Mathematics

(Chapter – 4) (Determinants) (Exercise 4.1)
(Class – XII)

Evaluate the determinants in Exercises 1 and 2.

Question 1:

$\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$

Answer 1:

$\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$ Expanding along R_1, we get

$= 2 \times (-1) - 4 \times (-5) = -2 + 20 = 18$

Question 2:

(i) $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$ **(ii)** $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$

Answer 2:

(i) $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} = \cos\theta \times \cos\theta - \sin\theta \times (-\sin\theta) = \cos^2\theta + \sin^2\theta = 1$

(ii) $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix} = (x^2 - x + 1) \times (x + 1) - (x - 1) \times (x + 1)$

$= x^3 + x^2 - x^2 - x + x + 1 - (x^2 + x - x - 1)$

$= x^3 - x^2 + 2$

Question 3:

If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then show that $|2A| = 4|A|$

Answer 3:

$|2A| = \begin{vmatrix} 2 & 4 \\ 8 & 4 \end{vmatrix}$ Expanding along R_1, we get

$= 2 \times 4 - 4 \times 8 = 8 - 32 = -24$ ... (1)

$4|A| = 4\begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix}$ Expanding along R_1, we get

$= 4(1 \times 2 - 2 \times 4) = 4(-6) = -24$ ... (2)

From the equation (1) and (2), we get, $|2A| = 4|A|$

Question 4:

If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, then show that $|3A| = 27|A|$

Answer 4:

$|3A| = \begin{vmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{vmatrix}$ Expanding along R_1, we get

$= 3(36 - 0) - 0(0 - 0) + 1(0 - 0) = 108$ ... (1)

$27|A| = 27\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{vmatrix}$ Expanding along R_1, we get

$= 27\{1(4 - 0) - 0(0 - 0) + 1(0 - 0)\} = 27(4) = 108$ ... (2)

From the equation (1) and (2), we get, $|3A| = 27|A|$

Question 5:

Evaluate the determinants:

(i) $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$ **(ii)** $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$ **(iii)** $\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$ **(iv)** $\begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

Answer 5:

(i) $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$ Expanding along R_1, we get

$= 3(0-5) + 1(0+3) - 2(0-0) = -15 + 3 - 0 = -12$

(ii) $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$ Expanding along R_1, we get

$= 3(1+6) + 4(1+4) + 5(3-2) = 21 + 20 + 5 = 46$

(iii) $\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$ Expanding along R_1, we get

$= 0(0+9) - 1(0-6) + 2(-3-0) = 0 + 6 - 6 = 0$

(iv) $\begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$ Expanding along R_1, we get

$= 2(0-5) + 1(0+3) - 2(0-6) = -10 + 3 + 12 = 5$

Question 6:

If $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$, find $|A|$.

Answer 6:

$|A| = \begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{vmatrix}$ Expanding along R_1, we get

$= 1(-9+12) - 1(-18+15) - 2(8-5) = 3 + 3 - 6 = 0$

Question 7:

Find values of x, if

(i) $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$ **(ii)** $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

Answer 7:

(i) $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

$\Rightarrow 2 - 20 = 2x^2 - 24 \quad \Rightarrow x^2 = 3 \quad \Rightarrow x = \pm\sqrt{3}$

(ii) $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

$\Rightarrow 10 - 12 = 5x - 6x \quad \Rightarrow -2 = -x \quad \Rightarrow x = 2$

Question 8:

If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, then x is equal to:

(A) 6 (B) ± 6 (C) -6 (D) 0

Answer 8:

$\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$

$\Rightarrow x^2 - 36 = 36 - 36 \quad \Rightarrow x^2 = 36 \quad \Rightarrow x = \pm 6$

Hence, the option (B) is correct.

Mathematics

(Chapter – 4) (Determinants) (Exercise 4.2)
(Class – XII)

Question 1:

Find area of the triangle with vertices at the point given in each of the following:

(i) $(1,0),(6,0),(4,3)$
(ii) $(2,7),(1,1),(10,8)$
(iii) $(-2,-3),(3,2),(-1,-8)$

Answer 1:

$$\text{Area of triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

(i) $A(1,0), B(6,0), C(4,3)$

$$\text{Area of triangle } ABC = \tfrac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[1(0-3) - 0(6-4) + 1(18-0)] = \frac{1}{2}(15) = 7.5 \text{ square units}$$

(ii) $A(2,7),\ B(1,1),\ C(10,8)$

$$\text{Area of triangle } ABC = \tfrac{1}{2}\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[2(1-8) - 7(1-10) + 1(8-10)] = \frac{1}{2}(47) = 25.5 \text{ square units}$$

(iii) $A(-2,-3), B(3,2), C(-1,-8)$

$$\text{Area of triangle } ABC = \tfrac{1}{2}\begin{vmatrix} -2 & -3 & 1 \\ 3 & 2 & 1 \\ -1 & -8 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[-2(2+8) + 3(3+1) + 1(-24+2)] = \frac{1}{2}(-30) = -15$$

Area of triangle $ABC = 15$ square units

Question 2:

Show that points $A(a, b+c), B(b, c+a), C(c, a+b)$ are collinear.

Answer 2:

If the points $A(a, b+c), B(b, c+a)$ and $C(c, a+b)$ are collinear, the area of triangle ABC will be zero.

$$\text{Area of triangle } ABC = \tfrac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} a & a+b+c & 1 \\ b & a+b+c & 1 \\ c & a+b+c & 1 \end{vmatrix} \qquad [\text{Applying } C_2 \to C_1 + C_2]$$

$$= \frac{1}{2}(a+b+c)\begin{vmatrix} a & 1 & 1 \\ b & 1 & 1 \\ c & 1 & 1 \end{vmatrix} \qquad [\text{Taking } a+b+c \text{ as common from } C_2]$$

$$= 0 \qquad [\because\ C_1 = C_3\ ]$$

Hence, the points $A(a, b+c), B(b, c+a)$ and $C(c, a+b)$ are collinear.

Question 3:

Find values of k if area of triangle is 4 sq. units and vertices are

(i) $(k, 0), (4, 0), (0, 2)$

(ii) $(-2, 0), (0, 4), (0, k)$

Answer 3:

(i) $A(k, 0), B(4, 0), C(0, 2)$

Area of triangle $ABC = \frac{1}{2}\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix}$

$= \dfrac{1}{2}[k(0-2) - 0(4-0) + 1(8-0)] = \dfrac{1}{2}(-2k+8) = -k+4$

According to question, Area of triangle $ABC = 4$ square units

Therefore, $|-k+4| = 4 \qquad \Rightarrow -k+4 = \pm 4$

$\Rightarrow -k+4 = 4 \qquad$ or $\qquad -k+4 = -4$

$\Rightarrow k = 0 \qquad$ or $\qquad k = 8$

Hence, the value of k are 0 and 8.

(ii) $A(-2, 0), B(0, 4), C(0, k)$

Area of triangle $ABC = \frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix}$

$= \dfrac{1}{2}[-2(4-k) - 0(0-0) + 1(0-0)] = \dfrac{1}{2}(-8+2k) = -4+k$

According to question, Area of triangle $ABC = 4$ square units

Therefore, $|-4+k| = 4 \qquad \Rightarrow -4+k = \pm 4$

$\Rightarrow -4+k = 4 \qquad$ or $\qquad -4+k = -4$

$\Rightarrow k = 8 \qquad$ or $\qquad k = 0$

Hence, the value of k are 0 and 8.

Question 4:

(i) Find equation of line joining (1, 2) and (3, 6) using determinants.

(ii) Find equation of line joining (3, 1) and (9, 3) using determinants.

Answer 4:

(i) Let, $P(x, y)$ be any point lie on the line joining $A(1, 2)$ and $B(3, 6)$. Hence, the points A, B and P will be collinear and area of triangle ABC will be zero.

