Mathematics

(Chapter – 3) (Matrices) (Exercise 3.1)
(Class – XII)

Question 1:

In the matrix $A = \begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$, write:

(i) The order of the matrix **(ii)** The number of elements

(iii) Write the elements $a_{13}, a_{21}, a_{33}, a_{24}, a_{23}$

Answer 1:

(i) The order of the matrix = number of row × number of column $= 3 \times 4$

(ii) The number of elements $= 3 \times 4 = 12$

(iii) Elements: $a_{13} = 19,\ a_{21} = 35, a_{33} = -5,\ a_{24} = 12, a_{23} = \frac{5}{2}$

Question 2:

If a matrix has 24 elements, what are the possible orders it can have? What, if it has 13 elements?

Answer 2:

The number of elements $= 24$

Therefore, the possible orders are as follows: $1 \times 24, 2 \times 12, 3 \times 8, 4 \times 6, 6 \times 4, 8 \times 3, 12 \times 2$ and 24×1

If it has 13 elements, then the possible orders: 13×1 and 1×13

Question 3:

If a matrix has 18 elements, what are the possible orders it can have? What, if it has 5 elements?

Answer 3:

Total number of elements in matrix $= 18$

Therefore, the possible orders are as follows: $1 \times 18, 2 \times 9, 3 \times 6, 6 \times 3, 9 \times 2$ and 18×1

If it has 5 elements, then the possible orders: 5×1 and 1×5

Question 4:

Construct a 2×2 matrix $A = [a_{ij}]$, whose elements are given by:

(i) $a_{ij} = \frac{(i+j)^2}{2}$ **(ii)** $a_{ij} = \frac{i}{j}$ **(iii)** $a_{ij} = \frac{(i+2j)^2}{2}$

Answer 4:

(i) Here, $a_{ij} = \frac{(i+j)^2}{2}$, therefore, the elements of matrix are:

$$a_{11} = \frac{(1+1)^2}{2} = 2, \qquad a_{12} = \frac{(1+2)^2}{2} = \frac{9}{2},$$

$$a_{21} = \frac{(2+1)^2}{2} = \frac{9}{2}, \qquad a_{22} = \frac{(2+2)^2}{2} = 8,$$

Therefore, matrix $= \begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$

(ii) Here, $a_{ij} = \frac{i}{j}$, therefore, the elements of matrix are:

$$a_{11} = \frac{1}{1} = 1, \qquad a_{12} = \frac{1}{2},$$

$$a_{21} = \frac{2}{1} = 2, \qquad a_{22} = \frac{2}{2} = 1$$

Therefore, matrix $= \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}$

(iii) Here, $a_{ij} = \frac{(i+2j)^2}{2}$, therefore, the elements of matrix are:

$$a_{11} = \frac{(1+2)^2}{2} = \frac{9}{2}, \qquad a_{12} = \frac{(1+4)^2}{2} = \frac{25}{2},$$

$$a_{21} = \frac{(2+2)^2}{2} = 8, \qquad a_{22} = \frac{(2+4)^2}{2} = 18$$

Therefore, matrix $= \begin{bmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{bmatrix}$

Question 5:

Construct a 3×4 matrix, whose elements are given by:

(i) $a_{ij} = \frac{1}{2}|-3i + j|$ **(ii)** $a_{ij} = 2i - j$

Answer 5:

(i) Here, $a_{ij} = \frac{1}{2}|-3i + j|$, therefore, the elements of matrix are:

$$a_{11} = \frac{1}{2}|-3+1| = 1, \quad a_{12} = \frac{1}{2}|-3+2| = \frac{1}{2}, \quad a_{13} = \frac{1}{2}|-3+3| = 0, \quad a_{14} = \frac{1}{2}|-3+4| = \frac{1}{2},$$

$$a_{21} = \frac{1}{2}|-6+1| = \frac{5}{2}, \quad a_{22} = \frac{1}{2}|-6+2| = 2, \quad a_{23} = \frac{1}{2}|-6+3| = \frac{3}{2}, \quad a_{24} = \frac{1}{2}|-6+4| = 1,$$

$$a_{31} = \frac{1}{2}|-9+1| = 4, \quad a_{32} = \frac{1}{2}|-9+2| = \frac{7}{2}, \quad a_{33} = \frac{1}{2}|-9+3| = 3, \quad a_{34} = \frac{1}{2}|-9+4| = \frac{5}{2},$$

$$\text{Therefore, matrix} = \begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$$

(ii) Here, $a_{ij} = 2i - j$, therefore, the elements of matrix are:

$$a_{11} = 2 - 1 = 1, \quad a_{12} = 2 - 2 = 0, \quad a_{13} = 2 - 3 = -1, \quad a_{14} = 2 - 4 = -2,$$

$$a_{21} = 4 - 1 = 3, \quad a_{22} = 4 - 2 = 2, \quad a_{23} = 4 - 3 = 1, \quad a_{24} = 4 - 4 = 0,$$

$$a_{31} = 6 - 1 = 5, \quad a_{32} = 6 - 2 = 4, \quad a_{33} = 6 - 3 = 3, \quad a_{34} = 6 - 4 = 2,$$

$$\text{Therefore, matrix} = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$$

Question 6:

Find the values of x, y and z from the following equations:

(i) $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$ **(ii)** $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$ **(iii)** $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

Answer 6:

(i) $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$

If two matrices are equal, then their corresponding elements are also equal. Therefore

$4 = y, \quad 3 = z$ and $x = 1$

(ii) $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

If two matrices are equal, then their corresponding elements are also equal. Therefore

$x + y = 6, \quad 5 + z = 5$ and $xy = 8$

On solving, $x = 2, y = 4$ and $z = 0$ or $x = 4, y = 2$ and $z = 0$

(iii) $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

If two matrices are equal, then their corresponding elements are also equal. Therefore

$x+y+z=9, \quad x+z=5$ and $y+z=7$

On solving, $x=2,\ y=4$ and $z=3$

Question 7:

Find the value of a, b, c and d from the equation:

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}.$$

Answer 7:

If two matrices are equal, then their corresponding elements are also equal. Therefore

$a-b=-1$ ... (1)
$2a-b=0$ ... (2)
$2a+c=5$ ... (3)
$3c+d=13$ ... (4)

Solving equation (1) and (2), we get, $a=1,\ b=2$

Putting the value of a in equation (3), we get, $c=3$

Putting the value of c in the equation (4), we get, $d=4$

Question 8:

$A=[a_{ij}]_{m\times n}$ is a square matrix, if

(A) $m<n$ (B) $m>n$ (C) $m=n$ (D) None of these

Answer 8:

In a square matrix the number of column is same as the number of rows, so the option (C) is correct.

Question 9:

Which of the given values of x and y make the following pair of matrices equal:

$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix}, \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

(A) $x=-\frac{1}{3}, y=7$ (B) Not possible to find

(C) $y=7, x=-\frac{2}{3}$ (D) $x=-\frac{1}{3}, y=-\frac{2}{3}$

Answer 9:

Here, $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

If two matrices are equal, then their corresponding elements are also equal. Therefore

$3x+7=0, \quad 5=y-2, \quad y-1=8$ and $2-3x=4$

On solving, $y=7,\ x=-\frac{7}{2}$ and $x=-\frac{2}{3}$, Here, the value of x is not unique, therefore the option (B) is correct.

