Mathematics

(Chapter - 12)*(Surface Areas and Volumes)*
(Class – X)

Exercise 12.1

Unless stated otherwise, take $\pi = \frac{22}{7}$.

Question 1:

2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboids.

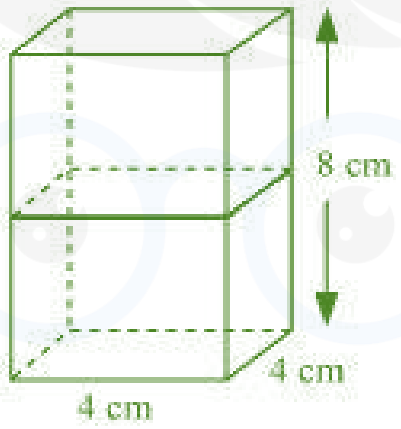
Answer 1:

Given that,

Volume of cubes = 64 cm^3

$(Edge)^3 = 64$

Edge = 4 cm



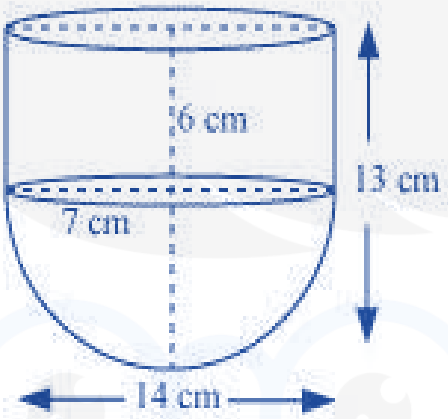
If cubes are joined end to end, the dimensions of the resulting cuboid will be 4 cm, 4 cm, 8 cm.

$$\therefore \text{Surface area of cuboids} = 2(lb+bh+lh)$$
$$= 2(4\times4+4\times8+4\times8)$$
$$= 2(16+32+32)$$
$$= 2(16+64)$$
$$= 2\times80 = 160 \text{ cm}^2$$

Question 2:

A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel. [Use $\pi = \frac{22}{7}$]

Answer 2:



It can be observed that radius (r) of the cylindrical part and the hemispherical part is the same (i.e., 7 cm).

Height of hemispherical part = Radius = 7 cm

Height of cylindrical part (h) = 13 −7 = 6 cm

Inner surface area of the vessel = CSA of cylindrical part + CSA of hemispherical part

$= 2\pi rh + 2\pi r^2$

$$\text{Inner surface area of vessel} = 2\times\frac{22}{7}\times 7\times 6 + 2\times\frac{22}{7}\times 7\times 7$$

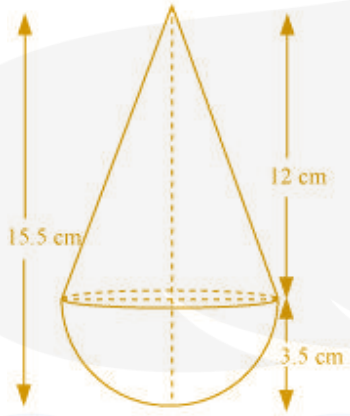
$$= 44(6+7) = 44\times 13$$

$$= 572 \text{ cm}^2$$

Question 3:

A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.[Use $\pi = \frac{22}{7}$]

Answer 3:



It can be observed that the radius of the conical part and the hemispherical part is same (i.e., 3.5 cm).

Height of hemispherical part = Radius (r) = 3.5 = 7/2 cm

Height of conical part (h) = 15.5 −3.5 = 12 cm

$$\text{Slant height } (l) \text{ of conical part} = \sqrt{r^2 + h^2}$$

$$= \sqrt{\left(\frac{7}{2}\right)^2 + (12)^2} = \sqrt{\frac{49}{4} + 144} = \sqrt{\frac{49 + 576}{4}}$$