Therefore, Area of triangle $ABP = \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 3 & 6 & 1 \\ x & y & 1 \end{vmatrix} = 0$

$\Rightarrow \dfrac{1}{2}[1(6-y) - 2(3-x) + 1(3y-6x)] = 0$

$\Rightarrow 6 - y - 6 + 2x + 3y - 6x = 0$

$\Rightarrow -4x + 2y = 0$

$\Rightarrow 2x = y$

(ii) Let, $P(x, y)$ be any point lie on the line joining $A(3, 1)$ and $B(9, 3)$. Hence, the points A, B and P will be collinear and area of triangle ABC will be zero.

Therefore, Area of triangle $ABP = \frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ 9 & 3 & 1 \\ x & y & 1 \end{vmatrix} = 0$

$\Rightarrow \frac{1}{2}[3(3 - y) - 1(9 - x) + 1(9y - 3x)] = 0$

$\Rightarrow 9 - 3y - 9 + x + 9y - 3x = 0$

$\Rightarrow -2x + 6y = 0$

$\Rightarrow x = 3y$

Question 5:

If area of triangle is 35 sq. units with vertices (2, – 6), (5, 4) and (k, 4). Then k is

(A) 12 (B) -2 (C) $-12, -2$ (D) $12, -2$

Answer 5:

$A(2, -6), B(5, 4), C(k, 4)$

Area of triangle $ABC = \frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix}$

$= \frac{1}{2}[2(4 - 4) + 6(5 - k) + 1(20 - 4k)] = \frac{1}{2}(30 - 6k + 20 - 4k) = 25 - 5k$

According to question, Area of triangle $ABC = 35$ square units

Therefore, $|25 - 5k| = 35$

$\Rightarrow 25 - 5k = \pm 35$

$\Rightarrow 25 - 5k = 35$ or $25 - 5k = -35$

$\Rightarrow k = \frac{-10}{5} = -2$ or $k = \frac{60}{5} = 12$

Hence, the option (D) is correct.

Mathematics

(Chapter – 4) (Determinants) (Exercise 4.3)
(Class – XII)

Write Minors and Cofactors of the elements of following determinants:

Question 1:

(i) $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$ **(ii)** $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$

Answer 1:

(i) $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$

The minor of element a_{ij} is M_{ij} and the cofactor is $A_{ij} = (-1)^{i+j} M_{ij}$, therefore,

The minor of element a_{11} is M_{11}= 3 and the cofactor is $A_{11} = (-1)^{1+1} M_{11} = 3$

The minor of element a_{12} is M_{12}= 0 and the cofactor is $A_{12} = (-1)^{1+2} M_{12} = 0$

The minor of element a_{21} is M_{21}= −4 and the cofactor is $A_{21} = (-1)^{2+1} M_{21} = 4$

The minor of element a_{22} is M_{22}= 2 and the cofactor is $A_{22} = (-1)^{2+2} M_{22} = 2$

(ii) $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$

The minor of element a_{11} is M_{11}= d and the cofactor is $A_{11} = (-1)^{1+1} M_{11} = d$

The minor of element a_{12} is M_{12}= b and the cofactor is $A_{12} = (-1)^{1+2} M_{12} = -b$

The minor of element a_{21} is M_{21}= c and the cofactor is $A_{21} = (-1)^{2+1} M_{21} = -c$

The minor of element a_{22} is M_{22}= a and the cofactor is $A_{22} = (-1)^{2+2} M_{22} = a$

Question 2:

(i) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ **(ii)** $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$

Answer 2:

(i) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$,

Here,

$M_{11} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1,\quad M_{12} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 - 0 = 0,\quad M_{13} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 - 0 = 0$

$M_{21} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 - 0 = 0,\quad M_{22} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1,\quad M_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0 - 0 = 0$

$M_{31} = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0 - 0 = 0,\quad M_{32} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0 - 0 = 0,\quad M_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$

and $A_{ij} = (-1)^{i+j} M_{ij}$, therefore

$A_{11} = (-1)^{1+1} M_{11} = 1 \qquad A_{12} = (-1)^{1+2} M_{12} = 0 \qquad A_{13} = (-1)^{1+3} M_{13} = 0$

$A_{21} = (-1)^{2+1} M_{21} = 0 \qquad A_{22} = (-1)^{2+2} M_{22} = 1 \qquad A_{23} = (-1)^{2+3} M_{233} = 0$

$A_{31} = (-1)^{3+1} M_{31} = 0 \qquad A_{32} = (-1)^{3+2} M_{32} = 0 \qquad A_{33} = (-1)^{3+3} M_{33} = 1$

(ii) $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$,

Here,

$M_{11} = \begin{vmatrix} 5 & -1 \\ 1 & 2 \end{vmatrix} = 10 + 1 = 11,\ M_{12} = \begin{vmatrix} 3 & -1 \\ 0 & 2 \end{vmatrix} = 6 - 0 = 6,\ M_{13} = \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3 - 0 = 3$

$M_{21} = \begin{vmatrix} 0 & 4 \\ 1 & 2 \end{vmatrix} = 0 - 4 = -4,\quad M_{22} = \begin{vmatrix} 1 & 4 \\ 0 & 2 \end{vmatrix} = 2 - 0 = 2,\quad M_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$

$M_{31} = \begin{vmatrix} 0 & 4 \\ 5 & -1 \end{vmatrix} = 0 - 20 = -20, M_{32} = \begin{vmatrix} 1 & 4 \\ 3 & -1 \end{vmatrix} = -1 - 12 = -13\ M_{33} = \begin{vmatrix} 1 & 0 \\ 3 & 5 \end{vmatrix} = 5 - 0 = 5$

and $A_{ij} = (-1)^{i+j} M_{ij}$, therefore

$A_{11} = (-1)^{1+1} M_{11} = 11$ $\quad A_{12} = (-1)^{1+2} M_{12} = -6$ $\quad A_{13} = (-1)^{1+3} M_{13} = 3$
$A_{21} = (-1)^{2+1} M_{21} = 4$ $\quad A_{22} = (-1)^{2+2} M_{22} = 2$ $\quad A_{23} = (-1)^{2+3} M_{233} = -1$
$A_{31} = (-1)^{3+1} M_{31} = -20$ $\quad A_{32} = (-1)^{3+2} M_{32} = 13$ $\quad A_{33} = (-1)^{3+3} M_{33} = 5$

Question 3:

Using Cofactors of elements of second row, evaluate $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$.

Answer 3:

$$\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix} = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23}$$

Here, $a_{21} = 2,\ a_{22} = 0,\ a_{23} = 1$ and

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 3 & 8 \\ 2 & 3 \end{vmatrix} = -(9 - 16) = 7$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 5 & 8 \\ 1 & 3 \end{vmatrix} = 15 - 8 = 7$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = -(10 - 3) = -7$$

Therefore, $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix} = 2(7) + 0(7) + 1(-7) = 7$

Question 4:

Using Cofactors of elements of third column, evaluate $\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$.

Answer 4:

$$\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix} = a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33}$$

Here, $a_{13} = yz,\ a_{23} = zx,\ a_{33} = xy$ and

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 1 & y \\ 1 & z \end{vmatrix} = z - y$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & x \\ 1 & z \end{vmatrix} = -(z - x) = x - z$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & x \\ 1 & y \end{vmatrix} = y - x$$

Therefore, $\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix} = yz(z - y) + zx(x - z) + xy(y - x)$

$= yz^2 - y^2z + zx^2 - xz^2 + xy^2 - x^2y$
$= zx^2 - x^2y - xz^2 + xy^2 + yz^2 - y^2z$
$= x^2(z - y) - x(z^2 - y^2) + yz(z - y)$
$= (z - y)[x^2 - x(z + y) + yz]$
$= (z - y)[x^2 - xz - xy + yz]$
$= (z - y)[x(x - z) - y(x - z)]$
$= (x - z)(z - y)(x - y)$
$= (x - y)(y - z)(z - x)$

Question 5:

If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ A_{ij} is Cofactors of a_{ij}, then value of Δ is given by

(A) $a_{11}A_{11} + a_{12}A_{32} + a_{13}A_{33}$

(B) $a_{11}A_{11} + a_{12}A_{21} + a_{13}A_{31}$

(C) $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$

(D) $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

Answer 5:

The value of $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ is given by: $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

Hence, the option (D) is correct.