Question 10:

The number of all possible matrices of order 3×3 with each entry 0 or 1 is:

(A) 27 (B) 18 (C) 81 (D) 512

Answer 10:

The total number of elements in a matrix of order $3\times 3=9$

If each entry is 0 or 1, then total number of permutation for each element $=2$

Therefore, the total permutation for 9 elements $=2^9=512$

Hence, the option (D) is correct.

Mathematics

(Chapter – 3) (Matrices) (Exercise 3.2)
(Class – XII)

Question 1:

Let $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}, C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$. Find each of the following:

(i) $A + B$ **(ii)** $A - B$ **(iii)** $3A - C$

(iv) AB **(v)** BA

Answer 1:

(i) $A + B$

$$= \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2+1 & 4+3 \\ 3-2 & 2+5 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 1 & 7 \end{bmatrix}$$

(ii) $A - B$

$$= \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2-1 & 4-3 \\ 3+2 & 2-5 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 5 & -3 \end{bmatrix}$$

(iii) $3A - C$

$$= 3\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 6+2 & 12-5 \\ 9-3 & 6-4 \end{bmatrix} = \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$$

(iv) AB

$$= \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2\times 1 + 4\times(-2) & 2\times 3 + 4\times 5 \\ 3\times 1 + 2\times(-2) & 3\times 3 + 2\times 5 \end{bmatrix} = \begin{bmatrix} -6 & 26 \\ -1 & 19 \end{bmatrix}$$

(v) BA

$$= \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 1\times 2 + 3\times 3 & 1\times 4 + 3\times 2 \\ -2\times 2 + 5\times 3 & -2\times 4 + 5\times 2 \end{bmatrix} = \begin{bmatrix} 11 & 10 \\ 11 & 2 \end{bmatrix}$$

Question 2:

Compute the following:

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ **(ii)** $\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$

(iii) $\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$ **(iv)** $\begin{bmatrix} cos^2x & sin^2x \\ sin^2x & cos^2x \end{bmatrix} + \begin{bmatrix} sin^2x & cos^2x \\ cos^2x & sin^2x \end{bmatrix}$

Answer 2:

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} a+a & b+b \\ -b+b & a+a \end{bmatrix} = \begin{bmatrix} 2a & 2b \\ 0 & 2a \end{bmatrix}$

(ii) $\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$

$$= \begin{bmatrix} a^2+b^2+2ab & b^2+c^2+2bc \\ a^2+c^2-2ac & a^2+b^2-2ab \end{bmatrix} = \begin{bmatrix} (a+b)^2 & (b+c)^2 \\ (a-c)^2 & (a-b)^2 \end{bmatrix}$$

(iii) $\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$

$$= \begin{bmatrix} -1+12 & 4+7 & -6+6 \\ 8+8 & 5+0 & 16+5 \\ 2+3 & 8+2 & 5+4 \end{bmatrix} = \begin{bmatrix} 11 & 11 & 0 \\ 16 & 5 & 21 \\ 5 & 10 & 9 \end{bmatrix}$$

(iv) $\begin{bmatrix} cos^2x & sin^2x \\ sin^2x & cos^2x \end{bmatrix} + \begin{bmatrix} sin^2x & cos^2x \\ cos^2x & sin^2x \end{bmatrix} = \begin{bmatrix} cos^2x+sin^2x & sin^2x+cos^2x \\ sin^2x+cos^2x & cos^2x+sin^2x \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Question 3:

Compute the indicated products:

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ **(ii)** $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$ **(iii)** $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

(iv) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$ **(v)** $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$ **(vi)** $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$

Answer 3:

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix} = \begin{bmatrix} a\times a + b\times b & a\times(-b) + b\times a \\ -b\times a + a\times b & -b\times(-b) + a\times a \end{bmatrix} = \begin{bmatrix} a^2+b^2 & 0 \\ 0 & b^2+a^2 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} 2 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 1\times 2 & 1\times 3 & 1\times 4 \\ 2\times 2 & 2\times 3 & 2\times 4 \\ 3\times 2 & 3\times 3 & 3\times 4 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 6 & 9 & 12 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1\times 1 + (-2)\times 2 & 1\times 2 + (-2)\times 3 & 1\times 3 + (-2)\times 1 \\ 2\times 1 + 3\times 2 & 2\times 2 + 3\times 3 & 2\times 3 + 3\times 1 \end{bmatrix} = \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$

(iv) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$

$= \begin{bmatrix} 2\times 1 + 3\times 0 + 4\times 3 & 2\times(-3) + 3\times 2 + 4\times 0 & 2\times 5 + 3\times 4 + 4\times 5 \\ 3\times 1 + 4\times 0 + 5\times 3 & 3\times(-3) + 4\times 2 + 5\times 0 & 3\times 5 + 4\times 4 + 5\times 5 \\ 4\times 1 + 5\times 0 + 6\times 3 & 4\times(-3) + 5\times 2 + 6\times 0 & 4\times 5 + 5\times 4 + 6\times 5 \end{bmatrix} = \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$

(v) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2\times 1 + 1\times(-1) & 2\times 0 + 1\times 2 & 2\times 1 + 1\times 1 \\ 3\times 1 + 2\times(-1) & 3\times 0 + 2\times 2 & 3\times 1 + 2\times 1 \\ -1\times 1 + 1\times(-1) & -1\times 0 + 1\times 2 & -1\times 1 + 1\times 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$

(vi) $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$

$= \begin{bmatrix} 3\times 2 + (-1)\times 1 + 3\times 3 & 3\times(-3) + (-1)\times 0 + 3\times 1 \\ -1\times 2 + 0\times 1 + 2\times 3 & -1\times(-3) + 0\times 0 + 2\times 1 \end{bmatrix} = \begin{bmatrix} 14 & -6 \\ 4 & 5 \end{bmatrix}$

Question 4:

If $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$, then compute $(A+B)$ and $(B-C)$. Also, verify that $A + (B-C) = (A+B) - C$.

Answer 4:

$A + B = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1+3 & 2-1 & -3+2 \\ 5+4 & 0+2 & 2+5 \\ 1+2 & -1+0 & 1+3 \end{bmatrix} = \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix}$

$B - C = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 3-4 & -1-1 & 2-2 \\ 4-0 & 2-3 & 5-2 \\ 2-1 & 0-(-2) & 3-3 \end{bmatrix} = \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$

$$LHS = A + (B - C) = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$$

$$RHS = (A + B) - C = \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$$

Hence, $A + (B - C) = (A + B) - C = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$

Question 5:

If $A = \begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix}$ and $B = \begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix}$, then compute $3A - 5B$.

