

Mathematics

(Chapter – 7) (Co-ordinate Geometry)

(Class – X)

Exercise 7.1

Question 1:

Find the distance between the following pairs of points:

(i) (2, 3), (4, 1)

(ii) (-5, 7), (-1, 3)

(iii) (a, b), (-a, -b)

Answer 1:

(i) Distance between the two points is given by

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Therefore, distance between (2, 3) and (4, 1) is given by

$$\begin{aligned} l &= \sqrt{(2-4)^2 + (3-1)^2} = \sqrt{(-2)^2 + (2)^2} \\ &= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2} \end{aligned}$$

(ii) Distance between (-5, 7) and (-1, 3) is given by

$$\begin{aligned} l &= \sqrt{(-5 - (-1))^2 + (7-3)^2} = \sqrt{(-4)^2 + (4)^2} \\ &= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2} \end{aligned}$$

(iii) Distance between (a, b) and (-a, -b) is given by

$$\begin{aligned} l &= \sqrt{(a - (-a))^2 + (b - (-b))^2} \\ &= \sqrt{(2a)^2 + (2b)^2} = \sqrt{4a^2 + 4b^2} = 2\sqrt{a^2 + b^2} \end{aligned}$$

Question 2:

Find the distance between the points (0, 0) and (36, 15). Can you now find the distance between the two towns A and B discussed in Section 7.2.

Answer 2:

Distance between points (0, 0) and (36, 15).

$$\begin{aligned}
 &= \sqrt{(36-0)^2 + (15-0)^2} = \sqrt{36^2 + 15^2} \\
 &= \sqrt{1296 + 225} = \sqrt{1521} = 39
 \end{aligned}$$

Yes, we can find the distance between the given towns A and B.

Assume town A at origin point (0, 0).

Therefore, town B will be at point (36, 15) with respect to town A. And hence, as calculated above, the distance between town A and B will be 39 km.

Question 3:

Determine if the points (1, 5), (2, 3) and (-2, -11) are collinear.

Answer 3:

Let the points (1, 5), (2, 3), and (-2, -11) be representing the vertices A, B, and C of the given triangle respectively.

$$\text{Let } A = (1, 5), B = (2, 3), C = (-2, -11)$$

$$\therefore AB = \sqrt{(1-2)^2 + (5-3)^2} = \sqrt{5}$$

$$BC = \sqrt{(2-(-2))^2 + (3-(-11))^2} = \sqrt{4^2 + 14^2} = \sqrt{16 + 196} = \sqrt{212}$$

$$CA = \sqrt{(1-(-2))^2 + (5-(-11))^2} = \sqrt{3^2 + 16^2} = \sqrt{9 + 256} = \sqrt{265}$$

Since $AB + BC \neq CA$,

Therefore, the points (1, 5), (2, 3), and (-2, -11) are not collinear.

Question 4:

Check whether (5, -2), (6, 4) and (7, -2) are the vertices of an isosceles triangle.

Answer 4:

Let the points (5, -2), (6, 4), and (7, -2) are representing the vertices A, B, and C of the given triangle respectively.

$$AB = \sqrt{(5-6)^2 + (-2-4)^2} = \sqrt{(-1)^2 + (-6)^2} = \sqrt{1+36} = \sqrt{37}$$

$$BC = \sqrt{(6-7)^2 + (4-(-2))^2} = \sqrt{(-1)^2 + (6)^2} = \sqrt{1+36} = \sqrt{37}$$

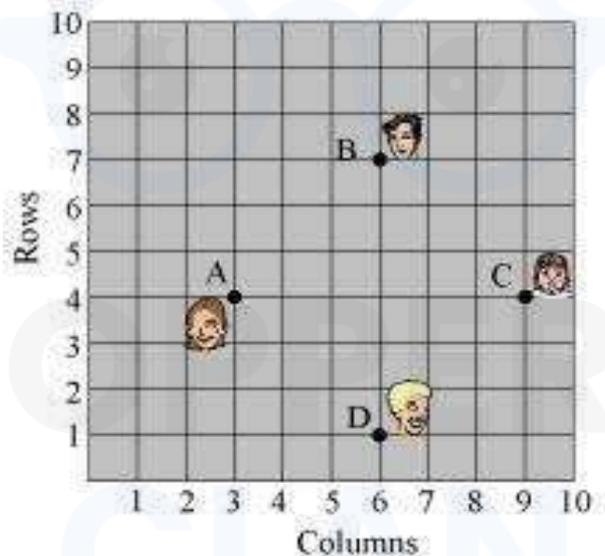
$$CA = \sqrt{(5-7)^2 + (-2-(-2))^2} = \sqrt{(-2)^2 + 0^2} = 2$$

Therefore, $AB = BC$

As two sides are equal in length, therefore, ABC is an isosceles triangle.