Mathematics

(Chapter – 4) (Determinants) (Exercise 4.4)
(Class – XII)

Question 1:

Find adjoint of each of the matrice: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Answer 1:

Here, $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, therefore,

$$A_{11} = 4 \qquad A_{12} = -3$$
$$A_{21} = -2 \qquad A_{22} = 1$$

Adjoint of matrix $A = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

Question 2:

Find adjoint of each of the matrice: $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$

Answer 2:

Here, $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$, therefore

$A_{11} = 3 \qquad A_{12} = -12 \qquad A_{13} = 6$
$A_{21} = 1 \qquad A_{22} = 5 \qquad A_{23} = 2$
$A_{31} = -11 \qquad A_{32} = -1 \qquad A_{33} = 5$

Adjoint of matrix $A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} 3 & 1 & -11 \\ -12 & 5 & -1 \\ 6 & 2 & 5 \end{bmatrix}$

Question 3:

Verify $A(adj\ A) = (adj\ A).A = |A|.I$ in $\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$

Answer 3:

Here, $A = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$, therefore,

$A_{11} = -6 \quad A_{12} = 4 \quad A_{21} = -3 \quad A_{22} = 2$
$|A| = -12 + 12 = 0$

$$adj\ A = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$$

$A(adj\ A) = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}\begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$

$= \begin{bmatrix} -12+12 & -6+6 \\ 24-24 & 12-12 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$(adj\ A).A = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$

$= \begin{bmatrix} -12+12 & -18+18 \\ 8-8 & 12-12 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$|A|.I = 0.\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Hence, $A(adj\ A) = (adj\ A).A = |A|.I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Question 4:

Verify $A(adj\ A) = (adj\ A).A = |A|.I$ in $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$

Answer 4:

Here, $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$, therefore, $|A| = 1(0-0) + 1(9+2) + 2(0-0) = 11$

$A_{11} = 0$ $\quad A_{12} = -11$ $\quad A_{13} = 0$

$A_{21} = 3$ $\quad A_{22} = 1$ $\quad A_{23} = -1$

$A_{31} = 2$ $\quad A_{32} = 8$ $\quad A_{33} = 3$

Adjoint of matrix $A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$

$A(adj\ A) = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 0+11+0 & 3-1-2 & 2-8+6 \\ 0+0+0 & 9+0+2 & 6+0-6 \\ 0+0+0 & 3+0-3 & 2+0+9 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$

$(adj\ A).A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$

$= \begin{bmatrix} 0+9+2 & 0+0+0 & 0-6+6 \\ -11+3+8 & 11+0+0 & -22-2+24 \\ 0-3+3 & 0+0+0 & 0+2+9 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$

$|A|.I = 11.\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$

Hence, $A(adj\ A) = (adj\ A).A = |A|.I = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$

Question 5:

Find the inverse of each of the matrices (if it exists): $\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$

Answer 5:

Here, $A = \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$,

Therefore, $A_{11} = 3$ $\quad A_{12} = -4$ $\quad A_{21} = 2$ $\quad A_{22} = 2$

$|A| = 6 + 8 = 14 \neq 0 \quad \Rightarrow \quad A^{-1}$ exists.

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{|A|} \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \frac{1}{14} \begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$$

Question 6:

Find the inverse of each of the matrices (if it exists): $\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$

Answer 6:

Here, $A = \begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$,

Therefore, $A_{11} = 2$ $\quad A_{12} = 3$ $\quad A_{21} = -5$ $\quad A_{22} = -1$

$|A| = -2 + 15 = 13 \neq 0 \quad \Rightarrow \quad A^{-1}$ exists.

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{|A|} \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \frac{1}{13} \begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$$

Question 7:

Find the inverse of each of the matrices (if it exists): $\begin{bmatrix}1 & 2 & 3\\0 & 2 & 4\\0 & 0 & 5\end{bmatrix}$

Answer 7:

Here, $A=\begin{bmatrix}1 & 2 & 3\\0 & 2 & 4\\0 & 0 & 5\end{bmatrix}$,

Therefore, $|A| = 1(10-0)-2(0-0)+3(0-0) = 10 \neq 0 \Rightarrow A^{-1}$ exists.

$A_{11}=10$ $A_{12}=0$ $A_{13}=0$

$A_{21}=-10$ $A_{22}=5$ $A_{23}=0$

$A_{31}=2$ $A_{32}=-4$ $A_{33}=2$

$$A^{-1}=\frac{1}{|A|}adj\ A=\frac{1}{|A|}\begin{bmatrix}A_{11} & A_{21} & A_{31}\\A_{12} & A_{22} & A_{32}\\A_{13} & A_{23} & A_{33}\end{bmatrix}=\frac{1}{10}\begin{bmatrix}10 & -10 & 2\\0 & 5 & -4\\0 & 0 & 2\end{bmatrix}$$

Question 8:

Find the inverse of each of the matrices (if it exists): $\begin{bmatrix}1 & 0 & 0\\3 & 3 & 0\\5 & 2 & -1\end{bmatrix}$

Answer 8:

Here, $A=\begin{bmatrix}1 & 0 & 0\\3 & 3 & 0\\5 & 2 & -1\end{bmatrix}$,

Therefore, $|A| = 1(-3-0)-0(-3-0)+0(6-15) = -3 \neq 0 \Rightarrow A^{-1}$ exists.

$A_{11}=-3$ $A_{12}=3$ $A_{13}=-9$

$A_{21}=0$ $A_{22}=-1$ $A_{23}=-2$

$A_{31}=0$ $A_{32}=0$ $A_{33}=3$

$$A^{-1}=\frac{1}{|A|}adj\ A=\frac{1}{|A|}\begin{bmatrix}A_{11} & A_{21} & A_{31}\\A_{12} & A_{22} & A_{32}\\A_{13} & A_{23} & A_{33}\end{bmatrix}=\frac{1}{-3}\begin{bmatrix}-3 & 0 & 0\\3 & -1 & 0\\-9 & -2 & 3\end{bmatrix}$$

Question 9:

Find the inverse of each of the matrices (if it exists): $\begin{bmatrix}2 & 1 & 3\\4 & -1 & 0\\-7 & 2 & 1\end{bmatrix}$

Answer 9:

Here, $A=\begin{bmatrix}2 & 1 & 3\\4 & -1 & 0\\-7 & 2 & 1\end{bmatrix}$,

Therefore, $|A| = 2(-1-0)-1(4-0)+3(8-7) = -3 \neq 0 \Rightarrow A^{-1}$ exists.

$A_{11}=-1$ $A_{12}=-4$ $A_{13}=1$

$A_{21}=5$ $A_{22}=23$ $A_{23}=-11$

$A_{31}=3$ $A_{32}=12$ $A_{33}=-6$

$$A^{-1}=\frac{1}{|A|}adj\ A=\frac{1}{|A|}\begin{bmatrix}A_{11} & A_{21} & A_{31}\\A_{12} & A_{22} & A_{32}\\A_{13} & A_{23} & A_{33}\end{bmatrix}=\frac{1}{-3}\begin{bmatrix}-1 & 5 & 3\\-4 & 23 & 12\\1 & -11 & -6\end{bmatrix}$$

Question 10:

Find the inverse of each of the matrices (if it exists): $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$

Answer 10:

Here, $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$,

Therefore, $|A| = 1(8-6) + 1(0+9) + 2(0-6) = -1 \neq 0 \Rightarrow A^{-1}$ exists.

