

Mathematics

(Chapter – 6) (Triangles)

(Class – X)

Exercise 6.1

Question 1:

Fill in the blanks using correct word given in the brackets: –

- (i) All circles are _____. (congruent, similar)
- (ii) All squares are _____. (similar, congruent)
- (iii) All _____ triangles are similar. (isosceles, equilateral)
- (iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are _____ and (b) their corresponding sides are _____. (equal, proportional)

Answer 1:

- (i) Similar
- (ii) Similar
- (iii) Equilateral
- (iv) (a) Equal
(b) Proportional

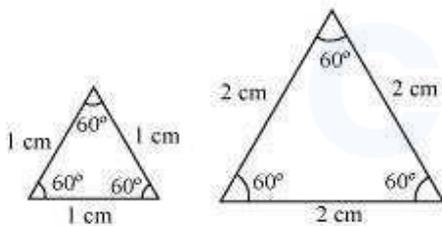
Question 2:

Give two different examples of pair of

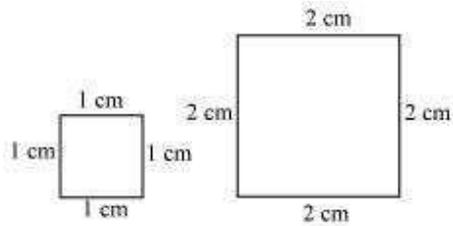
- (i) Similar figures (ii) Non-similar figures

Answer 2:

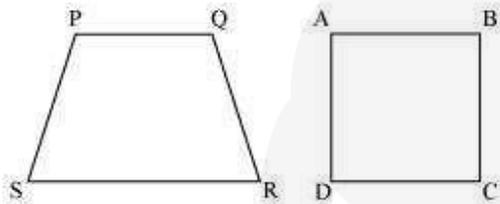
- (i) Two equilateral triangles with sides 1 cm and 2 cm



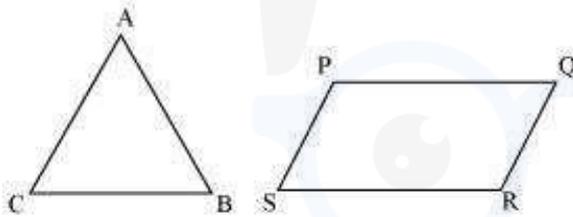
Two squares with sides 1 cm and 2 cm



(ii) Trapezium and square

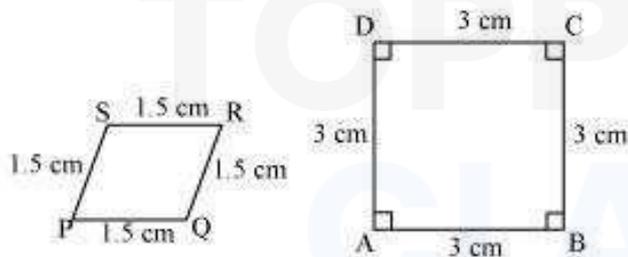


Triangle and parallelogram



Question 3:

State whether the following quadrilaterals are similar or not:



Answer 3:

Quadrilateral PQRS and ABCD are not similar as their corresponding sides are proportional, i.e. 1:2, but their corresponding angles are not equal.

Mathematics

(Chapter – 6) (Triangles)

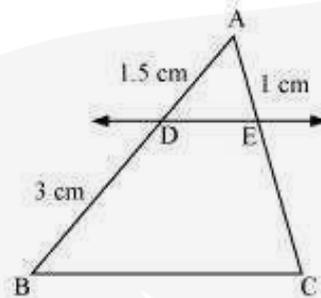
(Class – X)

Exercise 6.2

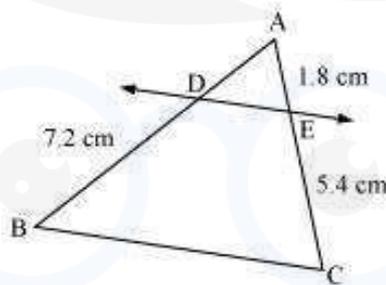
Question 1:

In figure.6.17. (i) and (ii), $DE \parallel BC$. Find EC in (i) and AD in (ii).

(i)

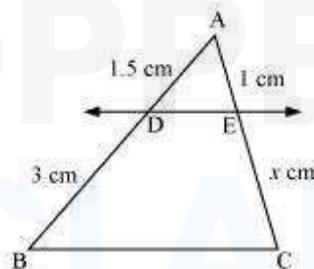


(ii)



Answer 1:

(i)



Let $EC = x$ cm

It is given that $DE \parallel BC$.

By using basic proportionality theorem, we obtain

$$\frac{AD}{DB} = \frac{AE}{EC}$$

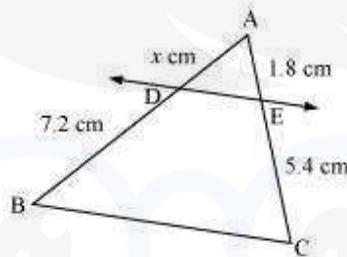
$$\frac{1.5}{3} = \frac{1}{x}$$

$$x = \frac{3 \times 1}{1.5}$$

$$x = 2$$

$$\therefore EC = 2 \text{ cm}$$

(ii)



Let $AD = x \text{ cm}$

It is given that $DE \parallel BC$.

By using basic proportionality theorem, we obtain

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{x}{7.2} = \frac{1.8}{5.4}$$

$$x = \frac{1.8 \times 7.2}{5.4}$$

$$x = 2.4$$

$$\therefore AD = 2.4 \text{ cm}$$

Question 2:

E and F are points on the sides PQ and PR respectively of a ΔPQR . For each of the following cases, state whether $EF \parallel QR$.