Question 5:

In a classroom, 4 friends are seated at the points A, B, C and D as shown in the following figure. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees. Using distance formula, find which of them is correct.



Answer 5:

It can be observed that A (3, 4), B (6, 7), C (9, 4), and D (6, 1) are the positions of these 4 friends.

$$AB = \sqrt{(3-6)^2 + (4-7)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

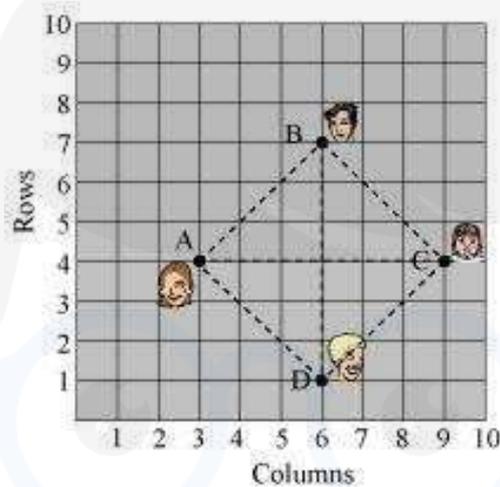
$$BC = \sqrt{(6-9)^2 + (7-4)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$CB = \sqrt{(9-6)^2 + (4-1)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$AD = \sqrt{(3-6)^2 + (4-1)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$\text{Diagonal AC} = \sqrt{(3-9)^2 + (4-4)^2} = \sqrt{(-6)^2 + 0^2} = 6$$

$$\text{Diagonal BD} = \sqrt{(6-6)^2 + (7-1)^2} = \sqrt{0^2 + (6)^2} = 6$$



It can be observed that all sides of this quadrilateral ABCD are of the same length and also the diagonals are of the same length.

Therefore, ABCD is a square and hence, Champa was correct

Question 6:

Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer:

(i) $(-1, -2), (1, 0), (-1, 2), (-3, 0)$

(ii) $(-3, 5), (3, 1), (0, 3), (-1, -4)$

(iii) $(4, 5), (7, 6), (4, 3), (1, 2)$

Answer 6:

(i) Let the points $(-1, -2)$, $(1, 0)$, $(-1, 2)$, and $(-3, 0)$ be representing the vertices A, B, C, and D of the given quadrilateral respectively.

$$\therefore AB = \sqrt{(-1-1)^2 + (-2-0)^2} = \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$BC = \sqrt{(1-(-1))^2 + (0-2)^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$CD = \sqrt{(-1-(-3))^2 + (2-0)^2} = \sqrt{(2)^2 + (2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$AD = \sqrt{(-1-(-3))^2 + (-2-0)^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$\text{Diagonal AC} = \sqrt{(-1-(-1))^2 + (-2-2)^2} = \sqrt{0^2 + (-4)^2} = \sqrt{16} = 4$$

$$\text{Diagonal BD} = \sqrt{(1-(-3))^2 + (0-0)^2} = \sqrt{(4)^2 + 0^2} = \sqrt{16} = 4$$

It can be observed that all sides of this quadrilateral are of the same length and also, the diagonals are of the same length. Therefore, the given points are the vertices of a square.

(ii) Let the points $(-3, 5)$, $(3, 1)$, $(0, 3)$, and $(-1, -4)$ be representing the vertices A, B, C, and D of the given quadrilateral respectively.

$$AB = \sqrt{(-3-3)^2 + (5-1)^2} = \sqrt{(-6)^2 + (4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$$

$$BC = \sqrt{(3-0)^2 + (1-3)^2} = \sqrt{(3)^2 + (-2)^2} = \sqrt{9+4} = \sqrt{13}$$

$$CD = \sqrt{(0-(-1))^2 + (3-(-4))^2} = \sqrt{(1)^2 + (7)^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$$

$$AD = \sqrt{(-3-(-1))^2 + (5-(-4))^2} = \sqrt{(-2)^2 + (9)^2} = \sqrt{4+81} = \sqrt{85}$$

It can be observed that all sides of this quadrilateral are of different lengths. Therefore, it can be said that it is only a general quadrilateral, and not specific such as square, rectangle, etc.