$A_{11} = 2$ $\quad A_{12} = -9$ $\quad A_{13} = -6$

$A_{21} = 0$ $\quad A_{22} = -2$ $\quad A_{23} = -1$

$A_{31} = -1$ $\quad A_{32} = 3$ $\quad A_{33} = 2$

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{|A|}\begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \frac{1}{-1}\begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

Question 11:

Find the inverse of each of the matrices (if it exists): $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$

Answer 11:

Here, $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$, therefore

$|A| = 1(-\cos^2\alpha - \sin^2\alpha) + 0(0-0) + 0(0-0) = -1 \neq 0$

$\Rightarrow A^{-1}$ exists.

$A_{11} = 1$ $\quad A_{12} = 0$ $\quad A_{13} = 0$

$A_{21} = 0$ $\quad A_{22} = -\cos\alpha$ $\quad A_{23} = -\sin\alpha$

$A_{31} = 0$ $\quad A_{32} = -\sin\alpha$ $\quad A_{33} = \cos\alpha$

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{|A|}\begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \frac{1}{-1}\begin{bmatrix} 1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$$

Question 12:

Let $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$. Verify that $(AB)^{-1} = B^{-1}A^{-1}$.

Answer 12:

Here, $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$, therefore, $A_{11} = 5 \quad A_{12} = -2 \quad A_{21} = -7 \quad A_{22} = 3$

$|A| = 15 - 14 = 1 \neq 0 \quad \Rightarrow A^{-1}$ exists.

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{|A|}\begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \frac{1}{1}\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

and $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$, therefore, $B_{11} = 9 \quad B_{12} = -7 \quad B_{21} = -8 \quad B_{22} = 6$

$|A| = 54 - 56 = -2 \neq 0 \quad \Rightarrow B^{-1}$ exists.

$$B^{-1} = \frac{1}{|B|} adj\ B = \frac{1}{|B|}\begin{bmatrix} B_{11} & B_{21} \\ B_{12} & B_{22} \end{bmatrix} = \frac{1}{-2}\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix} = \begin{bmatrix} -9/2 & 4 \\ 7/2 & -3 \end{bmatrix}$$

$$B^{-1}A^{-1} = \begin{bmatrix} -9/2 & 4 \\ 7/2 & -3 \end{bmatrix}\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} -\frac{45}{2} - 8 & \frac{63}{2} + 12 \\ \frac{35}{2} + 6 & -\frac{49}{2} - 9 \end{bmatrix} = \begin{bmatrix} -\frac{61}{2} & \frac{87}{2} \\ \frac{47}{2} & -\frac{67}{2} \end{bmatrix}$$

$$AB = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix} = \begin{bmatrix} 18+49 & 24+63 \\ 12+35 & 16+45 \end{bmatrix} = \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$$

$|AB| = 67 \times 61 - 87 \times 47 = 4087 - 4089 = -2 \neq 0 \quad \Rightarrow \ (AB)^{-1}$ exists.

$C_{11} = 61 \quad C_{12} = -47 \quad C_{21} = -87 \qquad C_{22} = 67$

$$(AB)^{-1} = \frac{1}{|AB|} adj \text{ AB} = \frac{1}{|AB|}\begin{bmatrix} C_{11} & C_{21} \\ C_{12} & C_{22} \end{bmatrix} = \frac{1}{-2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix} = \begin{bmatrix} -\frac{61}{2} & \frac{87}{2} \\ \frac{47}{2} & -\frac{67}{2} \end{bmatrix}$$

Hence, $(AB)^{-1} = B^{-1}A^{-1}$ is verified.

Question 13:

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ show that $A^2 - 5A + 7I = O$. Hence, find A^{-1}.

Answer 13:

LHS $= A^2 - 5A + 7I = AA - 5A + 7I$

$$= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \text{O} = \text{RHS}$$

$\Rightarrow A^2 - 5A + 7I = O$

$\Rightarrow A^2 - 5A = -7I$

Post multiplying by A^{-1} (because $|A| \neq 0$)

$AAA^{-1} - 5AA^{-1} = -7IA^{-1}$

$\Rightarrow AI - 5I = -7A^{-1}$ [Because $AA^{-1} = I$]

$$\Rightarrow 7A^{-1} = 5I - A = 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

Question 14:

For the matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find the numbers a and b such that $A^2 + aA + bI = O$.

Answer 14:

Given that: $A^2 + aA + bI = O$

$$\Rightarrow \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + a\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + b\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9+2 & 6+2 \\ 3+1 & 2+1 \end{bmatrix} + \begin{bmatrix} 3a & 2a \\ a & a \end{bmatrix} + \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 11+3a+b & 8+2a+0 \\ 4+a+0 & 3+a+b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow 4 + a = 0 \quad \Rightarrow a = -4 \qquad$ and $\qquad 3 + a + b = 0 \ \Rightarrow b = -3 - a = -3 + 4 = 1$

Hence, $a = -4, \ b = 1$

Question 15:

For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$, show that $A^3 - 6A^2 + 5A + 11\,I = O$. Hence, find A^{-1}.

Answer 15:

$$A^2 = A.A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1+2 & 1+2-1 & 1-3+3 \\ 1+2-6 & 1+4+3 & 1-6-9 \\ 2-1+6 & 2-2-3 & 2+3+9 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

$$A^3 = A^2.A = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2+2 & 4+4-1 & 4-6+3 \\ -3+8-28 & -3+16+14 & -3-24-42 \\ 7-3+28 & 7-6-14 & 7+9+42 \end{bmatrix} = \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$$

$\text{LHS} = A^3 - 6A^2 + 5A + 11\,I$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - 6\begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} + 5\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} + 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} + \begin{bmatrix} 5 & 5 & 5 \\ 5 & 10 & -15 \\ 10 & -5 & 15 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 8-24+5+11 & 7-12+5+0 & 1-6+5+0 \\ -23+18+5+0 & 27-48+10+11 & -69+84-15+0 \\ 32-42+10+0 & -13+18-5+0 & 58-84+15+11 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \text{O} = \text{RHS}$$

$\Rightarrow A^3 - 6A^2 + 5A + 11\,I = O \qquad \Rightarrow A^3 - 6A^2 + 5A = -11I$

Post multiplying by A^{-1} (because $|A| \neq 0$)

$A^2AA^{-1} - 6AAA^{-1} + 5AA^{-1} = -11IA^{-1}$

$\Rightarrow A^2I - 6AI + 5I = -11A^{-1}$ [Because $AA^{-1} = I$]

$\Rightarrow 11A^{-1} = -A^2 + 6A - 5I$

$$\Rightarrow 11A^{-1} = -\begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} + 6\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow 11A^{-1} = \begin{bmatrix} -4 & -2 & -1 \\ 3 & -8 & 14 \\ -7 & 3 & -14 \end{bmatrix} + \begin{bmatrix} 6 & 6 & 6 \\ 6 & 12 & -18 \\ 12 & -6 & 18 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$\Rightarrow 11A^{-1} = \begin{bmatrix} -4+6-5 & -2+6+0 & -1+6+0 \\ 3+6-0 & -8+12-5 & 14-18+0 \\ -7+12+0 & 3-6+0 & -14+18-5 \end{bmatrix} = \begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{11}\begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$$

Question 16:

If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, verify that $A^3 - 6A^2 + 9A - 4I = O$ and hence find A^{-1}.