Answer 5:

$$3A - 5B = 3\begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix} - 5\begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix} = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-2 & 3-3 & 5-5 \\ 1-1 & 2-2 & 4-4 \\ 7-7 & 6-6 & 2-2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$

Question 6:

Simplify, $cos\theta\begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix} + sin\theta\begin{bmatrix} sin\theta & -cos\theta \\ cos\theta & sin\theta \end{bmatrix}$

Answer 6:

$$cos\theta\begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix} + sin\theta\begin{bmatrix} sin\theta & -cos\theta \\ cos\theta & sin\theta \end{bmatrix} = \begin{bmatrix} cos^2\theta & sin\theta cos\theta \\ -sin\theta cos\theta & cos^2\theta \end{bmatrix} + \begin{bmatrix} sin^2\theta & -sin\theta cos\theta \\ sin\theta cos\theta & sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} cos^2\theta + sin^2\theta & sin\theta cos\theta - sin\theta cos\theta \\ -sin\theta cos\theta + sin\theta cos\theta & cos^2\theta + sin^2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Question 7:

Find X and Y, if

(i) $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

(ii) $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and $3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$

Answer 7:

(i) $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ ... (1)

and $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ ... (2)

Adding equation (1) and (2), we get

$$2X = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} \Rightarrow X = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$$

Putting the value of X in equation (1), we get

$$\begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix} + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \Rightarrow Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

(ii) $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ ... (1)

and $3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$ ... (2)

Multiply equation (1) by 3 and equation (2) by 2, on subtracting, we get

$3(2X + 3Y) - 2(3X + 2Y) = 3\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - 2\begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$

$\Rightarrow 6X + 9Y - 6X - 4Y = \begin{bmatrix} 6 & 9 \\ 12 & 0 \end{bmatrix} - \begin{bmatrix} 4 & -4 \\ -2 & 10 \end{bmatrix}$

$\Rightarrow 5Y = \begin{bmatrix} 2 & 13 \\ 14 & -10 \end{bmatrix} \quad \Rightarrow Y = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$

Putting the value of Y in equation (1), we get

$2X + \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$

$\Rightarrow 2X = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix} = \begin{bmatrix} 2 - \frac{6}{5} & 3 - \frac{39}{5} \\ 4 - \frac{42}{5} & 0 - 6 \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & -\frac{24}{5} \\ -\frac{22}{5} & 6 \end{bmatrix}$

$\Rightarrow X = \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix}$

Question 8:

Find X, if $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$

Answer 8:

$2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$

$\Rightarrow 2X + \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$ $\left[\because Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}\right]$

$\Rightarrow 2X = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix} \Rightarrow X = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$

Question 9:

Find x and y , if $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

Answer 9:

$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 + y & 6 + 0 \\ 0 + 1 & 2x + 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

If two matrices are equal, then their corresponding elements are also equal. Therefore,

$2 + y = 5$ and $2x + 2 = 8$

$\Rightarrow y = 3$ and $x = 3$

Question 10:

Solve the equation for x, y, z and t, if $2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

Answer 10:

$2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x & 2z \\ 2y & 2t \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x+3 & 2z-3 \\ 2y+0 & 2t+6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$

If two matrices are equal, then their corresponding elements are also equal. Therefore,

$2x + 3 = 9, 2z - 3 = 15, 2y = 12$ and $2t + 6 = 18$

$\Rightarrow x = 3, z = 9, y = 6$ and $t = 6$

Question 11:

If $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, find the value of x and y.

Answer 11:

$x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x \\ 3x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix} \quad \Rightarrow \begin{bmatrix} 2x-y \\ 3x+y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

If two matrices are equal, then their corresponding elements are also equal. Therefore,

$2x - y = 10$ and $3x + y = 5$

Adding the both equations, we get, $5x = 15 \quad \Rightarrow x = 3$

Putting the value of x in the equation $3x + y = 5$, we get, $3(3) + y = 5 \quad \Rightarrow y = -4$

Question 12:

Given $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$, find the value of x, y, z and w.

Answer 12:

$3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix} = \begin{bmatrix} x+4 & 6+x+y \\ -1+z+w & 2w+3 \end{bmatrix}$

If two matrices are equal, then their corresponding elements are also equal. Therefore,

$3x = x + 4, 3y = 6 + x + y, 3z = -1 + z + w$ and $3w = 2w + 3$

$\Rightarrow x = 2, \quad y = 4, \quad z = 1$ and $w = 3$

Question 13:

If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that $F(x)F(y) = F(x+y)$.

Answer 13:

$\text{LHS} = F(x)F(y)$

$= \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} \cos x\cos y - \sin x\sin y + 0 & -\cos x\sin y - \sin x\cos y + 0 & 0+0+0 \\ \sin x\cos y + \cos x\sin y + 0 & -\sin x\sin y + \cos x\cos y + 0 & 0+0+0 \\ 0+0+0 & 0+0+0+ & 0+0+1 \end{bmatrix}$

$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix} = F(x+y) = \text{RHS}$

Question 14:

Show that:

(i) $\begin{bmatrix}5 & -1\\6 & 7\end{bmatrix}\begin{bmatrix}2 & 1\\3 & 4\end{bmatrix} \neq \begin{bmatrix}2 & 1\\3 & 4\end{bmatrix}\begin{bmatrix}5 & -1\\6 & 7\end{bmatrix}$

(ii) $\begin{bmatrix}1 & 2 & 3\\0 & 1 & 0\\1 & 1 & 0\end{bmatrix}\begin{bmatrix}-1 & 1 & 0\\0 & -1 & 1\\2 & 3 & 4\end{bmatrix} \neq \begin{bmatrix}-1 & 1 & 0\\0 & -1 & 1\\2 & 3 & 4\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\0 & 1 & 0\\1 & 1 & 0\end{bmatrix}$

Answer 14:

(i) $LHS = \begin{bmatrix}5 & -1\\6 & 7\end{bmatrix}\begin{bmatrix}2 & 1\\3 & 4\end{bmatrix}$

$= \begin{bmatrix}5\times 2+(-1)\times 3 & 5\times 1+(-1)\times 4\\ 6\times 2+7\times 3 & 6\times 1+7\times 4\end{bmatrix} = \begin{bmatrix}7 & 1\\33 & 34\end{bmatrix}$

$RHS = \begin{bmatrix}2 & 1\\3 & 4\end{bmatrix}\begin{bmatrix}5 & -1\\6 & 7\end{bmatrix} = \begin{bmatrix}2\times 5+1\times 6 & 2\times(-1)+1\times 7\\3\times 5+4\times 6 & 3\times(-1)+4\times 7\end{bmatrix} = \begin{bmatrix}16 & 5\\39 & 25\end{bmatrix}$

Hence, $\begin{bmatrix}5 & -1\\6 & 7\end{bmatrix}\begin{bmatrix}2 & 1\\3 & 4\end{bmatrix} \neq \begin{bmatrix}2 & 1\\3 & 4\end{bmatrix}\begin{bmatrix}5 & -1\\6 & 7\end{bmatrix}$

(ii) $LHS = \begin{bmatrix}1 & 2 & 3\\0 & 1 & 0\\1 & 1 & 0\end{bmatrix}\begin{bmatrix}-1 & 1 & 0\\0 & -1 & 1\\2 & 3 & 4\end{bmatrix}$

