Mathematic

(Chapter – 4) (Quadratic Equations)

(Class X)

Exercise 4.1

Question 1:

Check whether the following are quadratic equations:

(i) $(x+1)^2 = 2(x-3)$ (ii) $x^2 - 2x = (-2)(3-x)$

(iii) $(x-2)(x+1) = (x-1)(x+3)$ (iv) $(x-3)(2x+1) = x(x+5)$

(v) $(2x-1)(x-3) = (x+5)(x-1)$ (vi) $x^2 + 3x + 1 = (x-2)^2$

(vii) $(x+2)^3 = 2x(x^2-1)$ (viii) $x^3 - 4x^2 - x + 1 = (x-2)^3$

Answer 1:

(i) $(x+1)^2 = 2(x-3) \Rightarrow x^2 + 2x + 1 = 2x - 6 \Rightarrow x^2 + 7 = 0$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(ii) $x^2 - 2x = (-2)(3-x) \Rightarrow x^2 - 2x = -6 + 2x \Rightarrow x^2 - 4x + 6 = 0$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(iii) $(x-2)(x+1) = (x-1)(x+3) \Rightarrow x^2 - x - 2 = x^2 + 2x - 3 \Rightarrow 3x - 1 = 0$

It is not of the form $ax^2 + bx + c = 0$.

Hence, the given equation is not a quadratic equation.

(iv) $(x-3)(2x+1) = x(x+5) \Rightarrow 2x^2 - 5x - 3 = x^2 + 5x \Rightarrow x^2 - 10x - 3 = 0$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(v) $(2x-1)(x-3) = (x+5)(x-1) \Rightarrow 2x^2 - 7x + 3 = x^2 + 4x - 5 \Rightarrow x^2 - 11x + 8 = 0$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(vi) $x^2 + 3x + 1 = (x-2)^2 \Rightarrow x^2 + 3x + 1 = x^2 + 4 - 4x \Rightarrow 7x - 3 = 0$

It is not of the form $ax^2 + bx + c = 0$.

Hence, the given equation is not a quadratic equation.

(vii) $(x+2)^3 = 2x(x^2-1) \Rightarrow x^3+8+6x^2+12x = 2x^3-2x \Rightarrow x^3-14x-6x^2-8=0$

It is not of the form $ax^2+bx+c=0$.

Hence, the given equation is not a quadratic equation.

(viii) $x^3-4x^3-x+1=(x-2)^3 \Rightarrow x^3-4x^2-x+1=x^3-8-6x^2+12x \Rightarrow 2x^2-13x+9=0$

It is of the form $ax^2+bx+c=0$.

Hence, the given equation is a quadratic equation.

Question 2:

Represent the following situations in the form of quadratic equations.

(i) The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

(ii) The product of two consecutive positive integers is 306. We need to find the integers.

(iii) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

Answer 2:

(i) Let the breadth of the plot be x m.

Hence, the length of the plot is $(2x + 1)$ m.

Area of a rectangle = Length × Breadth

∴ $528 = x(2x + 1)$

$\Rightarrow 2x^2 + x - 528 = 0$

(ii) Let the consecutive integers be x and $x + 1$.

It is given that their product is 306.

$\therefore\ x(x+1)=306 \Rightarrow x^2+x-306=0$

(iii) Let Rohan's age be *x*.

Hence, his mother's age = *x* + 26

3 years hence,

Rohan's age = *x* + 3

Mother's age = *x* + 26 + 3 = *x* + 29

It is given that the product of their ages after 3 years is 360.

$\therefore (x+3)(x+29)=360$

$\Rightarrow x^2+32x-273=0$

(iv) Let the speed of train be *x* km/h.

Time taken to travel 480 km = $\frac{480}{x}$ hrs

In second condition, let the speed of train = $(x-8)$ km/h

It is also given that the train will take 3 hours to cover the same distance.

Therefore, time taken to travel 480 km = $\left(\frac{480}{x}+3\right)$ hrs

Speed × Time = Distance

$(x-8)\left(\frac{480}{x}+3\right)=480$

$\Rightarrow 480+3x-\frac{3840}{x}-24=480$

$\Rightarrow 3x-\frac{3840}{x}=24$

$\Rightarrow 3x^2-24x+3840=0$

$\Rightarrow x^2-8x+1280=0$

Mathematic

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(Class X)

Exercise 4.2

Question 1:

Find the roots of the following quadratic equations by factorisation:

(i) $x^2-3x-10=0$ (ii) $2x^2+x-6=0$

(iii) $\sqrt{2}x^2+7x+5\sqrt{2}=0$ (iv) $2x^2-x+\frac{1}{8}=0$

(v) $100x^2-20x+1=0$

Answer 1:

(i) $x^2-3x-10$

$=x^2-5x+2x-10$

$=x(x-5)+2(x-5)$

$=(x-5)(x+2)$

Roots of this equation are the values for which $(x-5)(x+2)=0$

$\therefore\ x-5=0$ or $x+2=0$

i.e., $x=5$ or $x=-2$

(ii) $2x^2+x-6$

$=2x^2+4x-3x-6$

$=2x(x+2)-3(x+2)$

$=(x+2)(2x-3)$

Roots of this equation are the values for which $(x+2)(2x-3)=0$

$\therefore$ x + 2 = 0 or 2x – 3 = 0

i.e., $x=-2$ or $x=3/2$

(iii) $\sqrt{2}x^2+7x+5\sqrt{2}$

$=\sqrt{2}x^2+5x+2x+5\sqrt{2}$

$=x\left(\sqrt{2}x+5\right)+\sqrt{2}\left(\sqrt{2}x+5\right)$

$=\left(\sqrt{2}x+5\right)\left(x+\sqrt{2}\right)$

Roots of this equation are the values for which $\left(\sqrt{2}x+5\right)\left(x+\sqrt{2}\right)$= 0

$\sqrt{2}x+5$ = 0 or $x+\sqrt{2}$ = 0

i.e., x = $\frac{-5}{\sqrt{2}}$ or x = $-\sqrt{2}$

(iv) $2x^2-x+\frac{1}{8}$

$$=\frac{1}{8}\left(16x^2-8x+1\right)$$
$$=\frac{1}{8}\left(16x^2-4x-4x+1\right)$$
$$=\frac{1}{8}\left(4x(4x-1)-1(4x-1)\right)$$
$$=\frac{1}{8}(4x-1)^2$$

Roots of this equation are the values for which $(4x-1)^2$ = 0

Therefore, $(4x-1)=0$ or $(4x-1)=0$

$$x=\frac{1}{4}\text{ or }x=\frac{1}{4}$$

(v) $100x^2-20x+1$

$$=100x^2-10x-10x+1$$
$$=10x(10x-1)-1(10x-1)$$
$$=(10x-1)^2$$

Roots of this equation are the values for which $(10x-1)^2$ = 0

Therefore, $(10x-1)=0$ or $(10x-1)=0$

$$x=\frac{1}{10}\text{ or }x=\frac{1}{10}$$

Question 2:

(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. Find out how many marbles they had to start with. **(ii)** A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was Rs 750. Find out the number of toys produced on that day.

Answer 2:

(i) Let the number of John's marbles be x.

Therefore, number of Jivanti's marble = 45 − x

After losing 5 marbles,

Number of John's marbles = x − 5

Number of Jivanti's marbles = 45 − x − 5 = 40 − x

It is given that the product of their marbles is 124.

$$\therefore (x-5)(40-x)=124$$
$$\Rightarrow x^2-45x+324=0$$
$$\Rightarrow x^2-36x-9x+324=0$$
$$\Rightarrow x(x-36)-9(x-36)=0$$
$$\Rightarrow (x-36)(x-9)=0$$

X – 36 = 0 or x − 9 = 0

i.e., x = 36 or x = 9

If the number of John's marbles = 36,

Then, number of Jivanti's marbles = 45 − 36 = 9

If number of John's marbles = 9,

Then, number of Jivanti's marbles = 45 − 9 = 36

(ii) Let the number of toys produced be x.

∴ Cost of production of each toy = Rs (55 − x)

It is given that, total production of the toys = Rs 750

$\therefore x(55-x)=750$

$\Rightarrow x^2-55x+750=0$

$\Rightarrow x^2-25x-30x+750=0$

$\Rightarrow x(x-25)-30(x-25)=0$

$\Rightarrow (x-25)(x-30)=0$

$x - 25 = 0$ or $x - 30 = 0$

i.e., $x = 25$ or $x = 30$

Hence, the number of toys will be either 25 or 30.

Question 3:

Find two numbers whose sum is 27 and product is 182.

Answer 3:

Let the first number be x and the second number is $27 - x$.

Therefore, their product = $x(27 - x)$

It is given that the product of these numbers is 182.

Therefore, $x(27-x)=182$

$\Rightarrow x^2-27x+182=0$

$\Rightarrow x^2-13x-14x+182=0$

$\Rightarrow x(x-13)-14(x-13)=0$

$\Rightarrow (x-13)(x-14)=0$

$x - 13 = 0$ or $x - 14 = 0$

i.e., $x = 13$ or $x = 14$

If first number = 13, then

Other number = 27 − 13 = 14

If first number = 14, then

Other number = 27 − 14 = 13 Therefore,

the numbers are 13 and 14.