Answer 16:

$$A^2 = A.A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1+1 & -2-2-1 & 2+1+2 \\ -2-2-1 & 1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{bmatrix} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$A^3 = A^2.A = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 12+5+5 & -6-10-5 & 6+5+10 \\ -10-6-5 & 5+12+5 & -5-6-10 \\ 10+5+6 & -5-10-6 & 5+5+12 \end{bmatrix} = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

$\text{LHS} = A^3 - 6A^2 + 9A - 4I$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + 9\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - \begin{bmatrix} 36 & -30 & 30 \\ -30 & 36 & -30 \\ 30 & -30 & 36 \end{bmatrix} + \begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 22-36+18-4 & -21+30-9+0 & 21-30+9+0 \\ -21+30-9-0 & 22-36+18-4 & -21+30-9+0 \\ 21-30+9-0 & -21+30-9-0 & 22-36+18-4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \mathrm{O} = \text{RHS}$$

$\Rightarrow A^3 - 6A^2 + 9A - 4I = O \qquad \Rightarrow A^3 - 6A^2 + 9A = 4I$

Post multiplying by A^{-1} (because $|A| \neq 0$)

$A^2AA^{-1} - 6\mathrm{A}AA^{-1} + 9AA^{-1} = 4IA^{-1}$

$\Rightarrow A^2I - 6AI + 9I = 4A^{-1}$ [Because $AA^{-1} = I$]

$\Rightarrow 4A^{-1} = A^2 - 6A + 9I$

$$\Rightarrow 4A^{-1} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow 4A^{-1} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - \begin{bmatrix} 12 & -6 & 6 \\ -6 & 12 & -6 \\ 6 & -6 & 12 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\Rightarrow 4A^{-1} = \begin{bmatrix} 6-12+9 & -5+6+0 & 5-6+0 \\ -5+6+0 & 6-12+9 & -5+6+0 \\ 5-6+0 & -5+6+0 & 6-12+9 \end{bmatrix} = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

Question 17:

Let A be a non-singular square matrix of order 3 × 3. Then$|adj\ A|$ is equal to:

(A) $|A|$ (B) $|A|^2$ (C) $|A|^3$ (D) $3|A|$

Answer 17:

We know that $adj\ A = |A|I$

$$\Rightarrow (adj\ A)A = |A|\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$\Rightarrow |(adj\ A)A| = |A|\begin{vmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{vmatrix} = \begin{vmatrix}|A| & 0 & 0\\0 & |A| & 0\\0 & 0 & |A|\end{vmatrix} = |A|^3$$

$\Rightarrow |adj\ A| = |A|^2,$

Hence, the option (B) is correct.

Question 18:

If A is an invertible matrix of order 2, then $det(A^{-1})$ is equal to:

(A) $\det(A)$ (B) $\frac{1}{\det(A)}$ (C) 1 (D) 0

Answer 18:

Given that the matrix A is invertible, hence, $A^{-1} = \frac{1}{|A|}adj\ A$

The order of matrix is 2, so, let $A = \begin{bmatrix}a & b\\c & d\end{bmatrix}$

Therefore, $|A| = ad - bc$ and $adj\ A = \begin{bmatrix}d & -b\\-c & a\end{bmatrix}$

$$A^{-1} = \frac{1}{|A|}adj\ A = A^{-1} = \frac{1}{|A|}\begin{bmatrix}d & -b\\-c & a\end{bmatrix} = \begin{bmatrix}\frac{d}{|A|} & -\frac{b}{|A|}\\-\frac{c}{|A|} & \frac{a}{|A|}\end{bmatrix}$$

$$det(A^{-1}) = |A^{-1}| = \begin{vmatrix}\frac{d}{|A|} & -\frac{b}{|A|}\\-\frac{c}{|A|} & \frac{a}{|A|}\end{vmatrix}$$

$$= \frac{1}{|A|^2}\begin{vmatrix}d & -b\\-c & a\end{vmatrix} = \frac{1}{|A|^2}(ad - bc) = \frac{1}{|A|^2}|A| = \frac{1}{|A|}$$

Hence, the option (B) is correct.

Mathematics

(Chapter – 4) (Determinants) (Exercise 4.5)
(Class – XII)

Examine the consistency of the system of equations in Exercises 1 to 6.

Question 1:

$x + 2y = 2$
$2x + 3y = 3$

Answer 1:

The given system of equations: $\begin{array}{c} x + 2y = 2 \\ 2x + 3y = 3 \end{array}$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$|A| = 3 - 4 = -1 \neq 0$

$\Rightarrow$ A is non-singular and so A^{-1} exists.

Hence, the system of equations are consistent.

Question 2:

$2x - y = 5$
$x + y = 4$

Answer 2:

The given system of equations: $\begin{array}{c} 2x - y = 5 \\ x + y = 4 \end{array}$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

$|A| = 2 + 1 = 3 \neq 0$

$\Rightarrow$ A is non-singular and so A^{-1} exists.

Hence, the system of equations are consistent.

Question 3:

$x + 3y = 5$
$2x + 6y = 8$

Answer 3:

The given system of equations: $\begin{array}{c} x + 3y = 5 \\ 2x + 6y = 8 \end{array}$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

$|A| = 6 - 6 = 0$

$\Rightarrow$ A is a singular matrix and so A^{-1} does not exists.

Now,

$adj\ A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$

$(adj\ A)B = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 8 \end{bmatrix} = \begin{bmatrix} 30 - 24 \\ -10 + 8 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix} \neq O$

So, there is no solutions of the given system of equations.

Hence, the system of equations are inconsistent.

Question 4:

$x + y + z = 1$
$2x + 3y + 2z = 2$
$ax + ay + 2az = 4$

Answer 4:

The given system of equations: $\begin{array}{c} x + y + z = 1 \\ 2x + 3y + 2z = 2 \\ ax + ay + 2az = 4 \end{array}$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$|A| = 1(6a - 2a) - 1(4a - 2a) + 1(2a - 3a) = a \neq 0$

$\Rightarrow$ A is non-singular and so A^{-1} exists. Now,

Hence, the system of equations are consistent.

Question 5:

$3x - y - 2z = 2$
$2y - z = -1$
$3x - 5y = 3$

Answer 5:

The given system of equations: $\begin{array}{c} 3x - y - 2z = 2 \\ 2y - z = -1 \\ 3x - 5y = 3 \end{array}$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

$|A| = 3(0 - 5) + 1(0 + 3) - 2(0 - 6) = -15 + 3 + 12 = 0$

$\Rightarrow$ A is a singular matrix and so A^{-1} does not exists. Now,

$A_{11} = -5$	$A_{12} = -3$	$A_{13} = -6$
$A_{21} = 10$	$A_{22} = 6$	$A_{23} = 12$
$A_{31} = 5$	$A_{32} = 3$	$A_{33} = 6$

$$adj\ A = \begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix}$$

$$(adj\ A)B = \begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} -10 - 10 + 15 \\ -6 - 6 + 9 \\ -12 - 12 + 16 \end{bmatrix} = \begin{bmatrix} -5 \\ -3 \\ -6 \end{bmatrix} \neq O$$

So, there is no solutions of the given system of equations.

Hence, the system of equations are inconsistent.

Question 6:

$5x - y + 4z = 5$
$2x + 3y + 5z = 2$
$5x - 2y + 6z = -1$

Answer 6:

The given system of equations: $\begin{matrix} 5x - y + 4z = 5 \\ 2x + 3y + 5z = 2 \\ 5x - 2y + 6z = -1 \end{matrix}$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$$

$|A| = 5(18 + 10) + 1(12 - 25) + 4(-4 - 15) = 140 - 13 - 76 = 51 \neq 0$

$\Rightarrow$ A is non-singular and so A^{-1} exists. Hence, the system of equations are consistent.

Solve system of linear equations, using matrix method, in Exercises 7 to 14.

Question 7:

$5x + 2y = 4$
$7x + 3y = 5$

Answer 7:

The given system of equations: $\begin{matrix} 5x + 2y = 4 \\ 7x + 3y = 5 \end{matrix}$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$|A| = 15 - 14 = 1 \neq 0 \quad \Rightarrow$ A is non-singular and so A^{-1} exists.
Hence, the system of equations are consistent.
Now, $A_{11} = 3 \quad A_{12} = -7 \quad A_{21} = -2 \quad A_{22} = 5$

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{|A|} \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \frac{1}{1} \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$$

$X = A^{-1}B \quad \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 - 10 \\ -28 + 25 \end{bmatrix} \quad \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

$\Rightarrow x = 2, \quad y = -3$

Question 8:

$2x - y = -2$
$3x + 4y = 3$

Answer 8:

The given system of equations: $\begin{matrix} 2x - y = -2 \\ 3x + 4y = 3 \end{matrix}$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$|A| = 8 + 3 = 11 \neq 0 \quad \Rightarrow$ A is non-singular and so A^{-1} exists.
Hence, the system of equations are consistent.