$= \begin{bmatrix}1\times(-1)+2\times 0+3\times 2 & 1\times 1+2\times(-1)+3\times 3 & 1\times 0+2\times 1+3\times 4\\0\times(-1)+1\times 0+0\times 2 & 0\times 1+1\times(-1)+0\times 3 & 0\times 0+1\times 1+0\times 4\\1\times(-1)+1\times 0+0\times 2 & 1\times 1+1\times(-1)+0\times 3 & 1\times 0+1\times 1+0\times 4\end{bmatrix} = \begin{bmatrix}5 & 8 & 14\\0 & -1 & 1\\-1 & 0 & 1\end{bmatrix}$

$RHS = \begin{bmatrix}-1 & 1 & 0\\0 & -1 & 1\\2 & 3 & 4\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\0 & 1 & 0\\1 & 1 & 0\end{bmatrix}$

$= \begin{bmatrix}-1\times 1+1\times 0+0\times 1 & -1\times 2+1\times 1+0\times 1 & -1\times 3+1\times 0+0\times 0\\0\times 1+(-1)\times 0+1\times 1 & 0\times 2+(-1)\times 1+1\times 1 & 0\times 3+(-1)\times 0+1\times 0\\2\times 1+3\times 0+4\times 1 & 2\times 2+3\times 1+4\times 1 & 2\times 3+3\times 0+4\times 0\end{bmatrix} = \begin{bmatrix}-1 & -1 & -3\\1 & 0 & 0\\6 & 11 & 6\end{bmatrix}$

Hence, $\begin{bmatrix}1 & 2 & 3\\0 & 1 & 0\\1 & 1 & 0\end{bmatrix}\begin{bmatrix}-1 & 1 & 0\\0 & -1 & 1\\2 & 3 & 4\end{bmatrix} \neq \begin{bmatrix}-1 & 1 & 0\\0 & -1 & 1\\2 & 3 & 4\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\0 & 1 & 0\\1 & 1 & 0\end{bmatrix}$

Question 15:

Find $A^2 - 5A + 6I$, if $A = \begin{bmatrix}2 & 0 & 1\\2 & 1 & 3\\1 & -1 & 0\end{bmatrix}$

Answer 15:

$A = \begin{bmatrix}2 & 0 & 1\\2 & 1 & 3\\1 & -1 & 0\end{bmatrix} \Rightarrow A^2 = \begin{bmatrix}2 & 0 & 1\\2 & 1 & 3\\1 & -1 & 0\end{bmatrix}\begin{bmatrix}2 & 0 & 1\\2 & 1 & 3\\1 & -1 & 0\end{bmatrix}$

$= \begin{bmatrix}2\times 2+0\times 2+1\times 1 & 2\times 0+0\times 1+1\times(-1) & 2\times 1+0\times 3+1\times 0\\2\times 2+1\times 2+3\times 1 & 2\times 0+1\times 1+3\times(-1) & 2\times 1+1\times 3+3\times 0\\1\times 2+(-1)\times 2+0\times 1 & 1\times 0+(-1)\times 1+0\times(-1) & 1\times 1+(-1)\times 3+0\times 0\end{bmatrix} = \begin{bmatrix}5 & -1 & 2\\9 & -2 & 5\\0 & -1 & -2\end{bmatrix}$

Therefore, $A^2 - 5A + 6I$

$= \begin{bmatrix}5 & -1 & 2\\9 & -2 & 5\\0 & -1 & -2\end{bmatrix} - 5\begin{bmatrix}2 & 0 & 1\\2 & 1 & 3\\1 & -1 & 0\end{bmatrix} + 6\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}5 & -1 & 2\\9 & -2 & 5\\0 & -1 & -2\end{bmatrix} - \begin{bmatrix}10 & 0 & 5\\10 & 5 & 15\\5 & -5 & 0\end{bmatrix} + \begin{bmatrix}6 & 0 & 0\\0 & 6 & 0\\0 & 0 & 6\end{bmatrix}$

$= \begin{bmatrix}5-10+6 & -1-0+0 & 2-5+0\\9-10+0 & -2-5+6 & 5-15+0\\0-5+0 & -1+5+0 & -2-0+6\end{bmatrix} = \begin{bmatrix}1 & -1 & -3\\-1 & -1 & -10\\-5 & 4 & 4\end{bmatrix}$

Question 16:

If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = O$

Answer 16:

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1+0\times0+2\times2 & 1\times0+0\times2+2\times0 & 1\times2+0\times1+2\times3 \\ 0\times1+2\times0+1\times2 & 0\times0+2\times2+1\times0 & 0\times2+2\times1+1\times3 \\ 2\times1+0\times0+3\times2 & 2\times0+0\times2+3\times0 & 2\times2+0\times1+3\times3 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

$$A^3 = A^2A = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5\times1+0\times0+8\times2 & 5\times0+0\times2+8\times0 & 5\times2+0\times1+8\times3 \\ 2\times1+4\times0+5\times2 & 2\times0+4\times2+5\times0 & 2\times2+4\times1+5\times3 \\ 8\times1+0\times0+13\times2 & 8\times0+0\times2+13\times0 & 8\times2+0\times1+13\times3 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

Therefore, $LHS = A^3 - 6A^2 + 7A + 2I$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6\begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 21-30+7+2 & 0-0+0+0 & 34-48+14+0 \\ 12-12+0+0 & 8-24+14+2 & 23-30+7+0 \\ 34-48+14+0 & 0-0+0+0 & 55-78+21+2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O \quad = RHS$$

Question 17:

If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = kA - 2I$.

Answer 17:

Given: $A^2 = kA - 2I$

$$\Rightarrow \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3\times3+(-2)\times4 & 3\times(-2)+(-2)\times(-2) \\ 4\times3+(-2)\times4 & 4\times(-2)+(-2)\times(-2) \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$$

$\Rightarrow 4k = 4 \qquad \Rightarrow k = 1$

Question 18:

If $A=\begin{bmatrix}0 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 0\end{bmatrix}$ and I is the identity matrix of order 2, show that

$I+A=(I-A)\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$

Answer 18:

$\text{LHS}=I+A$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}0 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 0\end{bmatrix}=\begin{bmatrix}1 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 1\end{bmatrix}$$