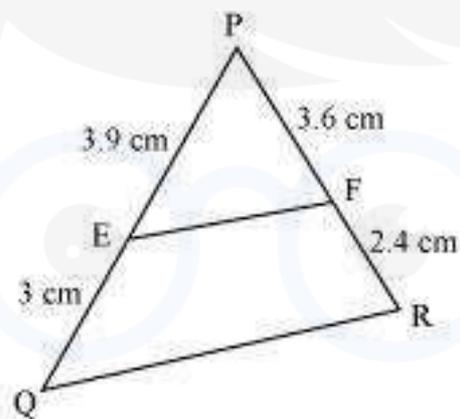
(i) $PE = 3.9$ cm, $EQ = 3$ cm, $PF = 3.6$ cm and $FR = 2.4$ cm

(ii) $PE = 4$ cm, $QE = 4.5$ cm, $PF = 8$ cm and $RF = 9$ cm

(iii) $PQ = 1.28$ cm, $PR = 2.56$ cm, $PE = 0.18$ cm and $PF = 0.36$ cm

Answer 2:

(i)



Given that, $PE = 3.9$ cm, $EQ = 3$ cm, $PF = 3.6$ cm, $FR = 2.4$ cm

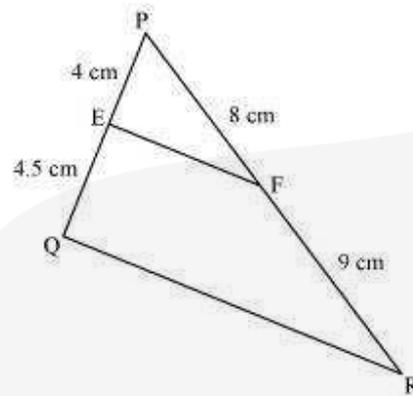
$$\frac{PE}{EQ} = \frac{3.9}{3} = 1.3$$

$$\frac{PF}{FR} = \frac{3.6}{2.4} = 1.5$$

$$\text{Hence, } \frac{PE}{EQ} \neq \frac{PF}{FR}$$

Therefore, EF is not parallel to QR .

(ii)



PE = 4 cm, QE = 4.5 cm, PF = 8 cm, RF = 9 cm

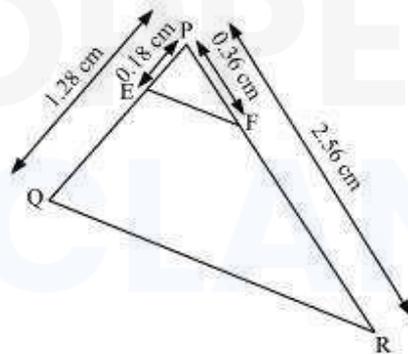
$$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$$

$$\frac{PF}{FR} = \frac{8}{9}$$

Hence, $\frac{PE}{EQ} = \frac{PF}{FR}$

Therefore, EF is parallel to QR.

(iii)



PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm, PF = 0.36 cm

$$\frac{PE}{PQ} = \frac{0.18}{1.28} = \frac{18}{128} = \frac{9}{64}$$

$$\frac{PF}{PR} = \frac{0.36}{2.56} = \frac{9}{64}$$

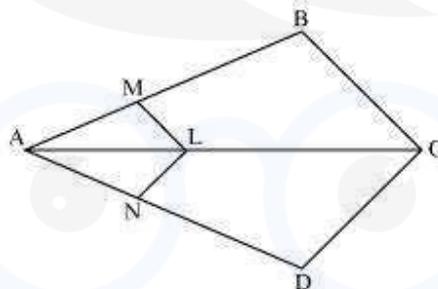
$$\text{Hence, } \frac{PE}{PQ} = \frac{PF}{PR}$$

Therefore, EF is parallel to QR.

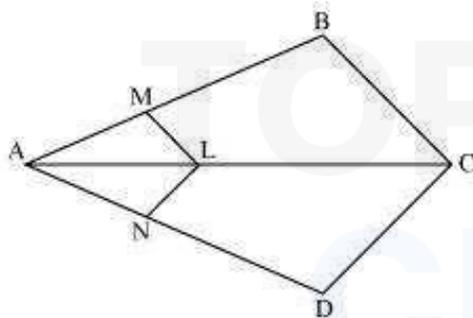
Question 3:

In the following figure, if $LM \parallel CB$ and $LN \parallel CD$, prove that

$$\frac{AM}{AB} = \frac{AN}{AD}$$



Answer 3:



In the given figure, $LM \parallel CB$

By using basic proportionality theorem, we obtain

$$\frac{AM}{AB} = \frac{AL}{AC} \quad (i)$$

Similarly, $LN \parallel CD$

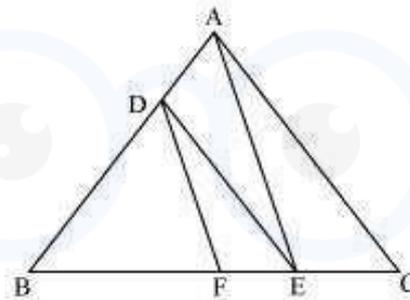
$$\therefore \frac{AN}{AD} = \frac{AL}{AC} \quad (ii)$$

From (i) and (ii), we obtain

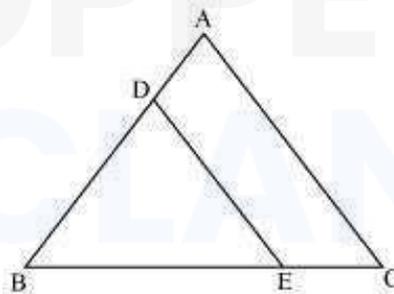
$$\frac{AM}{AB} = \frac{AN}{AD}$$

Question 4:

In the following figure, $DE \parallel AC$ and $DF \parallel AE$. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$.

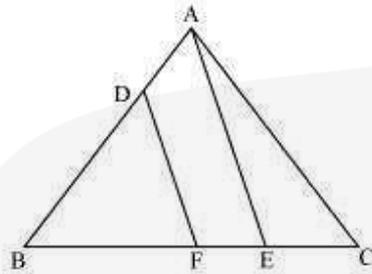


Answer 4:



In $\triangle ABC$, $DE \parallel AC$

$$\therefore \frac{BD}{DA} = \frac{BE}{EC} \quad \text{(Basic Proportionality Theorem)} \quad (i)$$



In $\triangle BAE$, $DF \parallel AE$

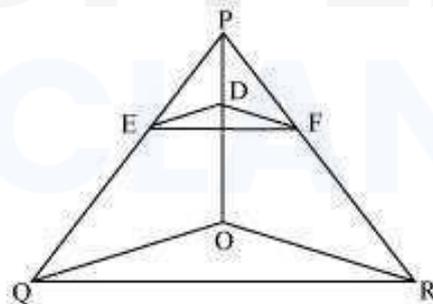
$$\therefore \frac{BD}{DA} = \frac{BF}{FE} \quad \text{(Basic Proportionality Theorem)} \quad (ii)$$

From (i) and (ii), we obtain

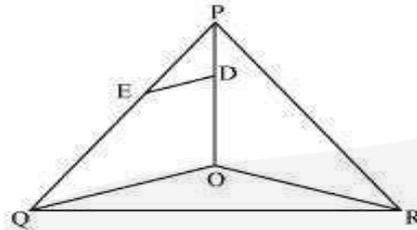
$$\frac{BE}{EC} = \frac{BF}{FE}$$

Question 5:

In the following figure, $DE \parallel OQ$ and $DF \parallel OR$, show that $EF \parallel QR$.