Now, $A_{11} = 4 \quad A_{12} = -3 \quad A_{21} = 1 \quad A_{22} = 2$

$$A^{-1} = \frac{1}{|A|} adj\, A = \frac{1}{|A|}\begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$$

$X = A^{-1}B \quad \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -8+3 \\ 6+6 \end{bmatrix} \quad \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{5}{11} \\ \frac{12}{11} \end{bmatrix} \quad \Rightarrow x = -\frac{5}{11}, \quad y = \frac{12}{11}$

Question 9:

$4x - 3y = 3$
$3x - 5y = 7$

Answer 9:

The given system of equations: $\begin{matrix} 4x - 3y = 3 \\ 3x - 5y = 7 \end{matrix}$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

$|A| = -20 + 9 = -11 \neq 0 \quad \Rightarrow A$ is non-singular and so A^{-1} exists.

Hence, the system of equations are consistent.

Now, $A_{11} = -5 \quad A_{12} = -3 \quad A_{21} = 3 \quad A_{22} = 4$

$$A^{-1} = \frac{1}{|A|} adj\, A = \frac{1}{|A|}\begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \frac{1}{-11}\begin{bmatrix} -5 & 3 \\ -3 & 4 \end{bmatrix}$$

$X = A^{-1}B \quad \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{11}\begin{bmatrix} -5 & 3 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 3 \\ 7 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{11}\begin{bmatrix} -15+21 \\ -9+28 \end{bmatrix} \quad \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{6}{11} \\ -\frac{19}{11} \end{bmatrix} \quad \Rightarrow x = -\frac{6}{11}, \quad y = -\frac{19}{11}$

Question 10:

$5x + 2y = 3$
$3x + 2y = 5$

Answer 10:

The given system of equations: $\begin{matrix} 5x + 2y = 3 \\ 3x + 2y = 5 \end{matrix}$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$|A| = 10 - 6 = 4 \neq 0 \quad \Rightarrow A$ is non-singular and so A^{-1} exists.

Hence, the system of equations are consistent.

Now, $A_{11} = 2 \quad A_{12} = -3 \quad A_{21} = -2 \quad A_{22} = 5$

$$A^{-1} = \frac{1}{|A|} adj\, A = \frac{1}{|A|}\begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$$

$X = A^{-1}B \quad \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 6-10 \\ -9+25 \end{bmatrix} \quad \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{4}{4} \\ \frac{16}{4} \end{bmatrix} \quad \Rightarrow x = -1, \quad y = 4$

Question 11:

$2x + y + z = 1$

$x - 2y - z = \frac{3}{2}$

$3y - 5z = 9$

Answer 11:

The given system of equations: $\begin{array}{c} 2x + y + z = 1 \\ x - 2y - z = \frac{3}{2} \\ 3y - 5z = 9 \end{array}$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 3/2 \\ 9 \end{bmatrix}$$

$|A| = 2(10 + 3) - 1(-5 - 0) + 1(3 - 0) = 26 + 5 + 3 = 34 \neq 0$

$\Rightarrow$ A is non-singular and so A^{-1} exists. Now,

$A_{11} = 13$ $\quad A_{12} = 5$ $\quad A_{13} = 3$

$A_{21} = 8$ $\quad A_{22} = -10$ $\quad A_{23} = -6$

$A_{31} = 1$ $\quad A_{32} = 3$ $\quad A_{33} = -5$

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{34}\begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}$$

$$X = A^{-1}B \quad \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{34}\begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}\begin{bmatrix} 1 \\ \frac{3}{2} \\ 9 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{34}\begin{bmatrix} 13 + 12 + 9 \\ 5 - 15 + 27 \\ 3 - 9 - 45 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{34}\begin{bmatrix} 34 \\ 17 \\ -51 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{1}{2} \\ -\frac{3}{2} \end{bmatrix}$$

$\Rightarrow x = 1, y = \frac{1}{2}, z = -\frac{3}{2}$

Question 12:

$x - y + z = 4$

$2x + y - 3z = 0$

$x + y + z = 2$

Answer 12:

The given system of equations: $\begin{array}{c} x - y + z = 4 \\ 2x + y - 3z = 0 \\ x + y + z = 2 \end{array}$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$|A| = 1(1 + 3) + 1(2 + 3) + 1(2 - 1) = 4 + 5 + 1 = 10 \neq 0$

$\Rightarrow$ A is non-singular and so A^{-1} exists. Now,

$A_{11} = 4$ $\quad A_{12} = -5$ $\quad A_{13} = 1$

$A_{21} = 2$ $\quad A_{22} = 0$ $\quad A_{23} = -2$

$A_{31} = 2$ $\quad A_{32} = 5$ $\quad A_{33} = 3$

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}, \text{therefore, } X = A^{-1}B \quad \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 16 + 0 + 4 \\ -20 + 0 + 10 \\ 4 + 0 + 6 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \Rightarrow x = 2, y = -1, z = 1$$

Question 13:

$2x + 3y + 3z = 5$
$x - 2y + z = -4$
$3x - y - 2z = 3$

Answer 13:

The given system of equations: $\begin{aligned} 2x + 3y + 3z &= 5 \\ x - 2y + z &= -4 \\ 3x - y - 2z &= 3 \end{aligned}$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$|A| = 2(4+1) - 3(-2-3) + 3(-1+6) = 10 + 15 + 15 = 40 \neq 0$

$\Rightarrow$ A is non-singular and so A^{-1} exists. Now,

$A_{11} = 5$ $\quad$ $A_{12} = 5$ $\quad$ $A_{13} = 5$
$A_{21} = 3$ $\quad$ $A_{22} = -13$ $\quad$ $A_{23} = 11$
$A_{31} = 9$ $\quad$ $A_{32} = 1$ $\quad$ $A_{33} = -7$

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

$$X = A^{-1}B \quad \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 25 - 12 + 27 \\ 25 + 52 + 3 \\ 25 - 44 - 21 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \Rightarrow x = 1, y = 2, z = -1$$

Question 14:

$x - y + 2z = 7$
$3x + 4y - 5z = -5$
$2x - y + 3z = 12$

Answer 14:

The given system of equations: $\begin{aligned} x - y + 2z &= 7 \\ 3x + 4y - 5z &= -5 \\ 2x - y + 3z &= 12 \end{aligned}$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$|A| = 1(12-5) + 1(9+10) + 2(-3-8) = 7 + 19 - 22 = 4 \neq 0$

$\Rightarrow$ A is non-singular and so A^{-1} exists. Now,

$A_{11} = 7$ $\quad$ $A_{12} = -19$ $\quad$ $A_{13} = -11$
$A_{21} = 1$ $\quad$ $A_{22} = -1$ $\quad$ $A_{23} = -1$
$A_{31} = -3$ $\quad$ $A_{32} = 11$ $\quad$ $A_{33} = 7$

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$

$$X = A^{-1}B \quad \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}\begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 49 - 5 - 36 \\ -133 + 5 + 132 \\ -77 + 5 + 84 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$\Rightarrow x = 2, y = 1, z = 3$

Question 15:

If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations

$$\begin{aligned} 2x - 3y + 5z &= 11 \\ 3x + 2y - 4z &= -5 \\ x + y - 2z &= -3 \end{aligned}$$

Answer 15:

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

$|A| = 2(-4 + 4) + 3(-6 + 4) + 5(3 - 2) = 0 - 6 + 5 = -1 \neq 0$

$\Rightarrow$ A is non-singular and so A^{-1} exists. Now,

$A_{11} = 0$ $\quad A_{12} = 2$ $\quad A_{13} = 1$
$A_{21} = -1$ $\quad A_{22} = -9$ $\quad A_{23} = -5$
$A_{31} = 2$ $\quad A_{32} = 23$ $\quad A_{33} = 13$

$$A^{-1} = \frac{1}{|A|} adj\, A = \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

The given system of equations: $\begin{aligned} 2x - 3y + 5z &= 11 \\ 3x + 2y - 4z &= -5 \\ x + y - 2z &= -3 \end{aligned}$

This system of equations can be written as $AX = B$, where

$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

$$X = A^{-1}B \quad \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 - 5 + 6 \\ -22 - 45 + 69 \\ -11 - 25 + 39 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\Rightarrow x = 1, y = 2, z = 3$

Question 16:

The cost of 4 kg onion, 3 kg wheat and 2 kg rice is ₹ 60. The cost of 2 kg onion, 4 kg wheat and 6 kg rice is ₹ 90. The cost of 6 kg onion 2 kg wheat and 3 kg rice is ₹ 70. Find cost of each item per kg by matrix method.