$\text{RHS}=(I-A)\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$

$$=\left(\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}-\begin{bmatrix}0 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 0\end{bmatrix}\right)\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}=\begin{bmatrix}1 & \tan\frac{\alpha}{2}\\ -\tan\frac{\alpha}{2} & 1\end{bmatrix}\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$$

$$=\begin{bmatrix}\cos\alpha+\tan\frac{\alpha}{2}\sin\alpha & -\sin\alpha+\tan\frac{\alpha}{2}\cos\alpha\\ -\tan\frac{\alpha}{2}\cos\alpha+\sin\alpha & \tan\frac{\alpha}{2}\sin\alpha+\cos\alpha\end{bmatrix}$$

$$=\begin{bmatrix}\cos\alpha+\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}\sin\alpha & -\sin\alpha+\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}\cos\alpha\\ -\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}\cos\alpha+\sin\alpha & \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}\sin\alpha+\cos\alpha\end{bmatrix}$$

$$=\begin{bmatrix}\cos\alpha+\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}\sin\alpha & -\sin\alpha+\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}\cos\alpha\\ -\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}\cos\alpha+\sin\alpha & \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}\sin\alpha+\cos\alpha\end{bmatrix}$$

$$=\begin{bmatrix}\frac{\cos\alpha\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}\sin\alpha}{\cos\frac{\alpha}{2}} & \frac{-\sin\alpha\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}\cos\alpha}{\cos\frac{\alpha}{2}}\\ \frac{-\cos\alpha\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}\sin\alpha}{\cos\frac{\alpha}{2}} & \frac{\sin\alpha\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}\cos\alpha}{\cos\frac{\alpha}{2}}\end{bmatrix}$$

$$=\begin{bmatrix}\frac{\cos\left(\alpha-\frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} & \frac{-\sin\left(\alpha-\frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}}\\ \frac{\sin\left(\alpha-\frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} & \frac{\cos\left(\alpha-\frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}}\end{bmatrix}=\begin{bmatrix}1 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 1\end{bmatrix}=LHS$$

Question 19:

A trust fund has ₹30,000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide ₹30,000 among the two types of bonds. If the trust fund must obtain an annual total interest of:

(a) ₹1800 **(b)** ₹2000

Answer 19:

Let the amount invested in first bond = ₹x

Therefore, the amount invested in second bond = ₹$(30000 - x)$

(a) If the total annual interest is ₹1800, then

Investment in Bonds (in ₹)	Annual Interest Rate	Interest (in ₹)
$\begin{bmatrix} x & 30000-x \end{bmatrix}$	$\begin{bmatrix} 5\% \\ 7\% \end{bmatrix}$	$\begin{bmatrix} 1800 \end{bmatrix}$

On solving, $x \times 5\% + (30000 - x) \times 7\% = 1800$

$\Rightarrow \frac{5x}{100} + \frac{7}{100}(30000 - x) = 1800$

$\Rightarrow 5x + 210000 - 7x = 180000$

$\Rightarrow -2x = -30000$

$\Rightarrow x = 15000$

Therefore, the amount invested in first bond is ₹15000 and second bond is ₹15000.

(b) If the total annual interest is ₹2000,

Investment in Bonds (in ₹)	Annual Interest Rate	Interest (in ₹)
$\begin{bmatrix} x & 30000-x \end{bmatrix}$	$\begin{bmatrix} 5\% \\ 7\% \end{bmatrix}$	$\begin{bmatrix} 2000 \end{bmatrix}$

On solving $x \times 5\% + (30000 - x) \times 7\% = 2000$

$\Rightarrow \frac{5x}{100} + \frac{7}{100}(30000 - x) = 2000$

$\Rightarrow 5x + 210000 - 7x = 200000$

$\Rightarrow -2x = -10000 \qquad \Rightarrow x = 5000$

Therefore, the amount invested in first bond is ₹5000 and second bond is ₹25000.

Question 20:

The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are ₹80, ₹60 and ₹40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra.

Assume X, Y, Z, W and P are matrices of order $2 \times n$, $3 \times k$, $2 \times p$, $n \times 3$ and $p \times k$, respectively. Choose the correct answer in Exercises 21 and 22.

Answer 20:

Number of books (Chemistry Physics Economics)	Selling Price per book	Total amount (in ₹)
$\begin{bmatrix} 120 & 96 & 120 \end{bmatrix}$	$\begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix}$	$\begin{bmatrix} x \end{bmatrix}$

On solving, we get

$120 \times 80 + 96 \times 60 + 120 \times 40 = x$

$\Rightarrow x = 9600 + 5760 + 4800$

$\Rightarrow x = 20160$

Hence, the bookshop will receive ₹20160 from selling all the books.

Question 21:

The restriction on n, k and p so that PY + WY will be defined are:

(A) $k = 3, p = n$

(B) k is arbitrary, $p = 2$

(C) p is arbitrary, $k = 3$

(D) $k = 2, p = 3$

Answer 21:

Order of P $= p \times k$ and order of $Y = 3 \times k$.

So, PY will be defined, if, $k = 3$, hence, the order of PY is $p \times k$.

Order of W $= n \times 3$ and order of $Y = 3 \times k$.

According to order, WY is defined and its order is $n \times k$.

PY + WY is defined, if the order of PY and WY are equal.

$\Rightarrow p \times k = n \times k$

$\Rightarrow p = n$, hence, the option (A) is correct.

Question 22:

If $n = p$, then the order of the matrix $7X - 5Z$ is:

(A) $p \times 2$

(B) $2 \times n$

(C) $n \times 3$

(D) $p \times n$

Answer 22:

During the addition or subtraction, the order of matrix doesn't changes.

Therefore, the order of $7X - 5Z$ = order of X = order of Z = $2 \times n$,

Hence, the option (B) is correct.

Mathematics

(Chapter – 3) (Matrices) (Exercise 3.3)
(Class – XII)

Question 1:

Find the transpose of each of the following matrices:

(i) $\begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$ **(ii)** $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ **(iii)** $\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$

Answer 1:

(i) Let $A = \begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$ Hence, $A' = \begin{bmatrix} 5 & \frac{1}{2} & -1 \end{bmatrix}$

(ii) Let $B = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ Hence, $B' = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

(iii) Let $C = \begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$ Hence, $C' = \begin{bmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 3 \\ 6 & 6 & -1 \end{bmatrix}$

Question 2:

If $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$, then verify that

(i) $(A+B)' = A' + B'$ **(ii)** $(A-B)' = A' - B'$

Answer 2:

(i) $(A+B)$

$$= \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -1-4 & 2+1 & 3-5 \\ 5+1 & 7+2 & 9+0 \\ -2+1 & 1+3 & 1+1 \end{bmatrix} = \begin{bmatrix} -5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2 \end{bmatrix}$$

Therefore $(A+B)' = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$ ... (1)

$$A' + B' = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}' + \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}'$$

$$= \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1-4 & 5+1 & -2+1 \\ 2+1 & 7+2 & 1+3 \\ 3-5 & 9+0 & 1+1 \end{bmatrix} = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix} \quad \text{... (2)}$$

From the equation (1) and (2), we get

$$(A+B)' = A' + B'$$

(ii) $(A-B)$

$$= \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -1+4 & 2-1 & 3+5 \\ 5-1 & 7-2 & 9-0 \\ -2-1 & 1-3 & 1-1 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 8 \\ 4 & 5 & 9 \\ -3 & -2 & 0 \end{bmatrix}$$

Therefore $(A-B)' = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$ ... (1)

$$A' - B' = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}' - \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}'$$

$$= \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1+4 & 5-1 & -2-1 \\ 2-1 & 7-2 & 1-3 \\ 3+5 & 9-0 & 1-1 \end{bmatrix} = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix} \quad \text{... (2)}$$

From the equation (1) and (2), we get

$$(A+B)' = A' + B'$$

Question 3:

If $A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then verify that

(i) $(A + B)' = A' + B'$ **(ii)** $(A - B)' = A' - B'$

Answer 3:

(i) $A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$ [Since $(A')' = A$]

$(A + B) = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 3-1 & -1+2 & 0+1 \\ 4+1 & 2+2 & 1+3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 5 & 4 & 4 \end{bmatrix}$

Therefore $(A + B)' = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$ ... (1)

$A' + B' = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}' + \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}'$

$= \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 3-1 & 4+1 \\ -1+2 & 2+2 \\ 0+1 & 1+3 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$ ... (2)

From the equation (1) and (2), we get

$$(A + B)' = A' + B'$$

(ii) $A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$ [Since $(A')' = A$]

$(A - B) = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 3+1 & -1-2 & 0-1 \\ 4-1 & 2-2 & 1-3 \end{bmatrix} = \begin{bmatrix} 4 & -3 & -1 \\ 3 & 0 & -2 \end{bmatrix}$

Therefore $(A - B)' = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$ ... (1)

$A' - B' = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}' - \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}'$

$= \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 3+1 & 4-1 \\ -1-2 & 2-2 \\ 0-1 & 1-3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$ ... (2)

From the equation (1) and (2), we get

$$(A - B)' = A' - B'$$

Question 4:

If $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find $(A + 2B)'$

Answer 4:

$(A + 2B) = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix} + 2\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 2 & 4 \end{bmatrix}$

$= \begin{bmatrix} -2-2 & 3+0 \\ 1+2 & 2+4 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 3 & 6 \end{bmatrix}$

Therefore $(A + 2B)' = \begin{bmatrix} -4 & 3 \\ 3 & 6 \end{bmatrix}$

Question 5:

For the matrices A and B, verify that $(AB)' = B'A'$, where

(i) $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$ **(ii)** $A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$

Answer 5:

(i) Given: $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$

So, $AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}\begin{bmatrix} -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1\times(-1) & 1\times 2 & 1\times 1 \\ -4\times(-1) & -4\times 2 & -4\times 1 \\ 3\times(-1) & 3\times 2 & 3\times 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$

Therefore $AB' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$ ... (1)

$B' = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$ and $A' = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}' = \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}$

Therfore, $B'A' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & -4 & 3 \end{bmatrix}$

$= \begin{bmatrix} -1\times 1 & -1\times(-4) & -1\times 3 \\ 2\times 1 & 2\times(-4) & 2\times 3 \\ 1\times 1 & 1\times(-4) & 1\times 3 \end{bmatrix}$

$= \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$ ... (2)

From the equation (1) and (2), we get

$$(AB)' = B'A'$$

(ii) Given: $A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$

So, $AB = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 & 5 & 7 \end{bmatrix} = \begin{bmatrix} 0\times 1 & 0\times 5 & 0\times 7 \\ 1\times 1 & 1\times 5 & 1\times 7 \\ 2\times 1 & 2\times 5 & 2\times 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 5 & 7 \\ 2 & 10 & 14 \end{bmatrix}$

Therefore $AB' = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$ ... (1)

$B' = \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$ and $A' = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}' = \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}$

So, $B'A' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1\times 0 & 1\times 1 & 1\times 2 \\ 5\times 0 & 5\times 1 & 5\times 2 \\ 7\times 0 & 7\times 1 & 7\times 2 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$ ... (2)

From the equation (1) and (2), we get

$$(AB)' = B'A'$$

Question 6:

(i) If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then verify that $A'A = I$

(ii) If $A = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$, then verify that $A'A = I$

Answer 6:

(i) $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \Rightarrow A' = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

Therefore $A'A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$= \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & \cos\alpha\sin\alpha - \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha - \cos\alpha\sin\alpha & \sin^2\alpha + \cos^2\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

Hence, $A'A = I$

(ii) $A = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix} \Rightarrow A' = \begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix}$

Therefore $A'A = \begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix}\begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$

$= \begin{bmatrix} \sin^2\alpha + \cos^2\alpha & \sin\alpha\cos\alpha - \cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha - \sin\alpha\cos\alpha & \cos^2\alpha + \sin^2\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

Hence, $A'A = I$

Question 7:

(i) Show that the matrix $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ is a symmetric matrix.

(ii) Show that the matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ is a skew symmetric matrix.

Answer 7:

(i) $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}' = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix} = A$

$\Rightarrow A' = A$, therefore the matrix $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ is a symmetric matrix.

(ii) $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}' = -\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} = -A$

$\Rightarrow A' = -A$, therefore the matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ is a skew symmetric matrix.

Question 8:

For the matrix $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, verify that:

(i) $(A + A')$ is a symmetric matrix.

(ii) $(A - A')$ is a skew symmetric matrix.

Answer 8:

(i) Given that: $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}' = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$

Therefore, $(A + A') = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$

$(A+A')' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix} = A$

$\Rightarrow (A+A')' = (A+A')$, hence the matrix $(A+A')$ is a symmetric matrix.

(ii) $(A-A') = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$(A-A')' = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}' = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = -A$

$\Rightarrow (A+A')' = -(A+A')$, hence the matrix $(A-A')$ is a skew symmetric matrix.

Question 9:

Find $\frac{1}{2}(A+A')$ and $\frac{1}{2}(A-A')$ when $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

Answer 9:

Given that: $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}' = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$

So, $\frac{1}{2}(A+A') = \frac{1}{2}\left(\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} + \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

and, $\frac{1}{2}(A-A') = \frac{1}{2}\left(\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} - \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}\right)$

$= \frac{1}{2}\begin{bmatrix} 0 & 2a & 2b \\ -2a & 0 & 2c \\ -2b & -2c & 0 \end{bmatrix} = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

Question 10:

Express the following matrices as the sum of a symmetric and a skew symmetric matrix:

(i) $\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$

(ii) $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

(iv) $\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$

Answer 10:

(i) Given that: $A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$

Therefore, $A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A')$, let $P = \frac{1}{2}(A+A')$ and $Q = \frac{1}{2}(A-A')$

$P = \frac{1}{2}(A+A') = \frac{1}{2}\left(\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}$

$P' = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}' = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} = P \Rightarrow P$ is a symmetric matrix.

$Q = \frac{1}{2}(A-A') = \frac{1}{2}\left(\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

$Q' = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}' = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = -Q$

$\Rightarrow Q$ is a skew symmetric matrix.

Hence, $A = P + Q = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

(ii) Given that: $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

Therefore, $\mathrm{A} = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$

Let, $P = \frac{1}{2}(A + A')$ and $Q = \frac{1}{2}(A - A')$

$$P = \frac{1}{2}(A + A') = \frac{1}{2}\left(\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$P' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = P$$

$\Rightarrow P$ is a symmetric matrix.

$$Q = \frac{1}{2}(A - A') = \frac{1}{2}\left(\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} - \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$Q' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = -Q$$

$\Rightarrow Q$ is a skew symmetric matrix.