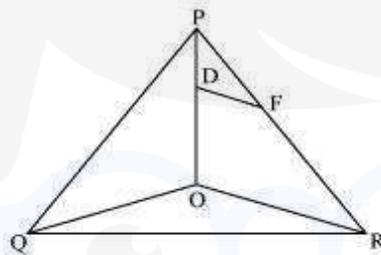
Answer 5:



In ΔPOQ , $DE \parallel OQ$

$$\therefore \frac{PE}{EQ} = \frac{PD}{DO}$$

(Basic proportionality theorem) (i)



In ΔPOR , $DF \parallel OR$

$$\therefore \frac{PF}{FR} = \frac{PD}{DO}$$

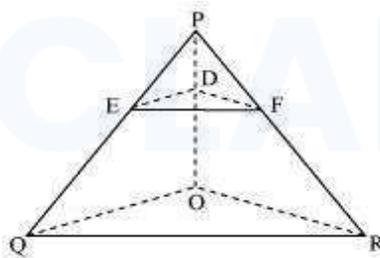
(Basic proportionality theorem) (ii)

From (i) and (ii), we obtain

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

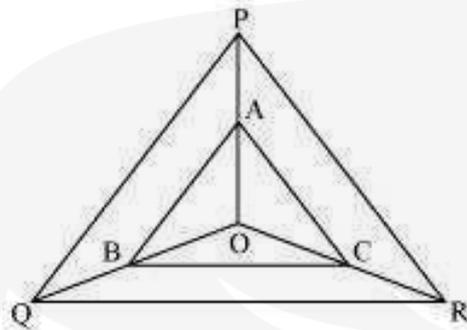
$\therefore EF \parallel QR$

(Converse of basic proportionality theorem)

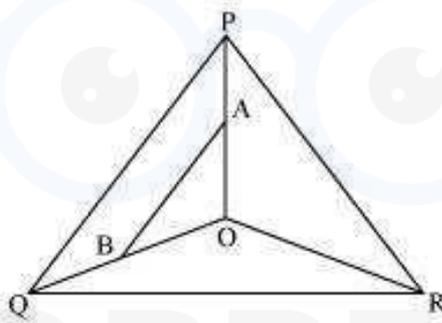


Question 6:

In the following figure, A, B and C are points on OP, OQ and OR respectively such that $AB \parallel PQ$ and $AC \parallel PR$. Show that $BC \parallel QR$.

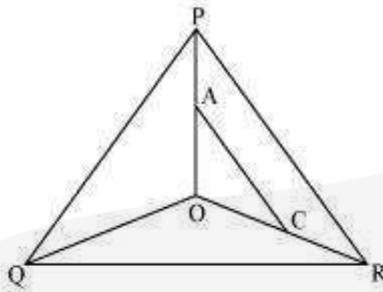


Answer 6:



In $\triangle POQ$, $AB \parallel PQ$

$$\therefore \frac{OA}{AP} = \frac{OB}{BQ} \quad \text{(Basic proportionality theorem)} \quad (i)$$



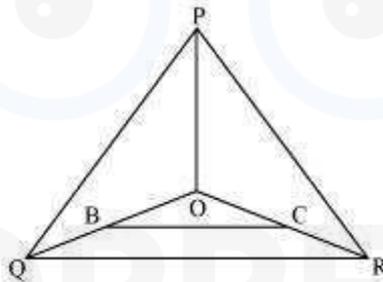
In ΔPOR , $AC \parallel PR$

$$\therefore \frac{OA}{AP} = \frac{OC}{CR} \quad (\text{By basic proportionality theorem}) \quad (ii)$$

From (i) and (ii), we obtain

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

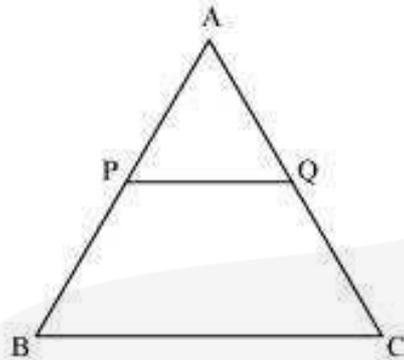
$$\therefore BC \parallel QR \quad (\text{By the converse of basic proportionality theorem})$$



Question 7:

Using Basic proportionality theorem, prove that a line drawn through the mid-points of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).

Answer 7:



Consider the given figure in which PQ is a line segment drawn through the mid-point P of line AB, such that $PQ \parallel BC$

By using basic proportionality theorem, we obtain

$$\frac{AQ}{QC} = \frac{AP}{PB}$$

$$\frac{AQ}{QC} = \frac{1}{1} \quad (\text{P is the mid-point of AB. } \therefore AP = PB)$$

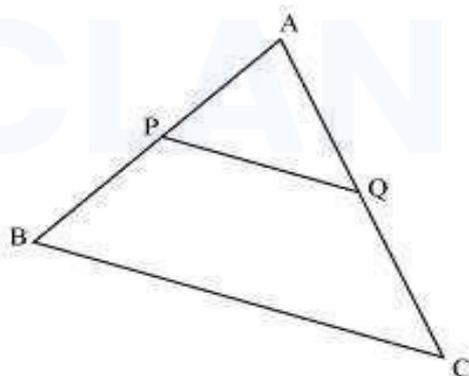
$$\Rightarrow AQ = QC$$

Or, Q is the mid-point of AC.

