2x(1 - sin^2x)}$$

$$= \frac{(sin^4x + cos^4x)(sin^2x - cos^2x)}{sin^4x + cos^4x} = -\cos 2x$$

Therefore, $I = \int \frac{sin^8x - cos^8x}{1 - 2\,sin^2x cos^2x} dx$

$$= \int -\cos 2x\, dx = -\frac{\sin 2x}{2} + C$$

**Question 11:**

$$\frac{1}{\cos(x+a)\cos(x+b)}$$

**Answer 11:**

$$\frac{1}{\cos(x+a)\cos(x+b)}$$

Multiplying and dividing by sin $(a - b)$, we have

$$= \frac{1}{\sin(a-b)}\left[\frac{\sin(a-b)}{\cos(x+a)\cos(x+b)}\right] = \frac{1}{\sin(a-b)}\left[\frac{\sin\{(x+a)-(x+b)\}}{\cos(x+a)\cos(x+b)}\right]$$

$$= \frac{1}{\sin(a-b)}\left[\frac{\sin(x+a)\cos(x+b) - \cos(x+a)\sin(x+b)}{\cos(x+a)\cos(x+b)}\right]$$

$$= \frac{1}{\sin(a-b)}\left[\frac{\sin(x+a)}{\cos(x+a)} - \frac{\sin(x+b)}{\cos(x+b)}\right] = \frac{1}{\sin(a-b)}[\tan(x+a) - \tan(x+b)]$$

Therefore, $I = \int \frac{1}{\cos(x+a)\cos(x+b)} dx = \frac{1}{\sin(a-b)} \int [\tan(x+a) - \tan(x+b)] dx$

$$= \frac{1}{\sin(a-b)}\left[-\log|\cos(x+a)| + \log|\cos(x+b)|\right] + C = \frac{1}{\sin(a-b)} \log\left|\frac{\cos(x+b)}{\cos(x+a)}\right| + C$$

**Question 12:**

$$\frac{x^3}{\sqrt{1-x^8}}$$

**Answer 12:**

$$I = \int \frac{x^3}{\sqrt{1-x^8}} dx$$

Let $x^4 = t \Rightarrow 4x^3 dx = dt$

$$I = \int \frac{x^3}{\sqrt{1-x^8}} = \frac{1}{4}\int \frac{dt}{\sqrt{1-t^2}}$$

$$= \frac{1}{4}\sin^{-1}t + C = \frac{1}{4}\sin^{-1}(x^4) + C$$

**Question 13:**

$$\frac{e^x}{(1+e^x)(2+e^x)}$$

**Answer 13:**

$$\frac{e^x}{(1+e^x)(2+e^x)}$$

Let $e^x = t \quad \Rightarrow \; e^x dx = dt$

$$I = \int \frac{e^x}{(1+e^x)(2+e^x)} dx = \int \frac{dt}{(t+1)(t+2)} = \int \left[\frac{1}{(t+1)} - \frac{1}{(t+2)}\right] dt$$

$$= \log|t+1| - \log|t+2| + C = \log\left|\frac{t+1}{t+2}\right| + C = \log\left|\frac{1+e^x}{2+e^x}\right| + C$$

**Question 14:**

$$\frac{1}{(x^2+1)(x^2+4)}$$

**Answer 14:**

$$\frac{1}{(x^2+1)(x^2+4)} = \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+4)} \qquad \ldots (1)$$

$\Rightarrow 1 = (Ax+B)(x^2+4) + (Cx+D)(x^2+1)$

$\Rightarrow 1 = Ax^3 + 4Ax + Bx^2 + 4B + Cx^3 + Cx + Dx^2 + D$

Equating the coefficients of $x^3, x^2, x$ and constant term, we have

A + C = 0, B + D = 0, 4A + C = 0 and 4B + D = 1

On solving these equations, we have

$$A = 0, \quad B = \frac{1}{3}, \quad C = 0 \quad \text{and} \quad D = -\frac{1}{3}$$

From equation (1), we have

$$\frac{1}{(x^2+1)(x^2+4)} = \frac{1}{3(x^2+1)} - \frac{1}{3(x^2+4)}$$

Therefore, $\quad I = \int \frac{1}{(x^2+1)(x^2+4)} dx = \frac{1}{3}\int \frac{1}{(x^2+1)} dx - \frac{1}{3}\int \frac{1}{(x^2+4)} dx$

$$= \frac{1}{3} tan^{-1}x - \frac{1}{3}.\frac{1}{2} tan^{-1}\frac{x}{2} + C = \frac{1}{3} tan^{-1}x - \frac{1}{6} tan^{-1}\frac{x}{2} + C$$

**Question 15:**

$$cos^3 x e^{\log \sin x}$$

**Answer 15:**

$I = \int cos^3 x e^{\log \sin x} dx = \int cos^3 x \sin x \, dx \qquad \left[\text{Since } e^{\log x} = x\right]$