Hence, $A = P + Q = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(iii) Given: $A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

Therefore, $\mathrm{A} = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$

Let, $P = \frac{1}{2}(A + A')$ and $Q = \frac{1}{2}(A - A')$

$$P = \frac{1}{2}(A + A') = \frac{1}{2}\left(\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 1/2 & -5/2 \\ 1/2 & -2 & -2 \\ -5/2 & -2 & 2 \end{bmatrix}$$

$$P' = \begin{bmatrix} 3 & 1/2 & -5/2 \\ 1/2 & -2 & -2 \\ -5/2 & -2 & 2 \end{bmatrix}' = \begin{bmatrix} 3 & 1/2 & -5/2 \\ 1/2 & -2 & -2 \\ -5/2 & -2 & 2 \end{bmatrix} = P$$

$\Rightarrow P$ is a symmetric matrix.

$$Q = \frac{1}{2}(A - A') = \frac{1}{2}\left(\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 5/2 & 3/2 \\ -5/2 & 0 & 3 \\ -3/2 & -3 & 0 \end{bmatrix}$$

$$Q' = \begin{bmatrix} 0 & 5/2 & 3/2 \\ -5/2 & 0 & 3 \\ -3/2 & -3 & 0 \end{bmatrix}' = \begin{bmatrix} 0 & -5/2 & -3/2 \\ 5/2 & 0 & -3 \\ 3/2 & 3 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 5/2 & 3/2 \\ -5/2 & 0 & 3 \\ -3/2 & -3 & 0 \end{bmatrix} = -Q$$

$\Rightarrow Q$ is a skew symmetric matrix.

Hence, $A = P + Q = \begin{bmatrix} 3 & 1/2 & -5/2 \\ 1/2 & -2 & -2 \\ -5/2 & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 5/2 & 3/2 \\ -5/2 & 0 & 3 \\ -3/2 & -3 & 0 \end{bmatrix}$

(iv) Given that: $A = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$

Therefore, $A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$

Let, $P = \frac{1}{2}(A + A')$ and $Q = \frac{1}{2}(A - A')$

$P = \frac{1}{2}(A + A') = \frac{1}{2}\left(\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$

$P' = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}' = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} = P$

$\Rightarrow P$ is a symmetric matrix.

$Q = \frac{1}{2}(A - A') = \frac{1}{2}\left(\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 0 & 6 \\ -6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$

$Q' = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}' = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix} = -Q$

$\Rightarrow Q$ is a skew symmetric matrix.

Hence, $A = P + Q = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$

Choose the correct answer in the Exercises 11 and 12.

Question 11:

If A, B are symmetric matrices of same order, then AB – BA is a

(A) Skew symmetric matrix (B) Symmetric matrix

(C) Zero matrix (D) Identity matrix

Answer 11:

$(AB - BA)' = (AB)' - (BA)'$ $[\because (X - Y)' = X' - Y']$

$= B'A' - A'B'$ $[\because (XY)' = Y'X']$

$= BA - AB$ $[\because \text{ Given: } A' = A,\ B' = B]$

$= -(AB - BA) \Rightarrow (AB - BA)' = -(AB - BA)$,

Therefore, the matrix (AB − BA) is a skew symmetric matrix.

Hence, the option (A) is correct.

Question 12:

If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ then $A + A' = I$ if the value of α is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) π (D) $\frac{3\pi}{2}$

Answer 12:

Given taht: $A + A' = I$

$\Rightarrow \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow 2\cos\alpha = 1 \quad \Rightarrow \cos\alpha = \frac{1}{2} \quad \Rightarrow \alpha = \frac{\pi}{3}$

Hence, the option (B) is correct.

Mathematics

(Chapter – 3) (Matrices) (Exercise 3.4)
(Class – XII)

Question 1:

Matrices A and B will be inverse of each other only if

(A) $AB = BA$

(B) $AB = BA = 0$

(C) $AB = 0$, $BA = I$

(D) $AB = BA = I$

Answer 1:

We know that $AA^{-1} = I$,

So, $AB = BA = I$.

Hence, option (D) is correct.

Mathematics

(Chapter – 3) (Matrices) (Miscellaneous Exercise)
(Class – XII)

Question 1:

If A and B are symmetric matrices, prove that AB – BA is a skew symmetric matrix.

Answer 1:

$(AB - BA)' = (AB)' - (BA)'$ $\quad [\because (X - Y)' = X' - Y']$

$= B'A' - A'B'$ $\quad [\because (AB)' = B'A']$

$= BA - AB$ $\quad [\because \text{Given } A' = A,\ B' = B]$

$= -(AB - BA)$

$\Rightarrow (AB - BA)' = -(AB - BA),$

Therefore, AB – BA is a skew symmetric matrix.

Question 2:

Show that the matrix B′AB is symmetric or skew symmetric according as A is symmetric or skew symmetric.

Answer 2:

If A is a symmetric matrix, then $A' = A$

Here, $(B'AB)' = (AB)'(B')'$ $\quad [\because (AB)' = B'A']$

$= (AB)'B$ $\quad [\because (B')' = B]$

$= B'A'B$ $\quad [\because (AB)' = B'A']$

$= B'AB$ $\quad [\because \text{Given } A' = A]$

$\Rightarrow (B'AB)' = B'AB,$

Hence, the matrix $B'AB$ is also symmetric.

If A is skew symmetric matrix, then $A' = -A$

Here, $(B'AB)' = (AB)'(B')'$ $\quad [\because (AB)' = B'A']$

$= (AB)'B$ $\quad [\because (B')' = B]$

$= B'A'B$ $\quad [\because (AB)' = B'A']$

$= -B'AB$ $\quad [\because \text{Given } A' = -A]$

$\Rightarrow (B'AB)' = -B'AB,$

Hence, the matrix $B'AB$ is also a skew-symmetric matrix.

Question 3:

Find the values of x, y, z if the matrix $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfy the equation $A'A = I$.

Answer 3:

Given that: $A'A = I$

$$\Rightarrow \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 + 4y^2 + z^2 & 0 + 2y^2 - z^2 & 0 - 2y^2 + z^2 \\ 0 + 2y^2 - z^2 & x^2 + y^2 + z^2 & x^2 - y^2 - z^2 \\ 0 - 2y^2 + z^2 & x^2 - y^2 - z^2 & x^2 + y^2 + z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

If two matrices are equal, then their corresponding elements are also equal. So

$4y^2 + z^2 = 1, \quad 2y^2 - z^2 = 0$ and $x^2 + y^2 + z^2 = 1$

On solving, we get $x = \pm\frac{1}{\sqrt{2}}, y = \pm\frac{1}{\sqrt{6}}$ and $z = \pm\frac{1}{\sqrt{3}}$

Question 4:

For what values of x: $\begin{bmatrix}1 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 0\\2 & 0 & 1\\1 & 0 & 2\end{bmatrix}\begin{bmatrix}0\\2\\x\end{bmatrix} = O$?