(iii) Let the points $(4, 5)$, $(7, 6)$, $(4, 3)$, and $(1, 2)$ be representing the vertices A, B, C, and D of the given quadrilateral respectively.

$$AB = \sqrt{(4-7)^2 + (5-6)^2} = \sqrt{(-3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$$

$$BC = \sqrt{(7-4)^2 + (6-3)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18}$$

$$CD = \sqrt{(4-1)^2 + (3-2)^2} = \sqrt{(3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10}$$

$$AD = \sqrt{(4-1)^2 + (5-2)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18}$$

$$\text{Diagonal AC} = \sqrt{(4-4)^2 + (5-3)^2} = \sqrt{(0)^2 + (2)^2} = \sqrt{0+4} = 2$$

$$\text{Diagonal BD} = \sqrt{(7-1)^2 + (6-2)^2} = \sqrt{(6)^2 + (4)^2} = \sqrt{36+16} = \sqrt{52} = 13\sqrt{2}$$

It can be observed that opposite sides of this quadrilateral are of the same length. However, the diagonals are of different lengths. Therefore, the given points are the vertices of a parallelogram.

Question 7:

Find the point on the x -axis which is equidistant from $(2, -5)$ and $(-2, 9)$.

Answer 7:

We have to find a point on x -axis. Therefore, its y -coordinate will be 0.

Let the point on x -axis be $(x, 0)$.

$$\text{Distance between } (x, 0) \text{ and } (2, -5) = \sqrt{(x-2)^2 + (0-(-5))^2} = \sqrt{(x-2)^2 + (5)^2}$$

$$\text{Distance between } (x, 0) \text{ and } (-2, 9) = \sqrt{(x-(-2))^2 + (0-(-9))^2} = \sqrt{(x+2)^2 + (9)^2}$$

By the given condition, these distances are equal in measure.

$$\sqrt{(x-2)^2 + (5)^2} = \sqrt{(x+2)^2 + (9)^2}$$

$$(x-2)^2 + 25 = (x+2)^2 + 81$$

$$x^2 + 4 - 4x + 25 = x^2 + 4 + 4x + 81$$

$$8x = 25 - 81$$

$$8x = -56$$

$$x = -7$$

Therefore, the point is $(-7, 0)$.

Question 8:

Find the values of y for which the distance between the points P $(2, -3)$ and Q $(10, y)$ is 10 units.

Answer 8:

It is given that the distance between $(2, -3)$ and $(10, y)$ is 10.

$$\text{Therefore, } \sqrt{(2-10)^2 + (-3-y)^2} = 10$$

$$\sqrt{(-8)^2 + (3+y)^2} = 10$$

$$64 + (y+3)^2 = 100$$

$$(y+3)^2 = 36$$

$$y+3 = \pm 6$$

$$y+3 = 6 \text{ or } y+3 = -6$$

$$\text{Therefore, } y = 3 \text{ or } -9$$

Question 9:

If Q (0, 1) is equidistant from P (5, - 3) and R (x, 6), find the values of x. Also find the distance QR and PR.

Answer 9:

$$PQ = QR$$

$$\sqrt{(5-0)^2 + (-3-1)^2} = \sqrt{(0-x)^2 + (1-6)^2}$$

$$\sqrt{(5)^2 + (-4)^2} = \sqrt{(-x)^2 + (-5)^2}$$

$$\sqrt{25+16} = \sqrt{x^2 + 25}$$

$$41 = x^2 + 25$$

$$16 = x^2$$

$$x = \pm 4$$

Therefore, point R is (4, 6) or (-4, 6).