Let $\cos x = t \quad \Rightarrow \sin x \, dx = dt$

$$I = -\int t^3 \, dt = -\int \frac{t^4}{4} + C = -\frac{cos^4 x}{4} + C$$

**Question 16:**

$$e^{3\log x}(x^4+1)^{-1}$$

**Answer 16:**

$$e^{3\log x}(x^4+1)^{-1} = e^{\log x^3}(x^4+1)^{-1} = \frac{x^3}{(x^4+1)} \qquad \left[\text{Since } e^{\log x} = x\right]$$

Let $x^4+1 = t \quad \Rightarrow 4x^3 dx = dt$

$$I = \int e^{3\log x}(x^4+1)^{-1}dx = \int \frac{x^3}{(x^4+1)}dx$$

$$= \frac{1}{4}\log|t| + C = \frac{1}{4}\log|x^4+1| + C = \frac{1}{4}\log(x^4+1) + C$$

**Question 17:**

$$f'(ax+b)\,[f(ax+b)]^n$$

**Answer 17:**

$$f'(ax+b)\left[f(ax+b)\right]^n$$

Put, $f(ax+b) = t \Rightarrow af'(ax+b)dx = dt$

$$\Rightarrow \int f'(ax+b)\left[f(ax+b)\right]^n dx = \frac{1}{a}\int t^n dt$$

$$= \frac{1}{a}\left[\frac{t^{n+1}}{n+1}\right] = \frac{1}{a(n+1)}\left(f(ax+b)\right)^{n+1} + C$$

**Question 18:**

$$\frac{1}{\sqrt{sin^3 x \sin(x+a)}}$$

**Answer 18:**

$$\frac{1}{\sqrt{sin^3 x \sin(x+a)}} = \frac{1}{\sqrt{sin^3 x(\sin x\cos a + \cos x \sin a)}} = \frac{1}{\sqrt{(sin^4 x\cos a + sin^3 x\cos x\sin a)}}$$

$$= \frac{1}{sin^2 x\sqrt{(\cos a + \cot x\sin a)}} = \frac{cosec^2 x}{\sqrt{(\cos a + \cot x \sin a)}}$$

Let $\cos a + \cot x \sin a = t \quad \Rightarrow -cosec^2 x \sin a\, dx = dt$

$$I = \int \frac{1}{\sqrt{sin^3 x\sin(x+a)}}dx = \int \frac{cosec^2 x}{\sqrt{(\cos a + \cot x\sin a)}}dx = \frac{-1}{\sin a}\int\frac{dt}{\sqrt{t}} = \frac{-1}{\sin a}\left[2\sqrt{t}\right] + C$$

$$= \frac{-2}{\sin a}\left[\sqrt{\cos a + \frac{\cos x\sin a}{\sin x}}\right] + C$$

$$= \frac{-2}{\sin a}\sqrt{\frac{\sin x\cos a + \cos x\sin a}{\sin x}} + C = \frac{-2}{\sin a}\sqrt{\frac{\sin(x+a)}{\sin x}} + C$$

## Question 19:

$$\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}$$

## Answer 19:

$$I=\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx$$

Let $x=cos^2\theta \quad \Rightarrow \; dx=-2\sin\theta\cos\theta\,d\theta$

$$I=\int\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\,(-2\sin\theta\cos\theta)d\theta=-\int\sqrt{\frac{2sin^2\frac{\theta}{2}}{2cos^2\frac{\theta}{2}}}\,(\sin 2\theta)d\theta=-\int\sqrt{\tan\frac{\theta}{2}}\,.2\sin\theta\cos\theta\,d\theta$$

$$=-2\int\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)\cos\theta\,d\theta=-4\int sin^2\frac{\theta}{2}\cos\theta\,d\theta=-4\int sin^2\frac{\theta}{2}\left(2cos^2\frac{\theta}{2}-1\right)d\theta$$

$$=-4\int\left(2sin^2\frac{\theta}{2}cos^2\frac{\theta}{2}-sin^2\frac{\theta}{2}\right)d\theta=-8\int sin^2\frac{\theta}{2}cos^2\frac{\theta}{2}d\theta+4\int sin^2\frac{\theta}{2}d\theta$$

$$=-2\int sin^2\theta\,d\theta+4\int sin^2\frac{\theta}{2}d\theta=-2\int\left(\frac{1-\cos 2\theta}{2}\right)d\theta+4\int\frac{1-\cos\theta}{2}d\theta$$

$$=-2\left[\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right]+4\left[\frac{\theta}{2}-\frac{\sin\theta}{2}\right]+C$$

$$=-\theta+\frac{\sin 2\theta}{2}+2\theta-2\sin\theta+C=\theta+\frac{\sin 2\theta}{2}-2\sin\theta+C$$