Question 8:

Using Converse of basic proportionality theorem, prove that the line joining the midpoints of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Answer 8:



Consider the given figure in which PQ is a line segment joining the mid-points P and Q of line AB and AC respectively.

i.e., $AP = PB$ and $AQ = QC$ It can be observed that

$$\frac{AP}{PB} = \frac{1}{1}$$

and $\frac{AQ}{QC} = \frac{1}{1}$

$$\therefore \frac{AP}{PB} = \frac{AQ}{QC}$$

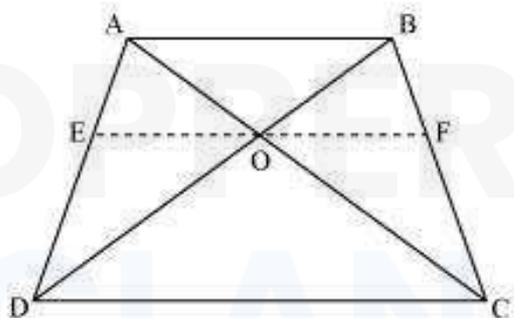
Hence, by using basic proportionality theorem, we obtain $PQ \parallel BC$

Question 9:

ABCD is a trapezium in which $AB \parallel DC$ and its diagonals intersect each other at the

point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Answer 9:



Draw a line EF through point O, such that $EF \parallel CD$

In $\triangle ADC$, $EO \parallel CD$

By using basic proportionality theorem, we obtain

$$\frac{AE}{ED} = \frac{AO}{OC} \quad (1)$$

In $\triangle ABD$, $OE \parallel AB$

So, by using basic proportionality theorem, we obtain

$$\begin{aligned} \frac{ED}{AE} &= \frac{OD}{BO} \\ \Rightarrow \frac{AE}{ED} &= \frac{BO}{OD} \quad (2) \end{aligned}$$

From equations (1) and (2), we obtain

$$\begin{aligned} \frac{AO}{OC} &= \frac{BO}{OD} \\ \Rightarrow \frac{AO}{BO} &= \frac{OC}{OD} \end{aligned}$$

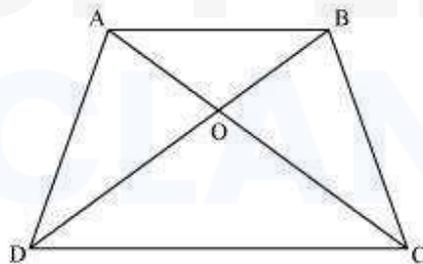
Question 10:

The diagonals of a quadrilateral ABCD intersect each other at the point O such that

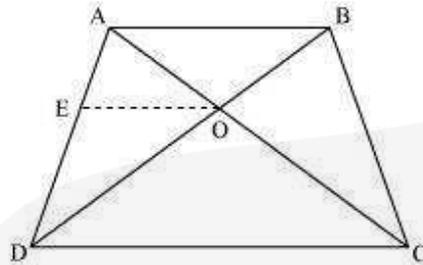
$$\frac{AO}{BO} = \frac{CO}{DO}. \text{ Show that ABCD is a trapezium.}$$

Answer 10:

Let us consider the following figure for the given question.



Draw a line $OE \parallel AB$



In $\triangle ABD$, $OE \parallel AB$

By using basic proportionality theorem, we obtain

$$\frac{AE}{ED} = \frac{BO}{OD} \quad (1)$$

However, it is given that

$$\frac{AO}{OC} = \frac{OB}{OD} \quad (2)$$

From equations (1) and (2), we obtain

$$\frac{AE}{ED} = \frac{AO}{OC}$$

$\Rightarrow EO \parallel DC$ [By the converse of basic proportionality theorem]

$\Rightarrow AB \parallel OE \parallel DC$

$\Rightarrow AB \parallel CD$

$\therefore ABCD$ is a trapezium.

TOPPERS
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Mathematics

(Chapter – 6) (Triangles)

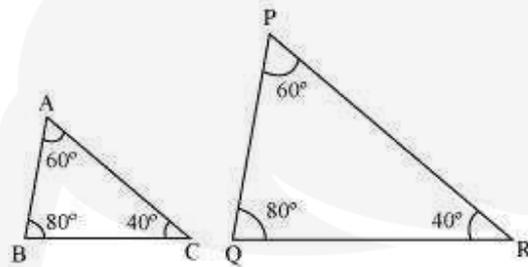
(Class – X)

Exercise 6.3

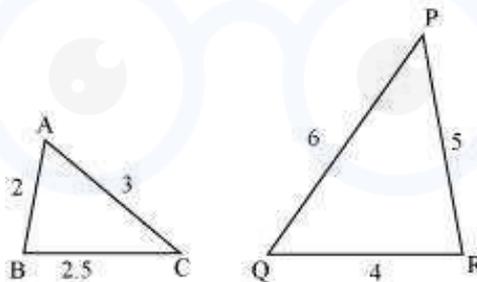
Question 1:

State which pairs of triangles in the following figure are similar? Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:

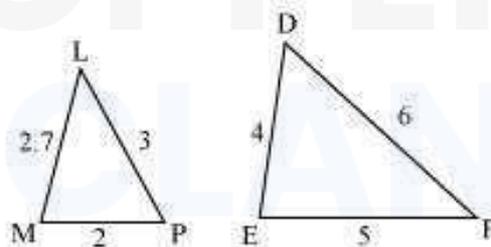
(i)



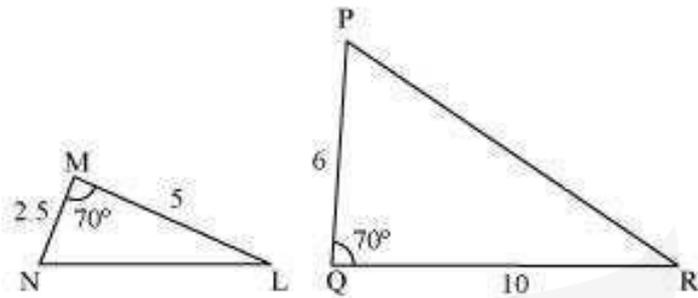
(ii)



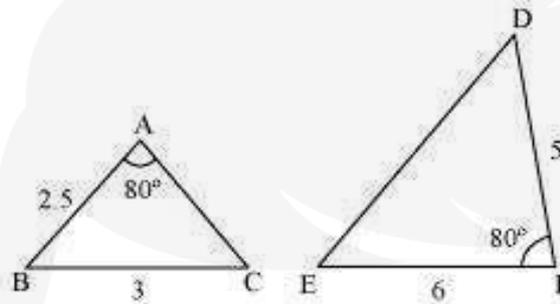
(iii)