When point R is (4, 6),

$$PR = \sqrt{(5-4)^2 + (-3-6)^2} = \sqrt{1^2 + (-9)^2} = \sqrt{1+81} = \sqrt{82}$$

$$QR = \sqrt{(0-4)^2 + (1-6)^2} = \sqrt{(-4)^2 + (-5)^2} = \sqrt{16+25} = \sqrt{41}$$

When point R is (-4, 6),

$$PR = \sqrt{(5-(-4))^2 + (-3-6)^2} = \sqrt{(9)^2 + (-9)^2} = \sqrt{81+81} = 9\sqrt{2}$$

$$QR = \sqrt{(0-(-4))^2 + (1-6)^2} = \sqrt{(4)^2 + (-5)^2} = \sqrt{16+25} = \sqrt{41}$$

Question 10:

Find a relation between x and y such that the point (x, y) is equidistant from the point $(3, 6)$ and $(-3, 4)$.

Answer 10:

Point (x, y) is equidistant from $(3, 6)$ and $(-3, 4)$.

$$\therefore \sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x-(-3))^2 + (y-4)^2}$$

$$\sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x+3)^2 + (y-4)^2}$$

$$(x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$$

$$x^2 + 9 - 6x + y^2 + 36 - 12y = x^2 + 9 + 6x + y^2 + 16 - 8y$$

$$36 - 16 = 6x + 6x + 12y - 8y$$

$$20 = 12x + 4y$$

$$3x + y = 5$$

$$3x + y - 5 = 0$$

TOPPERS
CLAN

Mathematics

(Chapter – 7) (Co-ordinate Geometry)

(Class – X)

Exercise 7.2

Question 1:

Find the coordinates of the point which divides the join of $(-1, 7)$ and $(4, -3)$ in the ratio 2:3.

Answer 1:

Let $P(x, y)$ be the required point. Using the section formula, we obtain

$$x = \frac{2 \times 4 + 3 \times (-1)}{2 + 3} = \frac{8 - 3}{5} = \frac{5}{5} = 1$$

$$y = \frac{2 \times (-3) + 3 \times 7}{2 + 3} = \frac{-6 + 21}{5} = \frac{15}{5} = 3$$

Therefore, the point is $(1, 3)$.

Question 2:

Find the coordinates of the points of trisection of the line segment joining $(4, -1)$ and $(-2, -3)$.

Answer 2:



Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ are the points of trisection of the line segment joining the given points i.e., $AP = PQ = QB$

Therefore, point P divides AB internally in the ratio 1:2.

$$x_1 = \frac{1 \times (-2) + 2 \times 4}{1+2}, \quad y_1 = \frac{1 \times (-3) + 2 \times (-1)}{1+2}$$

$$x_1 = \frac{-2+8}{3} = \frac{6}{3} = 2, \quad y_1 = \frac{-3-2}{3} = \frac{-5}{3}$$

$$\text{Therefore, } P(x_1, y_1) = \left(2, -\frac{5}{3} \right)$$

Point Q divides AB internally in the ratio 2:1.

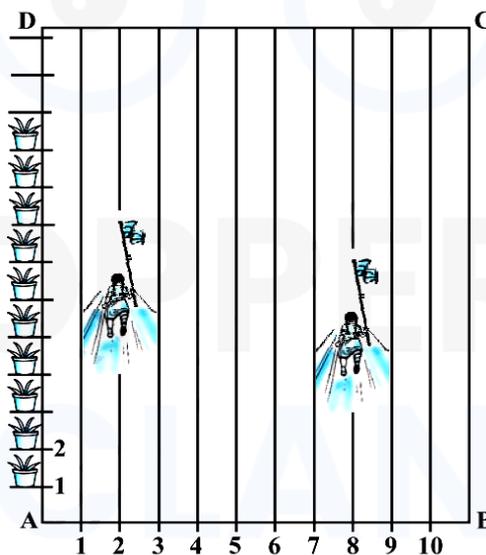
$$x_2 = \frac{2 \times (-2) + 1 \times 4}{2+1}, \quad y_2 = \frac{2 \times (-3) + 1 \times (-1)}{2+1}$$

$$x_2 = \frac{-4+4}{3} = 0, \quad y_2 = \frac{-6-1}{3} = \frac{-7}{3}$$

$$Q(x_2, y_2) = \left(0, -\frac{7}{3} \right)$$

Question 3:

To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in the following figure. Niharika runs $\frac{1}{4}$ th the distance AD on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}$ th the distance AD on the eighth line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?