$$=\theta+\frac{2\sin\theta\cos\theta}{2}-2\sin\theta+C=\theta+\sqrt{1-cos^2\theta}.\cos\theta-2\sqrt{1-cos^2\theta}+C$$

$$=cos^{-1}\sqrt{x}+\sqrt{1-x}\,.\sqrt{x}-2\sqrt{1-x}+C=-2\sqrt{1-x}+cos^{-1}\sqrt{x}+\sqrt{x(1-x)}+C$$

$$=-2\sqrt{1-x}+cos^{-1}\sqrt{x}+\sqrt{(x-x^2)}+C$$

## Question 20:

$$\frac{2+\sin 2x}{1+\cos 2x}\,e^x$$

## Answer 20:

$$I=\int\left(\frac{2+\sin 2x}{1+\cos 2x}\right)e^x dx=\int\left(\frac{2+2\sin x cos\,x}{2cos^2x}\right)e^x dx=\int\left(\frac{1+\sin x cos\,x}{cos^2x}\right)e^x dx$$

$$=\int(sec^2x+\tan x)e^x\,dx$$

Let $f(x)=\tan x \quad \Rightarrow \; f'(x)=sec^2x$

We know that: $I=\int\{f(x)+f'(x)\}e^x\,dx$

Therefore, $\quad I=\int(sec^2x+\tan x)e^x\,dx=e^xf(x)+C=e^x\tan x+C$

**Question 21:**

$$\frac{x^2+x+1}{(x+1)^2(x+2)}$$

**Answer 21:**

Let $\frac{x^2+x+1}{(x+1)^2(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+2)}$ ... (1)

$\Rightarrow \frac{x^2+x+1}{(x+1)^2(x+2)} = \frac{A(x+1)(x+2)+B(x+2)+C(x+1)^2}{(x+1)^2(x+2)}$

$\Rightarrow x^2+x+1 = A(x+1)(x+2)+B(x+2)+C(x^2+2x+1)$

$\Rightarrow x^2+x+1 = A(x^2+3x+2)+B(x+2)+C(x^2+2x+1)$

$\Rightarrow x^2+x+1 = (A+C)x^2+(3A+B+2C)x+(2A+2B+C)$

Equating the coefficients of $x^2$, x and constant term, we have

A + C = 1, 3A + B + 2C = 1 and 2A + 2B + C = 1

On solving these equations, we have

A = −2, B = 1 and C = 3

From equation (1), we have

$$\frac{x^2+x+1}{(x+1)^2(x+2)} = \frac{-2}{(x+1)} + \frac{1}{(x+1)^2} + \frac{3}{(x+2)}$$

Therefore, $I = \int \frac{x^2+x+1}{(x+1)^2(x+2)}dx = -2\int \frac{1}{(x+1)}dx + 3\int \frac{1}{(x+2)}dx + \int \frac{1}{(x+1)^2}dx$

$= -2\log|x+1| + 3\log|x+2| - \frac{1}{(x+1)} + C$

**Question 22:**

$$tan^{-1}\sqrt{\frac{1-x}{1+x}}\ dx$$

**Answer 22:**

$$I = \int tan^{-1}\sqrt{\frac{1-x}{1+x}}\ dx$$

Let $x = \cos\theta \quad \Rightarrow dx = -\sin\theta\, d\theta$

$I = \int tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\ (-\sin\theta\, d\theta) = -\int tan^{-1}\sqrt{\frac{2sin^2\frac{\theta}{2}}{2cos^2\frac{\theta}{2}}}\ \sin\theta\, d\theta = -\int tan^{-1}\tan\frac{\theta}{2}.\sin\theta\, d\theta$

$= -\frac{1}{2}\int \theta\sin\theta\, d\theta = -\frac{1}{2}\left[\theta(-\cos\theta) - \int 1.(-\cos\theta)\, d\theta\right] = -\frac{1}{2}[-\theta\cos\theta + sin\,\theta]$

$= \frac{1}{2}\theta\cos\theta - \frac{1}{2}\sqrt{1-\cos^2\theta} = \frac{1}{2}cos^{-1}x.x - \frac{1}{2}\sqrt{1-x^2} + C = \frac{x}{2}cos^{-1}x - \frac{1}{2}\sqrt{1-x^2} + C$

$= \frac{1}{2}\left(xcos^{-1}x - \sqrt{1-x^2}\right) + C$

## Question 23:

$$\frac{\sqrt{x^2+1}\ [\log(x^2+1)-2\log x]}{x^4}$$

## Answer 23:

$$\frac{\sqrt{x^2+1}\ [\log(x^2+1)-2\log x]}{x^4}=\frac{\sqrt{x^2+1}}{x^4}[\log(x^2+1)-2\log x]=\frac{\sqrt{x^2+1}}{x^4}\left[\log\left(\frac{x^2+1}{x^2}\right)\right]$$

$$=\frac{\sqrt{x^2+1}}{x^4}\left[log\left(1+\frac{1}{x^2}\right)\right]=\frac{1}{x^3}\sqrt{\frac{x^2+1}{x^2}}\left[log\left(1+\frac{1}{x^2}\right)\right]=\frac{1}{x^3}\sqrt{1+\frac{1}{x^2}}\left[log\left(1+\frac{1}{x^2}\right)\right]$$