$$= \sqrt{\frac{625}{4}} = \frac{25}{2}$$

Total surface area of toy = CSA of conical part + CSA of hemispherical part

$$= \pi r l + 2\pi r^2$$

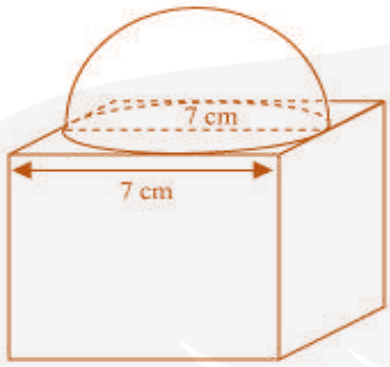
$$= \frac{22}{7} \times \frac{7}{2} \times \frac{25}{2} + 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$= 137.5 + 77 = 214.5 \text{cm}^2$$

Question 4:

A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid. [Use $\pi = \frac{22}{7}$]

Answer 4:



From the figure, it can be observed that the greatest diameter possible for such hemisphere is equal to the cube's edge, i.e., 7cm.

Radius (r) of hemispherical part = 7/2 = 3.5cm

Total surface area of solid = Surface area of cubical part + CSA of hemispherical part − Area of base of hemispherical part

$= 6\,(\text{Edge})^2 + 2\pi r^2 - \pi r^2 = 6\,(\text{Edge})^2 + \pi r^2$

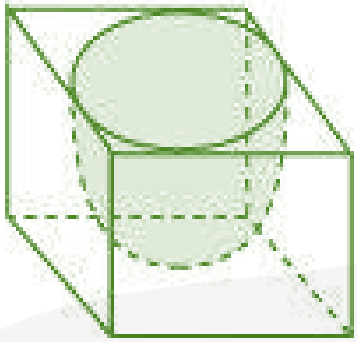
$$\text{Total surface area of solid} = 6(7)^2 + \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$= 294 + 38.5 = 332.5\,\text{cm}^2$$

Question 5:

A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Answer 5:



Diameter of hemisphere = Edge of cube = l

Radius of hemisphere = $l/2$

Total surface area of solid = Surface area of cubical part + CSA of hemispherical part − Area of base of hemispherical part

$= 6\,(\text{Edge})^2 + 2\pi r^2 - \pi r^2 = 6\,(\text{Edge})^2 + \pi r^2$

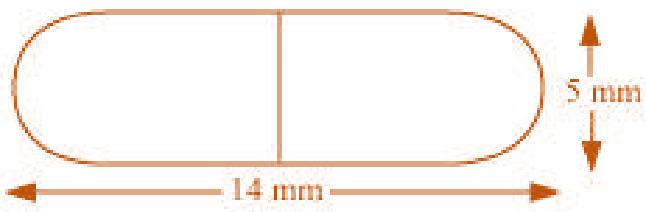
$$\text{Total surface area of solid} = 6l^2 + \pi \times \left(\frac{l}{2}\right)^2$$

$$= 6l^2 + \frac{\pi l^2}{4}$$

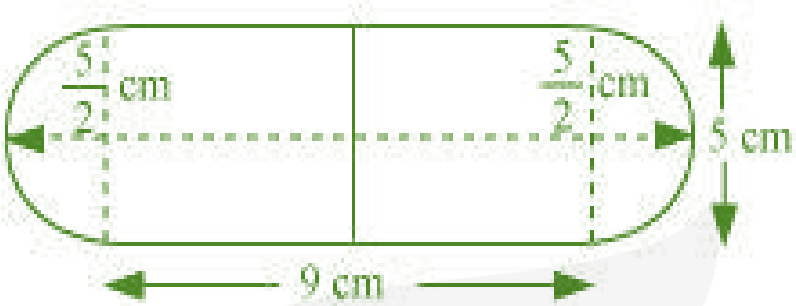
$$= \frac{1}{4}(24 + \pi)l^2 \text{ unit}^2$$

Question 6:

A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (see the given figure). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area. [Use $\pi = \frac{22}{7}$]



Answer 6:



It can be observed that

Radius (r) of cylindrical part = Radius (r) of hemispherical part

$$= \frac{\text{Diameter of the capsule}}{2} = \frac{5}{2}$$

Length of cylindrical part (h) = Length of the entire capsule $- 2 \times r$

$= 14 - 5 = 9$ cm

Surface area of capsule = 2×CSA of hemispherical part + CSA of cylindrical part

$$= 2 \times 2\pi r^2 + 2\pi rh$$

$$= 4\pi \left(\frac{5}{2}\right)^2 + 2\pi \left(\frac{5}{2}\right)(9)$$

$$= 25\pi + 45\pi$$

$$= 70\pi \text{ mm}^2$$

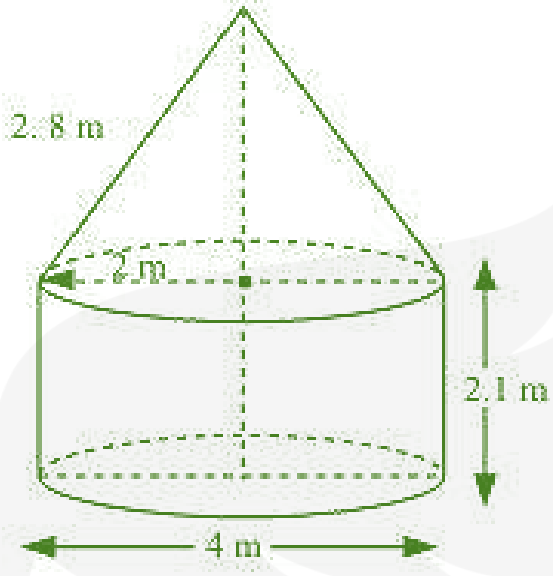
$$= 70 \times \frac{22}{7}$$

$$= 220 \text{ mm}^2$$

Question 7:

A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs 500 per m^2. (Note that the base of the tent will not be covered with canvas.)[Use $\pi = \frac{22}{7}$]

Answer 7:



Given that,

Height (h) of the cylindrical part = 2.1 m

Diameter of the cylindrical part = 4 m

Radius of the cylindrical part = 2 m

Slant height (l) of conical part = 2.8 m

Area of canvas used = CSA of conical part + CSA of cylindrical part

$$= \pi rl + 2\pi rh$$

$$= \pi \times 2 \times 2.8 + 2\pi \times 2 \times 2.1$$

$$= 2\pi\left[2.8 + 2 \times 2.1\right] = 2\pi\left[2.8 + 4.2\right] = 2 \times \frac{22}{7} \times 7$$

$$= 44 \text{ m}^2$$

Cost of 1 m^2 canvas = Rs 500

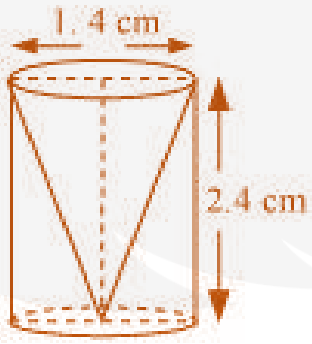
Cost of 44 m^2 canvas = 44 × 500 = 22000

Therefore, it will cost Rs 22000 for making such a tent.

Question 8:

From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2. [Use $\pi = \frac{22}{7}$]

Answer 8:



Given that,

Height (h) of the conical part = Height (h) of the cylindrical part = 2.4 cm

Diameter of the cylindrical part = 1.4 cm

Therefore, radius (r) of the cylindrical part = 0.7 cm

$$\text{Slant height } (l) \text{ of conical part} = \sqrt{r^2 + h^2}$$

$$= \sqrt{(0.7)^2 + (2.4)^2} = \sqrt{0.49 + 5.76}$$

$$= \sqrt{6.25} = 2.5$$

Total surface area of the remaining solid will be

= CSA of cylindrical part + CSA of conical part + Area of cylindrical base

$$= 2\pi rh + \pi rl + \pi r^2$$

$$= 2 \times \frac{22}{7} \times 0.7 \times 2.4 + \frac{22}{7} \times 0.7 \times 2.5 + \frac{22}{7} \times 0.7 \times 0.7$$