Answer 3:

It can be observed that Niharika posted the green flag at $\frac{1}{4}$ of the distance AD i.e.,

$$\left(\frac{1}{4} \times 100\right) \text{m} = 25$$

metre from the starting point of 2nd line. Therefore, the coordinates of this point G is (2, 25).

Similarly, Preet posted red flag at $\frac{1}{5}$ of the distance AD i.e.,

$$\left(\frac{1}{5} \times 100\right) \text{m} = 20$$

metre from the starting point of 8th line. Therefore, the coordinates of this point R are (8, 20).

Distance between these flags by using distance formula = GR

$$= \sqrt{(8-2)^2 + (25-20)^2} = \sqrt{36+25} = \sqrt{61} \text{ m}$$

The point at which Rashmi should post her blue flag is the mid-point of the line joining these points. Let this point be A (x, y).

$$x = \frac{2+8}{2}, \quad y = \frac{25+20}{2}$$

$$x = \frac{10}{2} = 5, \quad y = \frac{45}{2} = 22.5$$

$$\text{Hence, } A(x, y) = (5, 22.5)$$

Therefore, Rashmi should post her blue flag at 22.5m on 5th line

Question 4:

Find the ratio in which the line segment joining the points (-3, 10) and (6, -8) is divided by (-1, 6).

Answer 4:

Let the ratio in which the line segment joining (-3, 10) and (6, -8) is divided by point (-1, 6) be k:1.

$$\text{Therefore, } -1 = \frac{6k-3}{k+1}$$

$$-k-1 = 6k-3$$

$$7k = 2$$

$$k = \frac{2}{7}$$

Therefore, the required ratio is 2 : 7.

Question 5:

Find the ratio in which the line segment joining A (1, -5) and B (-4, 5) is divided by the x-axis. Also find the coordinates of the point of division.

Answer 5:

Let the ratio in which the line segment joining A (1, -5) and B (-4, 5) is divided by x - axis be k:1.

Therefore, the coordinates of the point of division is $\left(\frac{-4k+1}{k+1}, \frac{5k-5}{k+1}\right)$.

We know that y-coordinate of any point on x-axis is 0.

$$\therefore \frac{5k-5}{k+1} = 0$$

$$k = 1$$

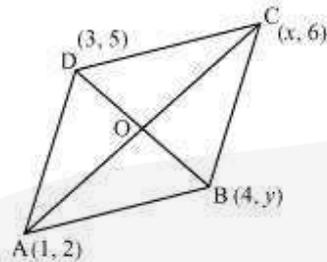
Therefore, x-axis divides it in the ratio 1:1.

$$\text{Division point} = \left(\frac{-4(1)+1}{1+1}, \frac{5(1)-5}{1+1}\right) = \left(\frac{-4+1}{2}, \frac{5-5}{2}\right) = \left(\frac{-3}{2}, 0\right)$$

Question 6:

If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

Answer 6:



Let $(1, 2)$, $(4, y)$, $(x, 6)$, and $(3, 5)$ are the coordinates of A, B, C, D vertices of a parallelogram ABCD. Intersection point O of diagonal AC and BD also divides these diagonals.

Therefore, O is the mid-point of AC and BD.

If O is the mid-point of AC, then the coordinates of O are

$$\left(\frac{1+x}{2}, \frac{2+6}{2}\right) \Rightarrow \left(\frac{x+1}{2}, 4\right)$$

If O is the mid-point of BD, then the coordinates of O are

$$\left(\frac{4+3}{2}, \frac{5+y}{2}\right) \Rightarrow \left(\frac{7}{2}, \frac{5+y}{2}\right)$$

Since both the coordinates are of the same point O,

$$\therefore \frac{x+1}{2} = \frac{7}{2} \text{ and } 4 = \frac{5+y}{2}$$

$$\Rightarrow x+1 = 7 \text{ and } 5+y = 8$$

$$\Rightarrow x = 6 \text{ and } y = 3$$

Question 7:

Find the coordinates of a point A, where AB is the diameter of circle whose centre is $(2, -3)$ and B is $(1, 4)$

Answer 7:

Let the coordinates of point A be (x, y) .