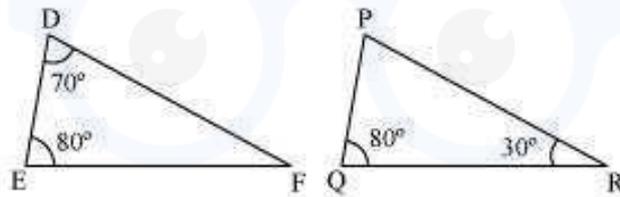
(iv)



(v)



(vi)



Answer 1:

(i) $\angle A = \angle P = 60^\circ$

$\angle B = \angle Q = 80^\circ$

$\angle C = \angle R = 40^\circ$

Therefore, $\triangle ABC \sim \triangle PQR$ [By AAA similarity criterion]

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{PR}$$

(ii)

$\therefore \triangle ABC \sim \triangle QRP$ [By SSS similarity criterion]

(iii) The given triangles are not similar as the corresponding sides are not proportional.

(iv) The given triangles are not similar as the corresponding sides are not proportional.

(v) The given triangles are not similar as the corresponding sides are not proportional.

(vi) In $\triangle DEF$,

$$\angle D + \angle E + \angle F = 180^\circ \text{ (Sum of the measures of the angles of a triangle is } 180^\circ\text{.)}$$

$$70^\circ + 80^\circ + \angle F = 180^\circ$$

$$\angle F = 30^\circ$$

Similarly, in $\triangle PQR$,

$$\angle P + \angle Q + \angle R = 180^\circ$$

(Sum of the measures of the angles of a triangle is 180° .)

$$\angle P + 80^\circ + 30^\circ = 180^\circ$$

$$\angle P = 70^\circ$$

In $\triangle DEF$ and $\triangle PQR$,

$$\angle D = \angle P \text{ (Each } 70^\circ\text{)}$$

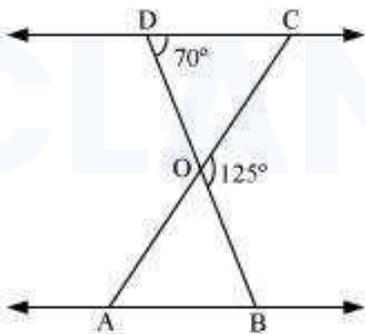
$$\angle E = \angle Q \text{ (Each } 80^\circ\text{)}$$

$$\angle F = \angle R \text{ (Each } 30^\circ\text{)}$$

$\therefore \triangle DEF \sim \triangle PQR$ [By AAA similarity criterion]

Question 2:

In the following figure, $\triangle ODC \sim \triangle OBA$, $\angle BOC = 125^\circ$ and $\angle CDO = 70^\circ$. Find $\angle DOC$, $\angle DCO$ and $\angle OAB$



Answer 2:

DOB is a straight line.

$$\therefore \angle DOC + \angle COB = 180^\circ$$

$$\Rightarrow \angle DOC = 180^\circ - 125^\circ = 55^\circ$$

In $\triangle DOC$,

$$\angle DCO + \angle CDO + \angle DOC = 180^\circ$$

(Sum of the measures of the angles of a triangle is 180° .)

$$\Rightarrow \angle DCO + 70^\circ + 55^\circ = 180^\circ$$

$$\Rightarrow \angle DCO = 55^\circ$$

It is given that $\triangle ODC \sim \triangle OBA$.

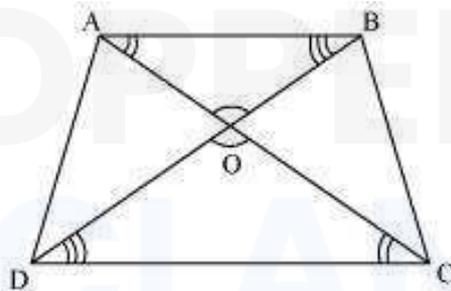
$$\therefore \angle OAB = \angle OCD \text{ [Corresponding angles are equal in similar triangles.]}$$

$$\Rightarrow \angle OAB = 55^\circ$$

Question 3:

Diagonals AC and BD of a trapezium ABCD with $AB \parallel DC$ intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{AO}{OC} = \frac{OB}{OD}$

Answer 3:



In $\triangle DOC$ and $\triangle BOA$,

$$\angle CDO = \angle ABO \text{ [Alternate interior angles as } AB \parallel DC]$$

$$\angle DCO = \angle BAO \text{ [Alternate interior angles as } AB \parallel DC]$$

$$\angle DOC = \angle BOA \text{ [Vertically opposite angles]}$$

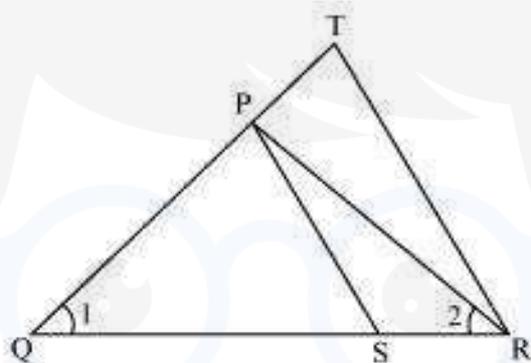
$\therefore \triangle DOC \sim \triangle BOA$ [AAA similarity criterion]

$$\therefore \frac{DO}{BO} = \frac{OC}{OA} \quad \text{[Corresponding sides are proportional]}$$

$$\Rightarrow \frac{OA}{OC} = \frac{OB}{OD}$$

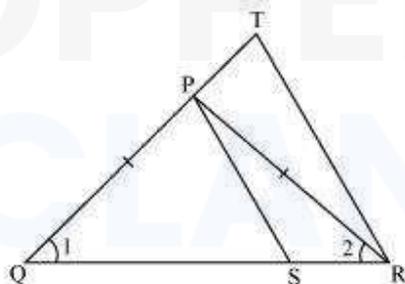
Question 4:

In the following figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$.