Let $1+\frac{1}{x^2}=t \quad \Rightarrow \frac{-2}{x^3}dx=dt$

Therefore, $\quad I=\int\frac{1}{x^3}\sqrt{1+\frac{1}{x^2}}\log\left(1+\frac{1}{x^2}\right)dx=-\frac{1}{2}\int\sqrt{t}\log t\,dt=-\frac{1}{2}\int t^{\frac{1}{2}}.\log t\,dt$

Integrating by parts, we have

$$I=-\frac{1}{2}\left[\log t.\int t^{\frac{1}{2}}dt-\left\{\left(\frac{d}{dt}\log t\right)\int t^{\frac{1}{2}}dt\right\}dt\right]=-\frac{1}{2}\left[\log t.\frac{t^{\frac{3}{2}}}{\frac{3}{2}}-\int\frac{1}{t}.\frac{t^{\frac{3}{2}}}{\frac{3}{2}}dt\right]$$

$$=-\frac{1}{2}\left[\frac{2}{3}t^{\frac{3}{2}}\log t-\frac{2}{3}\int t^{\frac{1}{2}}dt\right]=-\frac{1}{2}\left[\frac{2}{3}t^{\frac{3}{2}}\log t-\frac{4}{9}.t^{\frac{3}{2}}\right]=\frac{1}{3}t^{\frac{3}{2}}\log t-\frac{2}{9}.t^{\frac{3}{2}}$$

$$=\frac{1}{3}t^{\frac{3}{2}}\left[\log t-\frac{2}{3}\right]=\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{\frac{3}{2}}\left[\log\left(1+\frac{1}{x^2}\right)-\frac{2}{3}\right]+C$$

**Evaluate the definite integrals in Exercises 24 to 31.**

## Question 24:

$$\int_{\frac{\pi}{2}}^{\pi}e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx$$

## Answer 24:

$$I=\int_{\frac{\pi}{2}}^{\pi}e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx=\int_{\frac{\pi}{2}}^{\pi}e^x\left(\frac{1-2\sin\frac{x}{2}\cos\frac{x}{2}}{2sin^2\frac{x}{2}}\right)dx=\int_{\frac{\pi}{2}}^{\pi}e^x\left(\frac{\operatorname{cosec}\frac{x}{2}}{2}-\cot\frac{x}{2}\right)dx$$

Let $f(x)=-\cot\frac{x}{2} \quad \Rightarrow f'(x)=-\left(-\frac{1}{2}cosec^2\frac{x}{2}\right)=\frac{1}{2}cosec^2\frac{x}{2}$

Therefore, $\quad I=\int_{\frac{\pi}{2}}^{\pi}e^x\left(f(x)+f'(x)\right)dx$

$$=\left[e^x.\cot\frac{x}{2}\right]_{\frac{\pi}{2}}^{\pi}=-\left[e^{\pi}.\cot\frac{\pi}{2}-e^{\frac{\pi}{2}}.\cot\frac{\pi}{4}\right]$$

$$=-\left[e^{\pi}\times 0-e^{\frac{\pi}{2}}\times 1\right]=e^{\frac{\pi}{2}}$$

**Question 25:**

$$\int_0^{\frac{\pi}{4}} \frac{\sin x \cos x}{cos^4x + sin^4x}\, dx$$

**Answer 25:**

$$I = \int_0^{\frac{\pi}{4}} \frac{\sin x \cos x}{cos^4x + sin^4x}\, dx = \int_0^{\frac{\pi}{4}} \frac{\frac{(\sin x \cos)x}{cos^4x}}{\frac{(cos^4x + sin^4x)}{cos^4x}}\, dx = \int_0^{\frac{\pi}{4}} \frac{\tan x\ sec^2x}{1 + tan^4x}\, dx$$

Let $tan^2x = t \quad \Rightarrow 2\tan x\ sec^2x dx = dt$

When $x = 0$ , $t = 0$ and when $x = \frac{\pi}{4}$ , $t = 1$

$$I = \frac{1}{2}\int_0^1 \frac{dt}{1+t^2} = \frac{1}{2}[tan^{-1}t]_0^1 = \frac{1}{2}[tan^{-1}1 - tan^{-1}0] = \frac{1}{2}\left[\frac{\pi}{4}\right] = \frac{\pi}{8}$$