Mid-point of AB is $(2, -3)$, which is the center of the circle.

$$\therefore (2, -3) = \left(\frac{x+1}{2}, \frac{y+4}{2} \right)$$

$$\Rightarrow \frac{x+1}{2} = 2 \text{ and } \frac{y+4}{2} = -3$$

$$\Rightarrow x+1 = 4 \text{ and } y+4 = -6$$

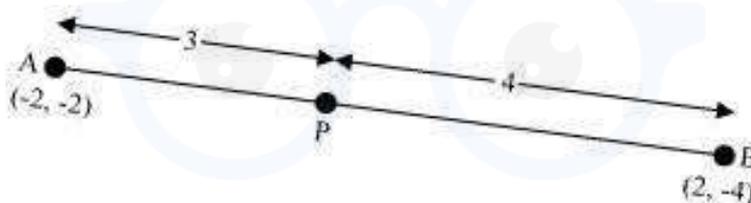
$$\Rightarrow x = 3 \text{ and } y = -10$$

Therefore, the coordinates of A are $(3, -10)$.

Question 8:

If A and B are $(-2, -2)$ and $(2, -4)$, respectively, find the coordinates of P such that $AP = \frac{3}{7}AB$ and P lies on the line segment AB.

Answer 8:



The coordinates of point A and B are $(-2, -2)$ and $(2, -4)$ respectively.

Since $AP = \frac{3}{7}AB$,

Therefore, $AP: PB = 3:4$

Point P divides the line segment AB in the ratio 3:4.

$$\begin{aligned} \text{Coordinates of P} &= \left(\frac{3 \times 2 + 4 \times (-2)}{3 + 4}, \frac{3 \times (-4) + 4 \times (-2)}{3 + 4} \right) \\ &= \left(\frac{6 - 8}{7}, \frac{-12 - 8}{7} \right) \\ &= \left(-\frac{2}{7}, -\frac{20}{7} \right) \end{aligned}$$

Question 9:

Find the coordinates of the points which divide the line segment joining A (– 2, 2) and B (2, 8) into four equal parts.

Answer 9:



From the figure, it can be observed that points P, Q, R are dividing the line segment in a ratio 1:3, 1:1, 3:1 respectively.

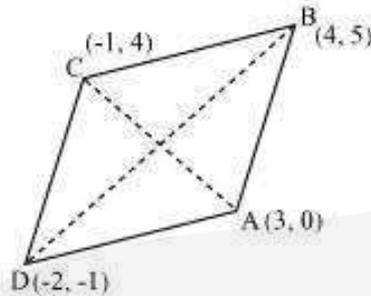
$$\begin{aligned}\text{Coordinates of P} &= \left(\frac{1 \times 2 + 3 \times (-2)}{1 + 3}, \frac{1 \times 8 + 3 \times 2}{1 + 3} \right) \\ &= \left(-1, \frac{7}{2} \right)\end{aligned}$$

$$\begin{aligned}\text{Coordinates of Q} &= \left(\frac{2 + (-2)}{2}, \frac{2 + 8}{2} \right) \\ &= (0, 5)\end{aligned}$$

$$\begin{aligned}\text{Coordinates of R} &= \left(\frac{3 \times 2 + 1 \times (-2)}{3 + 1}, \frac{3 \times 8 + 1 \times 2}{3 + 1} \right) \\ &= \left(1, \frac{13}{2} \right)\end{aligned}$$

Question 10:

Find the area of a rhombus if its vertices are $(3, 0)$, $(4, 5)$, $(-1, 4)$ and $(-2, -1)$ taken in order. [Hint: Area of a rhombus = $\frac{1}{2}$ (product of its diagonals)]

Answer 10:

Let $(3, 0)$, $(4, 5)$, $(-1, 4)$ and $(-2, -1)$ are the vertices A, B, C, D of a rhombus ABCD.

$$\begin{aligned}\text{Length of diagonal AC} &= \sqrt{[3 - (-1)]^2 + (0 - 4)^2} \\ &= \sqrt{16 + 16} = 4\sqrt{2}\end{aligned}$$

$$\begin{aligned}\text{Length of diagonal BD} &= \sqrt{[4 - (-2)]^2 + [5 - (-1)]^2} \\ &= \sqrt{36 + 36} = 6\sqrt{2}\end{aligned}$$

$$\begin{aligned}\text{Therefore, area of rhombus ABCD} &= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2} \\ &= 24 \text{ square units}\end{aligned}$$

TOPPERS
CLAN