Question 4:
Find two consecutive positive integers, sum of whose squares is 365.
Answer 4:
Let the consecutive positive integers be x and $x + 1$.

Given that $x^2 + (x+1)^2 = 365$

$\Rightarrow x^2 + x^2 + 1 + 2x = 365$

$\Rightarrow 2x^2 + 2x - 364 = 0$

$\Rightarrow x^2 + x - 182 = 0$

$\Rightarrow x^2 + 14x - 13x - 182 = 0$

$\Rightarrow x(x+14) - 13(x+14) = 0$

$\Rightarrow (x+14)(x-13) = 0$

Either $x + 14 = 0$ or $x - 13 = 0$, i.e., $x = -14$ or $x = 13$

Since the integers are positive, x can only be 13.

$\therefore x + 1 = 13 + 1 = 14$
Therefore, two consecutive positive integers will be 13 and 14.

Question 5:
The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Answer 5:
Let the base of the right triangle be x cm.
Its altitude = $(x - 7)$ cm
From pythagoras theorem,

$\text{Base}^2 + \text{Altitude}^2 = \text{Hypotenuse}^2$

$\therefore x^2 + (x-7)^2 = 13^2$

$\Rightarrow x^2 + x^2 + 49 - 14x = 169$

$\Rightarrow 2x^2 - 14x - 120 = 0$

$\Rightarrow x^2 - 7x - 60 = 0$

$\Rightarrow x^2 - 12x + 5x - 60 = 0$

$\Rightarrow x(x-12) + 5(x-12) = 0$

$\Rightarrow (x-12)(x+5) = 0$

Either $x - 12 = 0$ or $x + 5 = 0$, i.e., $x = 12$ or $x = -5$

Since sides are positive, x can only be 12.

Therefore, the base of the given triangle is 12 cm and the altitude of this triangle will be (12 − 7) cm = 5 cm.

Question 6:

A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs 90, find the number of articles produced and the cost of each article.

Answer 6:

Let the number of articles produced be x.

Therefore, cost of production of each article = Rs $(2x + 3)$

It is given that the total production is Rs 90.

$$\therefore x(2x+3)=90$$
$$\Rightarrow 2x^2+3x-90=0$$
$$\Rightarrow 2x^2+15x-12x-90=0$$
$$\Rightarrow x(2x+15)-6(2x+15)=0$$
$$\Rightarrow (2x+15)(x-6)=0$$

Either $2x + 15 = 0$ or $x - 6 = 0$, i.e., $x = \frac{-15}{2}$ or $x = 6$

As the number of articles produced can only be a positive integer, therefore, x can only be 6.

Hence, number of articles produced = 6

Cost of each article = $2 \times 6 + 3$ = Rs 15

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EXERCISE 4.3

Question 1:

Find the nature of the roots of the following quadratic equations.

If the real roots exist, find them;

(i) $2x^2 - 3x + 5 = 0$

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

(iii) $2x^2 - 6x + 3 = 0$

Answer 1:

We know that for a quadratic equation $ax^2 + bx + c = 0$, discriminant is $b^2 - 4ac$.

- If $b^2 - 4ac > 0$ → two distinct real roots
- If $b^2 - 4ac = 0$ → two equal real roots
- If $b^2 - 4ac < 0$ → no real roots

(i) $2x^2 - 3x + 5 = 0$

Comparing this equation with $ax^2 + bx + c = 0$,

we obtain $a = 2$, $b = -3$, $c = 5$

Discriminant $= b^2 - 4ac = (-3)^2 - 4(2)(5) = 9 - 40 = -31$

As $b^2 - 4ac < 0$,

Therefore, no real root is possible for the given equation.

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

Comparing this equation with $ax^2 + bx + c = 0$,

we obtain $a = 3,\ b = -4\sqrt{3}, c = 4$

Discriminant $= b^2 - 4ac = \left(-4\sqrt{3}\right)^2 - 4(3)(4) = 48 - 48 = 0$

As $b^2 - 4ac = 0$,

Therefore, real roots exist for the given equation and they are equal to each other.

And the roots will be $\frac{-b}{2a}$ and $\frac{-b}{2a}$.

$$\frac{-b}{2a}=\frac{-\left(-4\sqrt{3}\right)}{2\times3}=\frac{4\sqrt{3}}{6}=\frac{2\sqrt{3}}{3}=\frac{2}{\sqrt{3}}$$

Therefore, the roots are $\frac{2}{\sqrt{3}}$ and $\frac{2}{\sqrt{3}}$.