**Question 26:**

$$\int_0^{\frac{\pi}{2}} \frac{cos^2x}{cos^2x + 4sin^2x}\, dx$$

**Answer 26:**

$$I = \int_0^{\frac{\pi}{2}} \frac{cos^2x}{cos^2x + 4sin^2x}\, dx = \int_0^{\frac{\pi}{2}} \frac{cos^2x}{cos^2x + 4(1 - cos^2x)}\, dx = \int_0^{\frac{\pi}{2}} \frac{cos^2x}{cos^2x + 4-4cos^2x}\, dx$$

$$= \frac{-1}{3}\int_0^{\frac{\pi}{2}} \frac{4 - 3cos^2x - 4}{4-3cos^2x}\, dx = \frac{-1}{3}\int_0^{\frac{\pi}{2}} \frac{4 - 3cos^2x}{4-3cos^2x}\, dx + \frac{1}{3}\int_0^{\frac{\pi}{2}} \frac{4}{4-3cos^2x}\, dx$$

$$= \frac{-1}{3}\int_0^{\frac{\pi}{2}} 1\, dx + \frac{1}{3}\int_0^{\frac{\pi}{2}} \frac{4sec^2x}{4sec^2x - 3}\, dx = \frac{-1}{3}\int_0^{\frac{\pi}{2}} 1\, dx + \frac{1}{3}\int_0^{\frac{\pi}{2}} \frac{4sec^2x}{4(1 + tan^2x) - 3}\, dx$$

$$= \frac{-1}{3}\ [x]_0^{\frac{\pi}{2}} + \frac{1}{3}\int_0^{\frac{\pi}{2}} \frac{4sec^2x}{1+4tan^2x}\, dx = -\frac{\pi}{6} + \frac{2}{3}\int_0^{\frac{\pi}{2}} \frac{2sec^2x}{1+4tan^2x}\, dx \qquad ...(1)$$

For $\int_0^{\frac{\pi}{2}} \frac{2sec^2x}{1+4tan^2x}\, dx$ let $2\tan x = t \quad \Rightarrow 2sec^2x\, dx = dt$

When $x = 0, t = 0$ and $x = \frac{\pi}{2}, t = \infty$

$$\int_0^{\frac{\pi}{2}} \frac{2sec^2x}{1+4tan^2x}\, dx = \int_0^{\infty} \frac{dt}{1+t^2}$$

$$= [tan^{-1}t]_0^{\infty} = [tan^{-1}(\infty) - tan^{-1}(0)] = \frac{\pi}{2}$$

Therefore from (1), we have

$$I = -\frac{\pi}{6} + \frac{2}{3}\left[\frac{\pi}{2}\right] = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$$

**Question 27:**

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{\sin 2x}}\, dx$$

**Answer 27:**

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{\sin 2x}}\, dx I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{-(-sin\, 2x)}}\, dx I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{-(-1 + 1 - 2\sin x \cos x)}}\, dx$$

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{1 - (sin^2 x + cos^2 x - 2\sin x \cos x)}}\, dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{1 - (\sin x - \cos x)^2}}\, dx$$

Let $(\sin x - \cos x) = t$

$\Rightarrow\ (\sin x - \cos x)dx = dt$

When $x = \frac{\pi}{6}, t = \left(\frac{1 - \sqrt{3}}{2}\right)$ and when $x = \frac{\pi}{3}, t = \left(\frac{\sqrt{3} - 1}{2}\right)$

$$I = \int_{\left(\frac{1-\sqrt{3}}{2}\right)}^{\left(\frac{\sqrt{3}-1}{2}\right)} \frac{dt}{\sqrt{1 - (t)^2}}$$

$$\int_{-a}^{a} f(x)\, dx = 2\int_{0}^{a} f(x)\, dx \qquad \left[\text{As } \frac{1}{\sqrt{1 - (-t)^2}} = \frac{dt}{\sqrt{1 - t^2}}, \text{therefore } \frac{dt}{\sqrt{1 - t^2}} \text{ is an even function}\right]$$

Therefore, $\quad I = 2\int_{0}^{\left(\frac{\sqrt{3}-1}{2}\right)} \frac{dt}{\sqrt{1 - t^2}} = [2\, sin^{-1} t]_0^{\frac{\sqrt{3}-1}{2}} = 2\, sin^{-1}\left(\frac{\sqrt{3} - 1}{2}\right)$

**Question 28:**

$$\int_0^1 \frac{dx}{\sqrt{1 + x} - \sqrt{x}}$$

**Answer 28:**

$$I = \int_0^1 \frac{dx}{\sqrt{1 + x} - \sqrt{x}} dx$$

$$= \int_0^1 \frac{dx}{\sqrt{1 + x} - \sqrt{x}} \times \frac{(\sqrt{1 + x} + \sqrt{x})}{(\sqrt{1 + x} + \sqrt{x})}\, dx = \int_0^1 \frac{\sqrt{1 + x} + \sqrt{x}}{1 + x - x} dx$$

$$= \int_0^1 \sqrt{1 + x} dx + \int_0^1 \sqrt{x} dx = \left[\frac{2}{3}(1 + x)^{\frac{3}{2}}\right]_0^1$$