Answer 4:

Given that: $\begin{bmatrix}1 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 0\\2 & 0 & 1\\1 & 0 & 2\end{bmatrix}\begin{bmatrix}0\\2\\x\end{bmatrix} = O$

$\Rightarrow \begin{bmatrix}1+4+1 & 2+0+0 & 0+2+2\end{bmatrix}\begin{bmatrix}0\\2\\x\end{bmatrix} = O$

$\Rightarrow \begin{bmatrix}6 & 2 & 4\end{bmatrix}\begin{bmatrix}0\\2\\x\end{bmatrix} = O$

$\Rightarrow [0+4+4x] = [0]$

$\Rightarrow 4+4x = 0$

$\Rightarrow x = -1$

Question 5:

If $A = \begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}$, show that $A^2 - 5A + 7I = O$.

Answer 5:

$LHS = A^2 - 5A + 7I$

$= \begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}\begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix} - 5\begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix} + 7\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

$= \begin{bmatrix}9-1 & 3+2\\-3-2 & -1+4\end{bmatrix} - \begin{bmatrix}15 & 5\\-5 & 10\end{bmatrix} + \begin{bmatrix}7 & 0\\0 & 7\end{bmatrix}$

$= \begin{bmatrix}8 & 5\\-5 & 3\end{bmatrix} - \begin{bmatrix}15 & 5\\-5 & 10\end{bmatrix} + \begin{bmatrix}7 & 0\\0 & 7\end{bmatrix}$

$= \begin{bmatrix}8-15+7 & 5-5+0\\-5+5+0 & 3-10+7\end{bmatrix}$

$= \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix} = O$

$= RHS$

Question 6:

Find x, if $\begin{bmatrix}x & -5 & -1\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\0 & 2 & 1\\2 & 0 & 3\end{bmatrix}\begin{bmatrix}x\\4\\1\end{bmatrix} = O$.

Answer 6:

Given: $\begin{bmatrix}x & -5 & -1\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\0 & 2 & 1\\2 & 0 & 3\end{bmatrix}\begin{bmatrix}x\\4\\1\end{bmatrix} = O$

$\begin{bmatrix}x & -5 & -1\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\0 & 2 & 1\\2 & 0 & 3\end{bmatrix}\begin{bmatrix}x\\4\\1\end{bmatrix} \Rightarrow \begin{bmatrix}x+0-2 & 0-10+0 & 2x-5-3\end{bmatrix}\begin{bmatrix}x\\4\\1\end{bmatrix} = O$

$\Rightarrow \begin{bmatrix}x-2 & -10 & 2x-8\end{bmatrix}\begin{bmatrix}x\\4\\1\end{bmatrix} = O \Rightarrow [x^2 - 2x - 40 + 2x - 8] = [0]$

$\Rightarrow x^2 = 48 \quad \Rightarrow x = \pm 4\sqrt{3}$

Question 7:

A manufacturer produces three products x, y, z which he sells in two markets. Annual sales are indicated below:

Market	Products		
	x	y	z
I	10,000	2,000	18,000
II	6,000	20,000	8,000

(a) If unit sale prices of x, y and z are ₹2.50, ₹1.50 and ₹1.00, respectively, find the total revenue in each market with the help of matrix algebra.

(b) If the unit costs of the above three commodities are ₹2.00, ₹1.00 and 50 paise respectively. Find the gross profit.

Answer 7:

(a) If unit sale prices of x, y and z are ₹2.50, ₹1.50 and ₹1.00, then

$$\begin{array}{ccc} & \text{Products} & \text{Selling Price} \\ & \begin{array}{ccc} x & y & z \end{array} & \\ \begin{array}{c} \text{Market I} \\ \text{Market II} \end{array} & \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} & \begin{bmatrix} ₹2.50 \\ ₹1.50 \\ ₹1.00 \end{bmatrix} \end{array}$$

Total revenue of each market

$$= \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} ₹2.50 \\ ₹1.50 \\ ₹1.00 \end{bmatrix} = \begin{bmatrix} ₹25000 + ₹3000 + ₹18000 \\ ₹15000 + ₹30000 + ₹8000 \end{bmatrix} = \begin{bmatrix} ₹46000 \\ ₹53000 \end{bmatrix}$$

Hence the revenue of market I is ₹46000 and that of the market II is ₹53000.

(b) If the unit costs of the three commodities are ₹2.00, ₹1.00 and 50 paise, then

$$\begin{array}{ccc} & \text{Products} & \text{Costs} \\ & \begin{array}{ccc} x & y & z \end{array} & \\ \begin{array}{c} \text{Market I} \\ \text{Market II} \end{array} & \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} & \begin{bmatrix} ₹2.00 \\ ₹1.00 \\ ₹0.50 \end{bmatrix} \end{array}$$

Gross profit from each market

$$= \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} ₹2.00 \\ ₹1.00 \\ ₹0.50 \end{bmatrix} = \begin{bmatrix} ₹20000 + ₹2000 + ₹9000 \\ ₹12000 + ₹20000 + ₹4000 \end{bmatrix} = \begin{bmatrix} ₹31000 \\ ₹36000 \end{bmatrix}$$

Total revenue from market I is ₹46000 and cost is ₹31000.

Therefore, the gross profit= Revenue – Cost = ₹46000 − ₹31000 = ₹15000

Total revenue from market II is ₹53000 and cost is ₹36000.

Therefore, the gross profit = ₹53000 − ₹36000 = ₹17000

Question 8:

Find the matrix X, so that $X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$.

Answer 8:

Let, $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, therefore, $X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

If two matrices are equal, then their corresponding elements are also equal. So

$a + 4b = -7, \quad 2a + 5b = -8, c + 4d = 2$ and $2c + 5d = 4$

On solving, we get $a = 1, b = -2, c = 2$ and $d = 0$

Hence, $X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

Choose the correct answer in the following questions:

Question 9:

If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$, then

(A) $1 + \alpha^2 + \beta\gamma = 0$

(B) $1 - \alpha^2 + \beta\gamma = 0$

(C) $1 - \alpha^2 - \beta\gamma = 0$

(D) $1 + \alpha^2 - \beta\gamma = 0$

Answer 9:

Given that: $A^2 = I$

$\Rightarrow \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \alpha^2 + \beta\gamma & \alpha\beta - \beta\alpha \\ \alpha\gamma - \alpha\gamma & \beta\gamma + \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

On comparing, we get $\alpha^2 + \beta\gamma = 1$.

Hence the option (C) is correct.

Question 10:

If the matrix A is both symmetric and skew symmetric, then

(A) A is a diagonal matrix

(B) A is a zero matrix

(C) A is a square matrix

(D) None of these

Answer 10:

We know that only a zero matrix is always both symmetric and skew symmetric.

Hence, the option (B) is correct.

Question 11:

If A is square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to

(A) A

(B) $I - A$

(C) I

(D) $3A$

Answer 11:

$(I + A)^3 - 7A = I^3 + A^3 + 3I^2\text{A} + 3IA^2 - 7A$

$= I + A^2A + 3I\text{A} + 3IA^2 - 7A \qquad [\because\ I^3 = I^2 = I]$

$= I + AA + 3\text{A} + 3IA - 7A \qquad [\because\ A^2 = A]$

$= I + A + 3\text{A} + 3A - 7A = I \qquad [\because\ IA = A]$

Hence, the option (C) is correct.