Show that $\triangle PQS \sim \triangle TQR$

Answer 4:



In $\triangle PQR$, $\angle PQR = \angle PRQ$

$\therefore PQ = PR$ (i)

Given,

$$\frac{QR}{QS} = \frac{QT}{PR}$$

Using (i), we obtain

$$\frac{QR}{QS} = \frac{QT}{QP} \quad (ii)$$

In ΔPQS and ΔTQR ,

$$\frac{QR}{QS} = \frac{QT}{QP} \quad [\text{Using (ii)}]$$

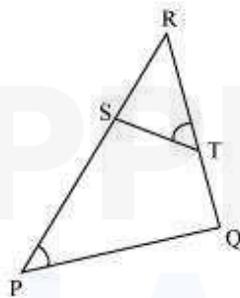
$$\angle Q = \angle Q$$

$$\therefore \Delta PQS \sim \Delta TQR \quad [\text{SAS similarity criterion}]$$

Question 5:

S and T are point on sides PR and QR of ΔPQR such that $\angle P = \angle RTS$. Show that $\Delta RPQ \sim \Delta RTS$.

Answer 5:



In ΔRPQ and ΔRTS ,

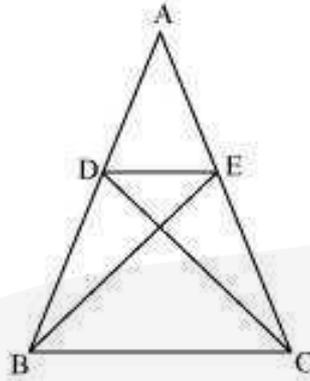
$$\angle RTS = \angle QPS \text{ (Given)}$$

$$\angle R = \angle R \text{ (Common angle)}$$

$$\therefore \Delta RPQ \sim \Delta RTS \text{ (By AA similarity criterion)}$$

Question 6:

In the following figure, if $\triangle ABE \cong \triangle ACD$, show that $\triangle ADE \sim \triangle ABC$.



Answer 6:

It is given that $\triangle ABE \cong \triangle ACD$.

$$\therefore AB = AC \text{ [By CPCT](1)}$$

$$\text{And, } AD = AE \text{ [By CPCT](2)}$$

In $\triangle ADE$ and $\triangle ABC$,

$$\frac{AD}{AB} = \frac{AE}{AC} \text{ [Dividing equation (2) by (1)]}$$

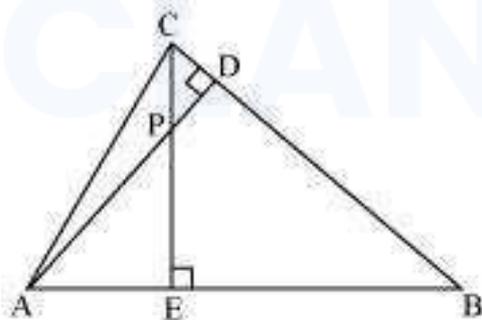
$$\angle A = \angle A \text{ [Common angle]}$$

$$\therefore \triangle ADE \sim \triangle ABC \text{ [By SAS similarity criterion]}$$

Question 7:

In the following figure, altitudes AD and CE of $\triangle ABC$ intersect each other at the point P.

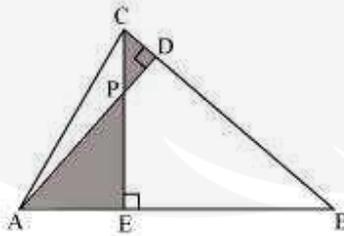
Show that:



- (i) $\triangle AEP \sim \triangle CDP$
- (ii) $\triangle ABD \sim \triangle CBE$
- (iii) $\triangle AEP \sim \triangle ADB$
- (v) $\triangle PDC \sim \triangle BEC$

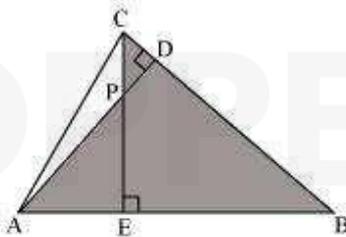
Answer 7:

(i)



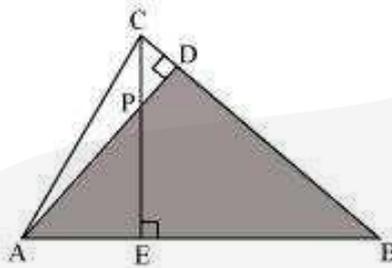
In $\triangle AEP$ and $\triangle CDP$,
 $\angle AEP = \angle CDP$ (Each 90°)
 $\angle APE = \angle CPD$ (Vertically opposite angles)
 Hence, by using AA similarity criterion,
 $\triangle AEP \sim \triangle CDP$

(ii)



In $\triangle ABD$ and $\triangle CBE$,
 $\angle ADB = \angle CEB$ (Each 90°)
 $\angle ABD = \angle CBE$ (Common)
 Hence, by using AA similarity criterion,
 $\triangle ABD \sim \triangle CBE$

(iii)



In $\triangle AEP$ and $\triangle ADB$,

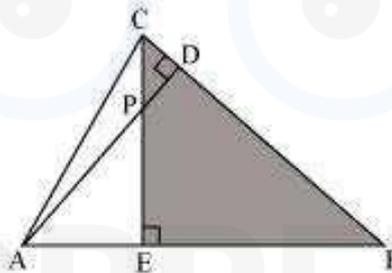
$$\angle AEP = \angle ADB \text{ (Each } 90^\circ\text{)}$$

$$\angle PAE = \angle DAB \text{ (Common)}$$

Hence, by using AA similarity criterion,

$$\triangle AEP \sim \triangle ADB$$

(iv)



In $\triangle PDC$ and $\triangle BEC$,

$$\angle PDC = \angle BEC \text{ (Each } 90^\circ\text{)}$$

$$\angle PCD = \angle BCE \text{ (Common angle)}$$

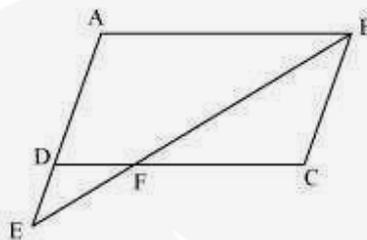
Hence, by using AA similarity criterion,

$$\triangle PDC \sim \triangle BEC$$

Question 8:

E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F.
Show that $\triangle ABE \sim \triangle CFB$

Answer 8:



In $\triangle ABE$ and $\triangle CFB$,

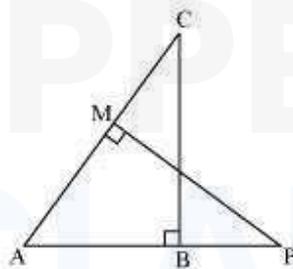
$\angle A = \angle C$ (Opposite angles of a parallelogram)

$\angle AEB = \angle CBF$ (Alternate interior angles as $AE \parallel BC$)

$\therefore \triangle ABE \sim \triangle CFB$ (By AA similarity criterion)

Question 9:

In the following figure, ABC and AMP are two right triangles, right angled at B and M respectively, prove that:



(i) $\triangle ABC \sim \triangle AMP$

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$

Answer 9:

In $\triangle ABC$ and $\triangle AMP$,

$$\angle ABC = \angle AMP \text{ (Each } 90^\circ\text{)}$$

$$\angle A = \angle A \text{ (Common)}$$

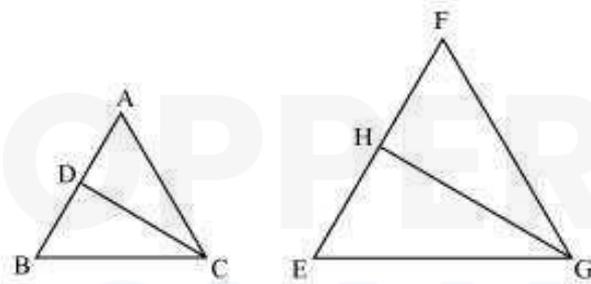
$\therefore \triangle ABC \sim \triangle AMP$ (By AA similarity criterion)

$$\Rightarrow \frac{CA}{PA} = \frac{BC}{MP} \quad \text{(Corresponding sides of similar triangles are proportional)}$$

Question 10:

CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$ such that D and H lie on sides AB and FE of $\triangle ABC$ and $\triangle EFG$ respectively. If $\triangle ABC \sim \triangle FEG$, Show that:

- (i) $\frac{CD}{GH} = \frac{AC}{FG}$
- (ii) $\triangle DCB \sim \triangle HGE$
- (iii) $\triangle DCA \sim \triangle HGF$

Answer 10:

It is given that $\triangle ABC \sim \triangle FEG$.

$$\therefore \angle A = \angle F, \angle B = \angle E, \text{ and } \angle ACB = \angle FGE$$

$$\angle ACB = \angle FGE$$

$$\therefore \angle ACD = \angle FGH \text{ (Angle bisector)}$$

And, $\angle DCB = \angle HGE$ (Angle bisector)

In $\triangle ACD$ and $\triangle FGH$,

$\angle A = \angle F$ (Proved above)

$\angle ACD = \angle FGH$ (Proved above)

$\therefore \triangle ACD \sim \triangle FGH$ (By AA similarity criterion)

$$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$$

In $\triangle DCB$ and $\triangle HGE$,

$\angle DCB = \angle HGE$ (Proved above)

$\angle B = \angle E$ (Proved above)

$\therefore \triangle DCB \sim \triangle HGE$ (By AA similarity criterion)

In $\triangle DCA$ and $\triangle HGF$,

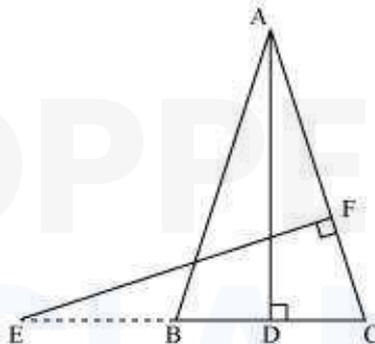
$\angle ACD = \angle FGH$ (Proved above)

$\angle A = \angle F$ (Proved above)

$\therefore \triangle DCA \sim \triangle HGF$ (By AA similarity criterion)

Question 11:

In the following figure, E is a point on side CB produced of an isosceles triangle ABC with $AB = AC$. If $AD \perp BC$ and $EF \perp AC$, prove that $\triangle ABD \sim \triangle ECF$



Answer 11:

It is given that ABC is an isosceles triangle.

$\therefore AB = AC$

$\Rightarrow \angle ABD = \angle ECF$

In $\triangle ABD$ and $\triangle ECF$,

$\angle ADB = \angle EFC$ (Each 90°)

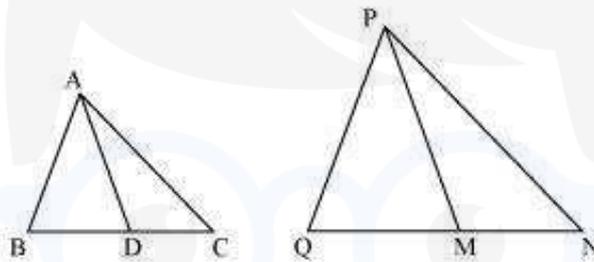
$\angle BAD = \angle CEF$ (Proved above)

$\therefore \triangle ABD \sim \triangle ECF$ (By using AA similarity criterion)

Question 12:

Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of $\triangle PQR$ (see the given figure). Show that $\triangle ABC \sim \triangle PQR$.

Answer 12:



Median divides the opposite side.

$$\therefore BD = \frac{BC}{2} \text{ and } QM = \frac{QR}{2}$$

Given that,

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

In $\triangle ABD$ and $\triangle PQM$,

$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM} \quad (\text{Proved above})$$

$\therefore \triangle ABD \sim \triangle PQM$ (By SSS similarity criterion)

$\Rightarrow \angle ABD = \angle PQM$ (Corresponding angles of similar triangles)

In $\triangle ABC$ and $\triangle PQR$,

$\angle ABD = \angle PQM$ (Proved above)

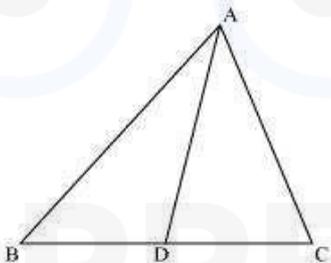
$$\frac{AB}{PQ} = \frac{BC}{QR}$$

$\therefore \triangle ABC \sim \triangle PQR$ (By SAS similarity criterion)

Question 13:

D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB \cdot CD$.