$$= \left[\frac{2}{3}(x)^{\frac{3}{2}}\right]_0^1 = \frac{2}{3}\left[(2)^{\frac{3}{2}}\right] + \frac{2}{3}[1]$$

$$= \frac{2}{3}(2)^{\frac{3}{2}} = \frac{2.2\sqrt{2}}{3} = \frac{4\sqrt{2}}{3}$$

## Question 29:

$$\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x}\, dx$$

## Answer 29:

$$I = \int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x}\, dx$$

Let $\sin x - \cos x = t \quad \Rightarrow (\cos x + \sin x)dx = dt$

When $x = 0, t = -1$ and when $x = \frac{\pi}{4}, t = 0$

Now, $\quad (\sin x - \cos x)^2 = t^2 \quad \Rightarrow sin^2 x + \ sin^2 x - 2\sin x \cos x = \ t^2 \Rightarrow 1 - \sin 2x = \ t^2$

$\Rightarrow sin\, 2x = 1 - \ t^2$

Therefore, $\quad I = \int_{-1}^{0} \frac{dt}{9 + 16 - 16t^2} = \int_{-1}^{0} \frac{dt}{25 - 16t^2} = \int_{-1}^{0} \frac{dt}{(5)^2 - (4t)^2}$

$= \frac{1}{4}\left[\frac{1}{2(5)} \log \left|\frac{5 + 4t}{5 - 4t}\right|\right]_{-1}^{0} = \frac{1}{40}\left[\log|1| - \log\left|\frac{1}{9}\right|\right] = \frac{1}{40} \log 9$

## Question 30:

$$\int_0^{\frac{\pi}{2}} \sin 2x\, tan^{-1}(\sin x)\, dx$$

## Answer 30:

$$I = \int_0^{\frac{\pi}{2}} \sin 2x\, tan^{-1}(\sin x)\, dx = \int_0^{\frac{\pi}{2}} 2 \sin x \cos x\, tan^{-1}(\sin x)\, dx$$

Let $\sin x = t$

$\Rightarrow \ \cos x\, dx = dt$

When $x = 0, t = 0$ and when $x = \frac{\pi}{2}, t = 1$

$I = 2\int_0^1 t\, tan^{-1}(t) dt \quad \ldots (1)$

Now $\int t\, tan^{-1} t\, dt = tan^{-1} t \int t\, dt - \int \left\{\frac{d}{dt}(tan^{-1} t) \int t\, dt\right\} dt = tan^{-1} t . \frac{t^2}{2} - \int \frac{1}{1 + t^2} . \frac{t^2}{2} dt$

$= \frac{t^2 tan^{-1} t}{2} - \frac{1}{2}\int \frac{t^2 + 1 - 1}{1 + t^2}\, dt$

$= \frac{t^2 tan^{-1} t}{2} - \frac{1}{2}\int 1\ dt + \frac{1}{2}\int \frac{1}{1 + t^2}\, dt = \frac{t^2 tan^{-1} t}{2} - \frac{1}{2} . t + \frac{1}{2} tan^{-1} t$

$\Rightarrow I = 2\int_0^1 t\, tan^{-1} t\, dt = 2\left[\frac{t^2 tan^{-1} t}{2} - \frac{t}{2} + \frac{1}{2} tan^{-1} t\right]_0^1$

$= \left[\frac{\pi}{4} - 1 + \frac{\pi}{4}\right] \quad = \frac{1}{2}\left[\frac{\pi}{2} - 1\right] = \frac{\pi}{2} - 1$

## Question 31:

$$\int_1^4 [|x-1|+|x-2|+|x-3|]dx$$

## Answer 31:

$$I=\int_1^4 [|x-1|+|x-2|+|x-3|]dx=\int_1^4 |x-1|dx+\int_1^4 |x-2|dx+\int_1^4 |x-3|dx$$

Let $I=I_1+I_2+I_3$ ... (1)

Where $I_1=\int_1^4 |x-1|dx,\quad I_2=\int_1^4 |x-2|dx\quad$ and $\quad I_3=\int_1^4 |x-3|dx$

$$I_1=\int_1^4 |x-1|dx$$

Here, $(x-1)>0$ if $x>4$

$$I_1=\int_1^4 (x-1)dx=\left[\frac{x^2}{x}-x\right]_1^4=\left[8-4-\frac{1}{2}+1\right]=\frac{9}{2} \qquad \text{... (2)}$$

$$I_2=\int_1^4 |x-2|\,dx$$

Here, $(x-2)>0$ if $x>2$ and $x-2<0$ if $x<2$

$$I_2=\int_1^2 (2-x)dx+\int_2^4 (x-2\,)dx=\left[2x-\frac{x^2}{x}\right]_1^2+\left[\frac{x^2}{x}-2x\right]_2^4$$