Answer 16:

Let the cost of 1 kg of onion = ₹ x,

Let the cost of 1 kg of wheat = ₹ y and

Let the cost of 1 kg rice = ₹ z

The cost of 4 kg onion, 3 kg wheat and 2 kg rice is ₹ 60. So $4x + 3y + 2z = 60$

The cost of 2 kg onion, 4 kg wheat and 6 kg rice is ₹ 90. So, $2x + 4y + 6z = 90$ and

The cost of 6 kg onion 2 kg wheat and 3 kg rice is ₹ 70. So, $6x + 2y + 3z = 70$

The given system of equations: $\begin{aligned} 4x + 3y + 2z &= 60 \\ 2x + 4y + 6z &= 90 \\ 6x + 2y + 3z &= 70 \end{aligned}$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$|A| = 4(12 - 12) - 3(6 - 36) + 2(4 - 24) = 0 + 90 - 40 = 50 \neq 0$

$\Rightarrow$ A is non-singular and so A^{-1} exists. Now,

$A_{11} = 0$ $\quad$ $A_{12} = 30$ $\quad$ $A_{13} = -20$

$A_{21} = -5$ $\quad$ $A_{22} = 0$ $\quad$ $A_{23} = 10$

$A_{31} = 10$ $\quad$ $A_{32} = -20$ $\quad$ $A_{33} = 10$

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

$$X = A^{-1}B \quad \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}\begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 0 - 450 + 700 \\ 1800 + 0 - 1400 \\ -1200 + 900 + 700 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

$\Rightarrow x = 5, y = 8, z = 8$

Hence, the cost of 1 kg of onion is ₹ 7, the cost of 1 kg of wheat is ₹ 8 and the cost of 1 kg rice is ₹ 8.

Mathematics

(Chapter – 4) (Determinants) (Miscellaneous Exercise)
(Class – XII)

Question 1:

Prove that the determinant $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ.

Answer 1:

$$\Delta = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$$

$= x(-x^2 - 1) - \sin\theta\,(-x\sin\theta - \cos\theta) + \cos\theta\,(-\sin\theta + x\cos\theta)$

$= -x^3 - x + x\sin^2\theta + \sin\theta\cos\theta - \cos\theta\sin\theta + x\cos^2\theta$

$= -x^3 - x + x(\sin^2\theta + \cos^2\theta) = -x^3 - x + x = -x^3$, which is independent of θ.

Question 2:

Evaluate $\begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$.

Answer 2:

$$\begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$$

$= -\sin\alpha\,(-\sin\alpha\sin^2\beta - \sin\alpha\cos^2\beta) - 0(\cos\alpha\cos\beta\sin\alpha\sin\beta - \cos\alpha\sin\beta\sin\alpha\cos\beta) +$
$\cos\alpha\,(\cos\alpha\cos^2\beta + \cos\alpha\sin^2\beta)$

[Expending along C_3]

$= \sin^2\alpha\,(\sin^2\beta + \cos^2\beta) + \cos^2\alpha\,(\cos^2\beta + \sin^2\beta)$

$= \sin^2\alpha + \cos^2\alpha = 1$

Question 3:

If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $(AB)^{-1}$.

Answer 3:

Here, $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$,

Therefore, $|B| = 1(3-0) - 2(-1-0) - 2(2-0) = 1 \neq 0 \Rightarrow B^{-1}$ exists.

$B_{11} = 3$ $\quad B_{12} = 1$ $\quad B_{13} = 2$

$B_{21} = 2$ $\quad B_{22} = 1$ $\quad B_{23} = 2$

$B_{31} = 6$ $\quad B_{32} = 2$ $\quad B_{33} = 5$

$$B^{-1} = \frac{1}{|B|}adj\,B = \frac{1}{1}\begin{bmatrix} B_{11} & B_{21} & B_{31} \\ B_{12} & B_{22} & B_{32} \\ B_{13} & B_{23} & B_{33} \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

We know that: $(\mathrm{AB})^{-1} = \mathrm{B}^{-1}\mathrm{A}^{-1}$, therefore

$$(\mathrm{AB})^{-1} = \mathrm{B}^{-1}\mathrm{A}^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-30+30 & -3+12-12 & 3-10+12 \\ 3-15+10 & -1+6-4 & 1-5+4 \\ 6-30+25 & -2+12-10 & 2-10+10 \end{bmatrix} = \begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

Question 4:

Let $A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$. Verify that

(i) $(adj\ A)^{-1} = adj(A^{-1})$ **(ii)** $(A^{-1})^{-1} = A$

Answer 4:

(i) Here, $A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$, therefore

$|A| = 1(15-1) + 2(-10-1) + 1(-2-3) = -13 \neq 0 \ \Rightarrow\ A^{-1}$ exists.

$A_{11} = 14$ $A_{12} = 11$ $A_{13} = -5$

$A_{21} = 11$ $A_{22} = 4$ $A_{23} = -3$

$A_{31} = -5$ $A_{32} = -3$ $A_{33} = -1$

$$adj\ A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{|A|}\begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \frac{1}{-13}\begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix} \qquad \dots(1)$$

Let, $B = adj\ A$, so, $B = \begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix}$, therefore

$|B| = 14(-4-9) - 11(-11-15) - 5(-33+20) = -182 + 286 + 65 = 169 \neq 0 \ \Rightarrow\ B^{-1}$ exists.

$B_{11} = -13$ $B_{12} = 26$ $B_{13} = -13$

$B_{21} = 26$ $B_{22} = -39$ $B_{23} = -13$

$B_{31} = -13$ $B_{32} = -13$ $B_{33} = -65$

$$B^{-1} = \frac{1}{|B|}\begin{bmatrix} B_{11} & B_{21} & B_{31} \\ B_{12} & B_{22} & B_{32} \\ B_{13} & B_{23} & B_{33} \end{bmatrix} = \frac{1}{169}\begin{bmatrix} -13 & 26 & -13 \\ 26 & -39 & -13 \\ -13 & -13 & -65 \end{bmatrix} = \frac{1}{13}\begin{bmatrix} -1 & 2 & -1 \\ 2 & -3 & -1 \\ -1 & -1 & -5 \end{bmatrix}$$

$$\Rightarrow (adj\ A)^{-1} = \frac{1}{13}\begin{bmatrix} -1 & 2 & -1 \\ 2 & -3 & -1 \\ -1 & -1 & -5 \end{bmatrix} \qquad \dots(2)$$

Let, $C = A^{-1}$, so, $C = \frac{1}{-13}\begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix} = \begin{bmatrix} -\frac{14}{13} & -\frac{11}{13} & \frac{5}{13} \\ -\frac{11}{13} & -\frac{4}{13} & \frac{3}{13} \\ \frac{5}{13} & \frac{3}{13} & \frac{1}{13} \end{bmatrix}$, therefore