$$= 4.4 \times 2.4 + 2.2 \times 2.5 + 2.2 \times 0.7$$

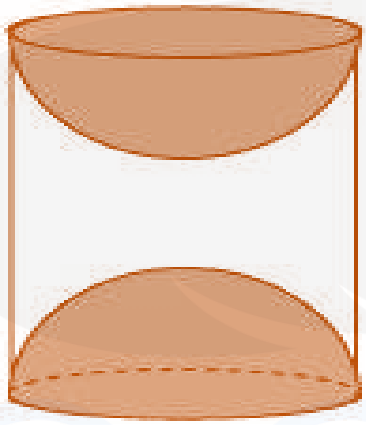
$$= 10.56 + 5.50 + 1.54 = 17.60 \text{ cm}^2$$

The total surface area of the remaining solid to the nearest cm^2 is 18 cm^2

Question 9:

A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in given figure. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article.

[Use $\pi = \frac{22}{7}$]



Answer 9:

Given that,

Radius (r) of cylindrical part = Radius (r) of hemispherical part = 3.5 cm

Height of cylindrical part (h) = 10 cm

Surface area of article = CSA of cylindrical part + 2 × CSA of hemispherical part

$= 2\pi rh + 2 \times 2\pi r^2$

$= 2\pi \times 3.5 \times 10 + 2 \times 2\pi \times 3.5 \times 3.5$

$= 70\pi + 49\pi$

$= 119\pi$

$= 17 \times 22 = 374 \text{ cm}^2$

Mathematics

(Chapter - 12) *(Surface Areas and Volumes)*
(Class – X)

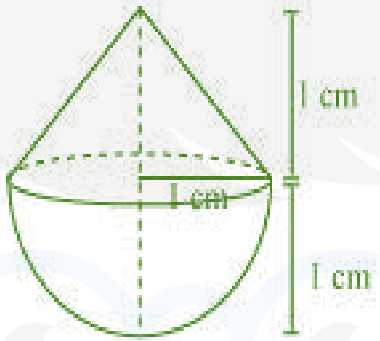
Exercise 12.2

Unless stated otherwise, take $\pi = \frac{22}{7}$.

Question 1:

A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.

Answer 1:



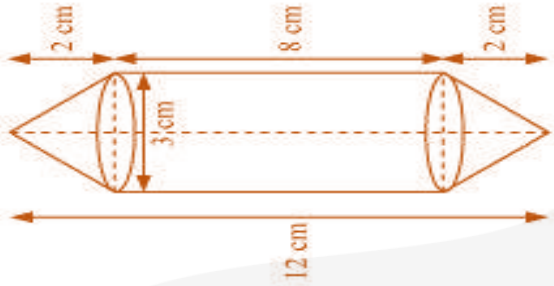
Given that,
Height (h) of conical part = Radius(r) of conical part = 1 cm
Radius(r) of hemispherical part = Radius of conical part (r) = 1 cm
Volume of solid = Volume of conical part + Volume of hemispherical part

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$
$$= \frac{1}{3}\pi.1^2.1 + \frac{2}{3}\pi.1^3 = \pi\ cm^3$$

Question 2:

Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. if each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.) [Use $\pi = \frac{22}{7}$]

Answer 2:



From the figure, it can be observed that
Height (h_1) of each conical part = 2 cm
Height (h_2) of cylindrical part = 12 − 2 × Height of conical part
= 12 − 2 ×2 = 8 cm
Radius (r) of cylindrical part = Radius of conical part = 3/2 cm
Volume of air present in the model = Volume of cylinder + 2 × Volume of cones

$$= \pi r^2 h_2 + 2 \times \frac{1}{3}\pi r^2 h_1$$

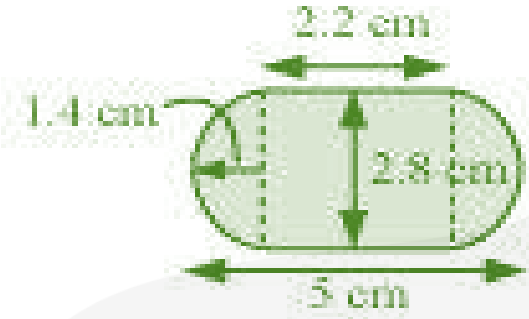
$$= \pi \left(\frac{3}{2}\right)^2 .8 + 2 \times \frac{1}{3}\pi \left(\frac{3}{2}\right)^2 .2$$

$$= 18\pi + 3\pi = 21\pi = 21 \times \frac{22}{7} = 66\ cm^3$$

Question 3:

A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm (see the given figure). [Use $\pi = \frac{22}{7}$]

Answer 3:



It can be observed that

Radius (r) of cylindrical part = Radius (r) of hemispherical part = $\frac{2.8}{2} = 1.4\,\text{cm}$

Length of each hemispherical part = Radius of hemispherical part = 1.4 cm

Length (h) of cylindrical part = 5 − 2 × Length of hemispherical part

= 5 − 2 × 1.4 = 2.2 cm

Volume of one gulab jamun = Vol. of cylindrical part + 2 × Vol. of hemispherical part

$$= \pi r^2 h + 2 \times \frac{2}{3}\pi r^3 = \pi r^2 h + \frac{4}{3}\pi r^3$$

$$= \pi \times (1.4)^2 \times 2.2 + \frac{4}{3}\pi (1.4)^3$$

$$= \frac{22}{7} \times 1.4 \times 1.4 \times 2.2 + \frac{4}{3} \times \frac{22}{7} \times 1.4 \times 1.4 \times 1.4$$

$$= 13.552 + 11.498 = 25.05\ \text{cm}^3$$

Volume of 45 gulab jamuns = 45×25.05 = 1,127.25 cm^3

Volume of sugar syrup = 30% of volume

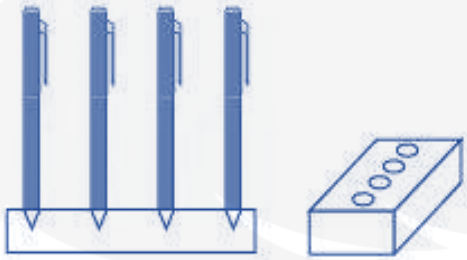
$$= \frac{30}{100} \times 1{,}127.25$$

$$= 338.17\ \text{cm}^3$$

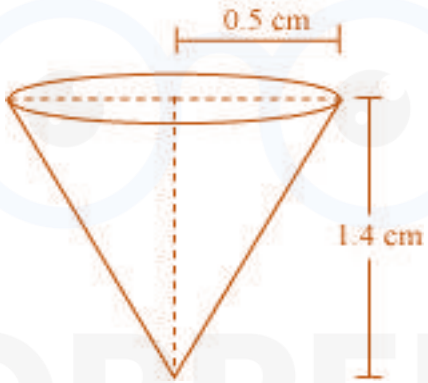
$$\simeq 338\ \text{cm}^3$$

Question 4:

A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboids are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand (see the following figure). [Use $\pi = \frac{22}{7}$]



Answer 4:



Depth (h) of each conical depression = 1.4 cm
Radius (r) of each conical depression = 0.5 cm
Volume of wood = Volume of cuboid − 4 × Volume of cones

$$= lbh - 4 \times \frac{1}{3}\pi r^2 h$$

$$= 15 \times 10 \times 3.5 - 4 \times \frac{1}{3} \times \frac{22}{7} \times \left(\frac{1}{2}\right)^2 \times 1.4$$