$$I_2=\left[4-2-2-\frac{1}{2}\right]+[8-8-2+4]=\frac{1}{2}+2=\frac{5}{2} \qquad \text{... (3)}$$

$$I_3=\int_1^4 |x-3|dx$$

Here, $(x-3)>0$ if $x>3$ and $x-3<0$ if $x<3$

$$I_3=\int_1^3 (3-x)dx+\int_3^4 (x-3)dx=\left[3x-\frac{x^2}{x}\right]_1^3+\left[\frac{x^2}{x}-3x\right]_3^4$$

$$I_3=\left[9-\frac{9}{2}-3+\frac{1}{2}\right]+\left[8-12-\frac{9}{2}+9\right]=[6-4]+\left[\frac{1}{2}\right]=\frac{5}{2} \qquad \text{... (4)}$$

From equations (1), (2), (3), and (4), we have

$$I=\frac{9}{2}+\frac{5}{2}+\frac{5}{2}=\frac{19}{2}$$

**Prove the following (Exercises 32 to 37)**

## Question 32:

Prove the following: $\displaystyle\int_1^3 \frac{dx}{x^2(x+1)}=\frac{2}{3}+\log\left(\frac{2}{3}\right)$

## Answer 32:

$$I=\int_1^3 \frac{dx}{x^2(x+1)}$$

Let $\displaystyle\frac{1}{x^2(x+1)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+1}=\frac{Ax(x+1)+B(x+1)+Cx^2}{x^2(x+1)}$

$\Rightarrow 1=Ax(x+1)+B(x+1)+C(x^2) \qquad \Rightarrow 1=Ax^2+Ax+Bx+Cx^2$

Equating the coefficients of $x^2$, $x$ and constant term, we have

A + C = 0, A + B = 0 and B = 1

On solving these equations, we have

A = −1, B = 1 and C = 1

$$\frac{1}{x^2(x+1)} = \frac{-1}{x} + \frac{1}{x^2} + \frac{1}{x+1}$$

$$I = \int_1^3 \left\{-\frac{1}{x} + \frac{1}{x^2} + \frac{1}{(x+1)}\right\} dx = \left[-\log x - \frac{1}{x} + \log(x+1)\right]_1^3 = \left[\log\left(\frac{x+1}{x}\right) - \frac{1}{x}\right]_1^3$$

$$= \log\left(\frac{4}{3}\right) - \frac{1}{3} - \log\left(\frac{2}{1}\right) + 1 = \log 4 - \log 3 - \log 2 + \frac{2}{3} = \log\left(\frac{2}{3}\right) + \frac{2}{3}$$

Hence, the result is proved.

## Question 33:

Prove the following: $\int_0^1 x\, e^x dx = 1$

## Answer 33:

$$I = \int_0^1 x\, e^x dx$$

Integrating by parts, we have

$$I = x\int_0^1 e^x\, dx - \int_0^1 \left\{\frac{d}{dx}(x)\int e^x\, dx\right\} dx$$

$$= [xe^x]_0^1 - [e^x]_0^1 = e - e + 1 \quad = 1$$

Hence, the result is proved.

## Question 34:

Prove the following: $\int_{-1}^1 x^{17} cos^4 x\, dx = 0$

## Answer 34:

$$I = \int_{-1}^1 x^{17} cos^4 x\, dx$$

Let $f(x) = x^{17} cos^4 x$

Therefore, $f(-x) = (-x)^{17} cos^4(-x) = -x^{17} cos^4 x = -f(x)$

Therefore, $f(x)$ is an odd function.

We know that if $f(x)$ is an odd function, then $\int_{-a}^{a} f(x)dx = 0$

Therefore, $I = \int_{-1}^1 x^{17} cos^4 x\, dx = 0$

Hence, the result is proved.

**Question 35:**

Prove the following: $\int_0^{\frac{\pi}{2}} sin^3x\, dx = \frac{2}{3}$

**Answer 35:**

$$I = \int_0^{\frac{\pi}{2}} sin^3x\, dx = \int_0^{\frac{\pi}{2}} sin^2x\ \sin x\, dx = \int_0^{\frac{\pi}{2}} (1 - cos^2x)\ \sin x\, dx = \int_0^{\frac{\pi}{2}} \sin x\, dx - \int_0^{\frac{\pi}{2}} cos^2x\ \sin x\, dx$$

$$I = [-\cos x]_0^{\frac{\pi}{2}} + \left[\frac{cos^3x}{3}\right]_0^{\frac{\pi}{2}} = 1 + \frac{1}{3}[-1] = 1 - \frac{1}{3} = \frac{2}{3}$$

Hence, the given result is proved.