Answer 13:



In $\triangle ADC$ and $\triangle BAC$,

$\angle ADC = \angle BAC$ (Given)

$\angle ACD = \angle BCA$ (Common angle)

$\therefore \triangle ADC \sim \triangle BAC$ (By AA similarity criterion)

We know that corresponding sides of similar triangles are in proportion.

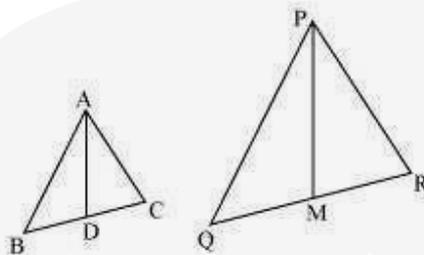
$$\therefore \frac{CA}{CB} = \frac{CD}{CA}$$

$$\Rightarrow CA^2 = CB \times CD$$

Question 14:

Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\triangle ABC \sim \triangle PQR$

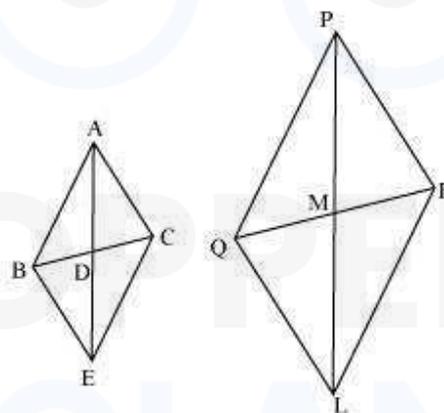
Answer 14:



Given that,

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

Let us extend AD and PM up to point E and L respectively, such that AD = DE and PM = ML. Then, join B to E, C to E, Q to L, and R to L.



We know that medians divide opposite sides.

Therefore, $BD = DC$ and $QM = MR$

Also, $AD = DE$ (By construction)

And, $PM = ML$ (By construction)

In quadrilateral ABEC, diagonals AE and BC bisect each other at point D.

Therefore, quadrilateral ABEC is a parallelogram.

$\therefore AC = BE$ and $AB = EC$ (Opposite sides of a parallelogram are equal)

Similarly, we can prove that quadrilateral PQLR is a parallelogram and $PR = QL$,

$PQ = LR$

It was given that

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{2AD}{2PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{AE}{PL}$$

$\therefore \triangle ABE \sim \triangle PQL$ (By SSS similarity criterion)

We know that corresponding angles of similar triangles are equal.

$$\therefore \angle BAE = \angle QPL \dots (1)$$

Similarly, it can be proved that $\triangle AEC \sim \triangle PLR$ and

$$\angle CAE = \angle RPL \dots (2)$$

Adding equation (1) and (2), we obtain

$$\angle BAE + \angle CAE = \angle QPL + \angle RPL$$

$$\Rightarrow \angle CAB = \angle RPQ \dots (3)$$

In $\triangle ABC$ and $\triangle PQR$,

$$\frac{AB}{PQ} = \frac{AC}{PR} \text{ (Given)}$$

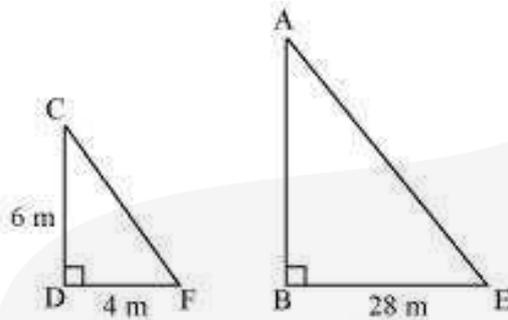
$$\angle CAB = \angle RPQ \text{ [Using equation (3)]}$$

$\therefore \triangle ABC \sim \triangle PQR$ (By SAS similarity criterion)

Question 15:

A vertical pole of length 6 m casts a shadow 4m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Answer 15:



Let AB and CD be a tower and a pole respectively.

Let the shadow of BE and DF be the shadow of AB and CD respectively.

At the same time, the light rays from the sun will fall on the tower and the pole at the same angle.

Therefore, $\angle DCF = \angle BAE$

And, $\angle DFC = \angle BEA$

$\angle CDF = \angle ABE$ (Tower and pole are vertical to the ground)

$\therefore \triangle ABE \sim \triangle CDF$ (AAA similarity criterion)

$$\Rightarrow \frac{AB}{CD} = \frac{BE}{DF}$$

$$\Rightarrow \frac{AB}{6 \text{ m}} = \frac{28}{4}$$

$$\Rightarrow AB = 42 \text{ m}$$

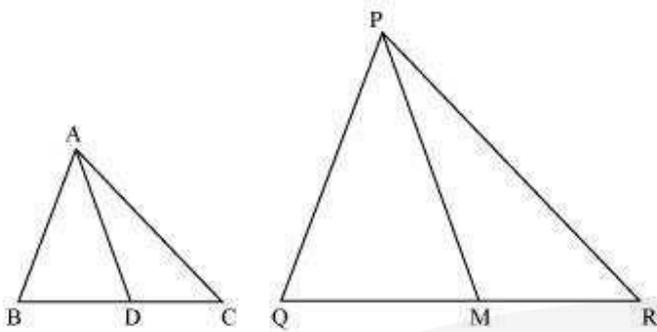
Therefore, the height of the tower will be 42 metres.

Question 16:

If AD and PM are medians of triangles ABC and PQR, respectively where

$\triangle ABC \sim \triangle PQR$ prove that $\frac{AB}{PQ} = \frac{AD}{PM}$

Answer 16:



It is given that $\triangle ABC \sim \triangle PQR$

We know that the corresponding sides of similar triangles are in proportion.

$$\therefore \frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR} \dots (1)$$

Also, $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R \dots (2)$

Since AD and PM are medians, they will divide their opposite sides.

$$\therefore BD = \frac{BC}{2} \text{ and } QM = \frac{QR}{2} \dots (3)$$

From equations (1) and (3), we obtain

$$\frac{AB}{PQ} = \frac{BD}{QM} \dots (4)$$

In $\triangle ABD$ and $\triangle PQM$,

$\angle B = \angle Q$ [Using equation (2)]

$$\frac{AB}{PQ} = \frac{BD}{QM} \text{ [Using equation (4)]}$$

$\therefore \triangle ABD \sim \triangle PQM$ (By SAS similarity criterion)

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$