(iii) $2x^2 - 6x + 3 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain a

$= 2$, $b = -6$, $c = 3$

Discriminant $= b^2 - 4ac = (-6)^2 - 4(2)(3)$

$= 36 - 24 = 12$

As $b^2 - 4ac > 0$,

Therefore, distinct real roots exist for this equation as follows.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(-6)\pm\sqrt{(-6)^2-4(2)(3)}}{2(2)}$$

$$=\frac{6\pm\sqrt{12}}{4}=\frac{6\pm2\sqrt{3}}{4}$$

$$=\frac{3\pm\sqrt{3}}{2}$$

Therefore, the roots are $\frac{3+\sqrt{3}}{2}$ or $\frac{3-\sqrt{3}}{2}$.

Question 2:

Find the values of k for each of the following quadratic equations, so that they have two equal roots.

(i) $2x^2 + kx + 3 = 0$

(ii) $kx(x - 2) + 6 = 0$

Answer 2:

We know that if an equation $ax^2 + bx + c = 0$ has two equal roots, its discriminant $(b^2 - 4ac)$ will be 0.

(i) $2x^2 + kx + 3 = 0$

Comparing equation with $ax^2 + bx + c = 0$,

we obtain $a = 2$, $b = k$, $c = 3$

Discriminant $= b^2 - 4ac = (k)^2 - 4(2)(3) = k^2 - 24$

For equal roots,

Discriminant = 0

$k^2 - 24 = 0$

$k^2 = 24$

$k = \pm\sqrt{24} = \pm 2\sqrt{6}$

(ii) $kx(x - 2) + 6 = 0$

or $kx^2 - 2kx + 6 = 0$

Comparing this equation with $ax^2 + bx + c = 0$,

we obtain $a = k$, $b = -2k$, $c = 6$

Discriminant $= b^2 - 4ac = (-2k)^2 - 4(k)(6) = 4k^2 - 24k$

For equal roots, $b^2 - 4ac = 0$

$4k^2 - 24k = 0$

$4k(k - 6) = 0$

Either $4k = 0$ or $k = 6 = 0$

$k = 0$ or $k = 6$

However, if $k = 0$, then the equation will not have the terms 'x^2' and 'x'.

Therefore, if this equation has two equal roots, k should be 6 only.

Question 3:

Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m^2?

If so, find its length and breadth.

Answer 3:

Let the breadth of mango grove be l.

Length of mango grove will be $2l$.

Area of mango grove = $(2l)$ (l)

= $2l^2$

$$2l^2 = 800$$

$$l^2 = \frac{800}{2} = 400$$

$$l^2 - 400 = 0$$

Comparing this equation with $al^2 + bl + c = 0$,

we obtain $a = 1$ $b = 0$, $c = 400$

Discriminant = $b^2 - 4ac = (0)^2 - 4 \times (1) \times (-400) = 1600$

Here, $b^2 - 4ac > 0$

Therefore, the equation will have real roots. And hence, the desired rectangular mango grove can be designed.

$$l = \pm 20$$

However, length cannot be negative.

Therefore, breadth of mango grove = 20 m

Length of mango grove = 2 × 20 = 40 m

Question 4:

Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Answer 4:

Let the age of one friend be x years.

Age of the other friend will be $(20 - x)$ years.

4 years ago, age of 1st friend = $(x - 4)$ years

And, age of 2nd friend = $(20 - x - 4) = (16 - x)$ years

Given that,

$(x - 4)(16 - x) = 48$

$16x - 64 - x^2 + 4x = 48$

$-x^2 + 20x - 112 = 0$

$x^2 - 20x + 112 = 0$

Comparing this equation with $ax^2 + bx + c = 0$,

we obtain $a = 1$, $b = -20$, $c = 112$

Discriminant = $b^2 - 4ac = (-20)^2 - 4(1)(112) = 400 - 448 = -48$

As $b^2 - 4ac < 0$,

Therefore, no real root is possible for this equation and hence, this situation is not possible.

Question 5:

Is it possible to design a rectangular park of perimeter 80 and area 400 m^2? If so find its length and breadth.

Answer 5:

Let the length and breadth of the park be l and b.

Perimeter = $2(l + b) = 80$ $l + b = 40$

Or, $b = 40 - l$

Area = $l \times b = l(40 - l) = 40l - l^2$

$40l - l^2 = 400$

$l^2 - 40l + 400 = 0$

Comparing this equation with $al^2 + bl + c = 0$,

we obtain $a = 1$, $b = -40$, $c = 400$

Discriminate = $b^2 - 4ac = (-40)^2 - 4(1)(400) = 1600 - 1600 = 0$

As $b^2 - 4ac = 0$,

Therefore, this equation has equal real roots. And hence, this situation is possible.

Root of this equation,

$$l = -\frac{b}{2a}$$

$$l = -\frac{(-40)}{2(1)} = \frac{40}{2} = 20$$

Therefore, length of park, $l = 20$ m

And breadth of park, $b = 40 - l = 40 - 20 = 20$ m