**Question 36:**

Prove the following: $\int_0^{\frac{\pi}{4}} 2tan^3x\, dx = 1 - \log 2$

**Answer 36:**

$$I = \int_0^{\frac{\pi}{4}} 2tan^3x\, dx = 2\int_0^{\frac{\pi}{4}} tan^2x \tan x\ \ dx = \int_0^{\frac{\pi}{4}} (sec^2x - 1) \tan x\, dx$$

$$= \int_0^{\frac{\pi}{4}} (sec^2x\ tan\ x)dx - 2\int_0^{\frac{\pi}{4}} \tan x\, dx = 2\left[\frac{tan^2x}{2}\right]_0^{\frac{\pi}{4}} + 2[\log\cos x]_0^{\frac{\pi}{4}}$$

$$= (1 - 0) + 2\left[\log\cos\frac{\pi}{4} - \log\cos 0\right] = 1 + 2\left[\log\frac{1}{\sqrt{2}} - \log 1\right] = 1 - \log 2 - 0 = 1 - \log 2$$

Hence, the result is proved.

**Question 37:**

Prove the following: $\int_0^1 sin^{-1}x\, dx = \frac{\pi}{2} - 1$

**Answer 37:**

$$I = \int_0^1 sin^{-1}x\, dx = \int_0^1 sin^{-1}x\ .1dx$$

Integrating by parts, we have

$$I = [xsin^{-1}x]_0^1 - \int_0^1 \frac{1}{\sqrt{1-x^2}}.x\ \ dx = [xsin^{-1}x]_0^1 + \frac{1}{2}\int_0^1 \frac{(-2x)}{\sqrt{1-x^2}}dx$$

Let $1 - x^2 = t \qquad \Rightarrow -2x\, dx = dt$

When $x = 0, t = 1$ and when $x = 1, t = 0$

$$I = [xsin^{-1}x]_0^1 + \frac{1}{2}\int_1^0 \frac{dt}{\sqrt{t}} = [xsin^{-1}x]_0^1 + \frac{1}{2}\left[2\sqrt{t}\right]_1^0$$

$$= sin^{-1}(1) + \left[-\sqrt{1}\right] = \frac{\pi}{2} - 1$$

Hence, the result is proved.

**Choose the correct answers in Exercises 38 to 40**

## Question 38:

$$\int \frac{dx}{e^x + e^{-x}} \quad \text{is equal to}$$

(A) $tan^{-1}(e^x) + C$

(B) $tan^{-1}(e^{-x}) + C$

(C) $tan(e^x - e^{-x}) + C$

(D) $tan(e^x + e^{-x}) + C$

## Answer 38:

$$I = \int \frac{1}{e^x + e^{-x}} dx = \int \frac{e^x}{e^{2x} + 1} dx$$

Let $e^x = t$

$\Rightarrow e^x dx = dt$

$$I = \int \frac{dt}{1 + t^2} = tan^{-1}t + C = tan^{-1}(e^x) + C$$

Hence, the correct answer is (A).

## Question 39:

$$\int \frac{\cos 2x}{(\sin x + \cos x)^2}\, dx \quad \text{is equal to}$$

(A) $\frac{-1}{\sin x + \cos x} + C$

(B) $\log|\sin x + \cos x| + C$

(C) $\log|\sin x - \cos x| + C$

(D) $\frac{1}{(\sin x + \cos x)^2}$

## Answer 39:

$$I = \int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$$

$$= \int \frac{cos^2x - sin^2x}{(\sin x + \cos x)^2} dx$$

$$= \int \frac{(\cos x + \sin x)(\cos x - \sin x)}{(\sin x + \cos x)^2} dx$$

$$\Rightarrow I = \int \frac{(\cos x - \sin x)}{(\sin x + \cos x)} dx$$

Let $\sin x + \cos x = t$

$\Rightarrow (\cos x - \sin x)\, dx = dt$

$$I = \int \frac{dt}{t} + C = \log|t| + C$$

$= \log|\cos x + \sin x| + C$

Hence, the correct answer is (B).

**Question 40:**

If $f(a+b-x)=f(x)$, then $\int_a^b xf(x)dx$ is equal to

(A) $\frac{a+b}{2}\int_a^b f(b-x)dx$

(B) $\frac{a+b}{2}\int_a^b f(b+x)dx$

(C) $\frac{b-a}{2}\int_a^b f(x)dx$

(D) $\frac{a+b}{2}\int_a^b f(x)dx$

**Answer 40:**

$$I=\int_a^b xf(x)dx \qquad \dots(1)$$

$I=\int_a^b (a+b-x)f(a+b-x)dx \quad \left[\text{Using property: }\int_a^b f(x)dx=\int_a^b f(a+b-x)dx\right]$

$I=\int_a^b (a+b-x)f(x)dx$

$I=(a+b)\int_a^b (x)dx-I \qquad$ [From (1)]

$\Rightarrow I+I=(a+b)\int_a^b (x)dx$

$\Rightarrow 2I=(a+b)\int_a^b (x)dx$

$\Rightarrow \ I=\left(\frac{a+b}{2}\right)\int_a^b (x)dx$

Hence, the correct answer is (D).