$C_{11} = -\frac{1}{13}$ $C_{12} = \frac{2}{13}$ $C_{13} = -\frac{1}{13}$

$C_{21} = \frac{2}{13}$ $C_{22} = -\frac{3}{13}$ $C_{23} = -\frac{1}{13}$

$C_{31} = -\frac{1}{13}$ $C_{32} = -\frac{1}{13}$ $C_{33} = -\frac{5}{13}$

$$Adj\ C = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix} = \begin{bmatrix} -\frac{1}{13} & \frac{2}{13} & -\frac{1}{13} \\ \frac{2}{13} & -\frac{3}{13} & -\frac{1}{13} \\ -\frac{1}{13} & -\frac{1}{13} & -\frac{5}{13} \end{bmatrix} = \frac{1}{13}\begin{bmatrix} -1 & 2 & -1 \\ 2 & -3 & -1 \\ -1 & -1 & -5 \end{bmatrix}$$

$\Rightarrow Adj\ C = adj(A^{-1}) = \frac{1}{13}\begin{bmatrix} -1 & 2 & -1 \\ 2 & -3 & -1 \\ -1 & -1 & -5 \end{bmatrix}$ ... (3)

From the equations (2) and (3), we have, $(adj\ A)^{-1} = adj(A^{-1})$

(ii) From the equation (1), we have,

$$A^{-1} = \frac{1}{-13}\begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix}$$

Let, $D = A^{-1}$, so, $D = \frac{1}{-13}\begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix} = \begin{bmatrix} -\frac{14}{13} & -\frac{11}{13} & \frac{5}{13} \\ -\frac{11}{13} & -\frac{4}{13} & \frac{3}{13} \\ \frac{5}{13} & \frac{3}{13} & \frac{1}{13} \end{bmatrix}$, therefore

$$|D| = -\left(\frac{1}{13}\right)^3 [14(-4-9) - 11(-11-15) - 5(-33+20)]$$

$= -\left(\frac{1}{13}\right)^3 (169) = -\frac{1}{13} \neq 0 \quad \Rightarrow D^{-1}$ exists.

$D_{11} = -\frac{1}{13}$ $\quad D_{12} = \frac{2}{13}$ $\quad D_{13} = -\frac{1}{13}$

$D_{21} = \frac{2}{13}$ $\quad D_{22} = -\frac{3}{13}$ $\quad D_{23} = -\frac{1}{13}$

$D_{31} = -\frac{1}{13}$ $\quad D_{32} = -\frac{1}{13}$ $\quad D_{33} = -\frac{5}{13}$

$$D^{-1} = \frac{1}{|D|}\begin{bmatrix} D_{11} & D_{21} & D_{31} \\ D_{12} & D_{22} & D_{32} \\ D_{13} & D_{23} & D_{33} \end{bmatrix} = \frac{1}{-1/13}\begin{bmatrix} -\frac{1}{13} & \frac{2}{13} & -\frac{1}{13} \\ \frac{2}{13} & -\frac{3}{13} & -\frac{1}{13} \\ -\frac{1}{13} & -\frac{1}{13} & -\frac{5}{13} \end{bmatrix} = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$$

$$\Rightarrow D^{-1} = (A^{-1})^{-1} = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix} = A$$

Question 5:

Evaluate $\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$.

Answer 5:

Given that: $\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$

$= \begin{vmatrix} 2(x+y) & y & x+y \\ 2(x+y) & x+y & x \\ 2(x+y) & x & y \end{vmatrix}$ [Applying $C_1 \to C_1 + C_2 + C_3$]

$= 2(x+y)\begin{vmatrix} 1 & y & x+y \\ 1 & x+y & x \\ 1 & x & y \end{vmatrix}$ [Taking $2(x+y)$ as common from C_1]

$= 2(x+y)\begin{vmatrix} 0 & -x & y \\ 0 & y & x-y \\ 1 & y & y+k \end{vmatrix}$ [Applying $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$]

$= 2(x+y)\{(-x)(x-y) - y.y\}$ [Expending along C_1]

$= 2(x+y)(-x^2 + xy - y^2) = -2(x+y)(x^2 - xy + y^2) = -2(x^3 + y^3)$

Question 6:

Evaluate $\begin{vmatrix} 1 & x & y \\ 1 & x+y & y \\ 1 & x & x+y \end{vmatrix}$.

Answer 6:

Given that: $\begin{vmatrix} 1 & x & y \\ 1 & x+y & y \\ 1 & x & x+y \end{vmatrix} = \begin{vmatrix} 0 & -y & 0 \\ 0 & y & -x \\ 1 & x & x+y \end{vmatrix}$ [Applying $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$]

$= \{(-y)(-x) - y.0\}$ [Expending along C_1]

$= xy$

Using properties of determinants prove that:

Question 7:

Solve the system of equations: $$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4$$ $$\frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1$$ $$\frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$$

Answer 7:

The given system of equations: $$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4$$ $$\frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1$$ $$\frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X = \begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$|A| = 2(120-45) - 3(-80-30) + 10(36+36) = 150 + 330 + 720 = 1200 \neq 0 \quad \Rightarrow A^{-1}$ exists.

$A_{11} = 75$ $\quad A_{12} = 110$ $\quad A_{13} = 72$

$A_{21} = 150$ $\quad A_{22} = -100$ $\quad A_{23} = 0$

$A_{31} = 75$ $\quad A_{32} = 30$ $\quad A_{33} = -24$

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$X = A^{-1}B \quad \Rightarrow \begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 300+150+150 \\ 440-100+60 \\ 288+0-48 \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

$\Rightarrow \frac{1}{x} = \frac{1}{2}, \quad \frac{1}{y} = \frac{1}{3}, \quad \frac{1}{z} = \frac{1}{5} \quad\quad \Rightarrow x = 2, y = 3, z = 5$

Choose the correct answer:

Question 8:

If x, y, z are nonzero real numbers, then the inverse of matrix $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ is:

(A) $\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

(B) $xyz \begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

(C) $\frac{1}{xyz} \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$

(D) $\frac{1}{xyz} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Answer 8:

$$A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$$

$|A| = x(yz - 0) - 0(0 - 0) + 0(0 - 0) = xyz \neq 0 \Rightarrow A^{-1}$ exists. Therefore,

$A_{11} = yz$ $\quad A_{12} = 0$ $\quad A_{13} = 0$

$A_{21} = 0$ $\quad A_{22} = xz$ $\quad A_{23} = 0$

$A_{31} = 0$ $\quad A_{32} = 0$ $\quad A_{33} = xy$

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{\text{xyz}} \begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix} = \begin{bmatrix} \frac{1}{x} & 0 & 0 \\ 0 & \frac{1}{y} & 0 \\ 0 & 0 & \frac{1}{z} \end{bmatrix} = \begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$$

Hence, the option (A) is correct.

Choose the correct answer:

Question 9:

Let $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$, where $0 \leq \theta \leq 2\pi$ then:

(A) $\det(A) = 0$

(B) $\det(A) \in (2, \infty)$

(C) $\det(A) \in (2, 4)$

(D) $\det(A) \in [2, 4]$.

Answer 9:

$$A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$$

$= 1(1 + sin^2\theta) + \sin\theta\,(-\sin\theta + \sin\theta) + 1(sin^2\theta + 1)$ $\quad$ [Expending along C_1]

$= 2(1 + sin^2\theta)$

Now, given that: $0 \leq \theta \leq 2\pi$

$\Rightarrow 0 \leq \sin\theta \leq 1$

$\Rightarrow 0 \leq sin^2\theta \leq 1$

$\Rightarrow 1 \leq 1 + sin^2\theta \leq 2$

$\Rightarrow 2 \leq 2(1 + sin^2\theta) \leq 4$

$\Rightarrow \det(A) \in [2, 4]$

Hence, the option (D) is correct.