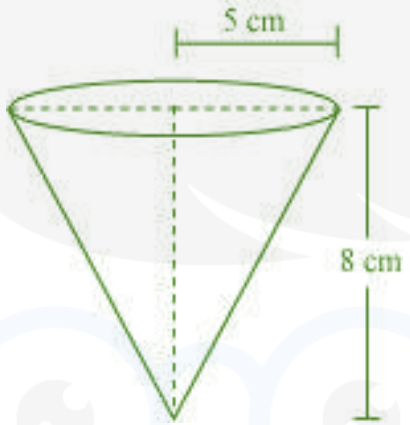
$$= 525 - 1.47$$

$$= 523.53 \text{ cm}^3$$

Question 5:

A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Answer 5:



Height (h) of conical vessel = 8 cm
Radius (r_1) of conical vessel = 5 cm
Radius (r_2) of lead shots = 0.5 cm
Let n number of lead shots were dropped in the vessel.
Volume of water spilled = Volume of dropped lead shots

$$\frac{1}{4}\times \text{Volume of cone} = n\times\frac{4}{3}\pi r_2^3$$

$$\frac{1}{4}\times\frac{1}{3}\pi r_1^2 h = n\times\frac{4}{3}\pi r_2^3$$

$$r_1^2 h = n\times 16 r_2^3$$

$$5^2\times 8 = n\times 16\times(0.5)^3$$

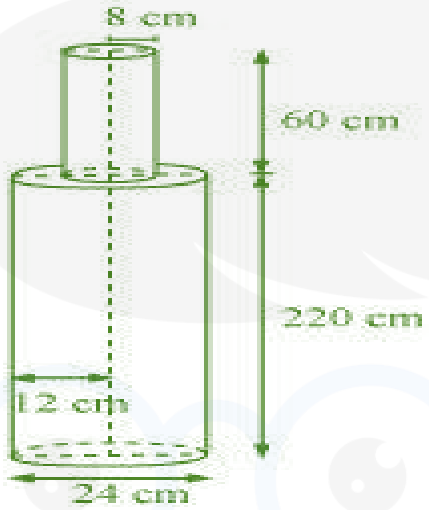
$$n = \frac{25\times 8}{16\times\left(\frac{1}{2}\right)^3} = 100$$

Hence, the number of lead shots dropped in the vessel is 100.

Question 6:

A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 g mass. [Use $\pi = 3.14$]

Answer 6:



From the figure, it can be observed that

Height (h_1) of larger cylinder = 220 cm

Radius (r_1) of larger cylinder = 24/2 = 12 cm

Height (h_2) of smaller cylinder = 60 cm

Radius (r_2) of smaller cylinder = 8 cm

$$\text{Total volume of pole} = \text{Volume of larger cylinder} + \text{Volume of smaller cylinder}$$

$$= \pi r_1^2 h_1 + \pi r_2^2 h_2$$

$$= \pi (12)^2 \times 220 + \pi (8)^2 \times 60$$

$$= \pi [144 \times 220 + 64 \times 60]$$

$$= 35520 \times 3.14 = 1,11,532.8 \text{ cm}^3$$

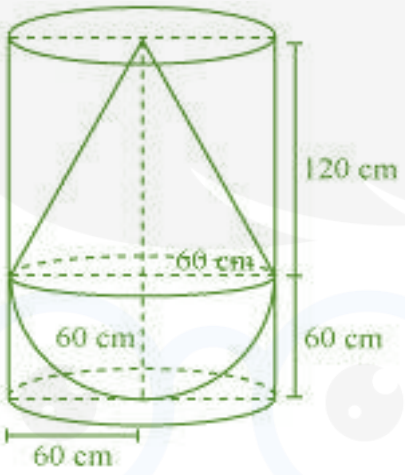
Mass of 1 cm^3 iron = 8g

Mass of 111532.8 cm^3 = 111532.8×8g = 892262.4g = 892.262 Kg

Question 7:

A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm. [Use $\pi = \frac{22}{7}$]

Answer 7:



Radius (r) of hemispherical part = Radius (r) of conical part = 60 cm
Height (h_2) of conical part of solid = 120 cm
Height (h_1) of cylinder = 180 cm
Radius (r) of cylinder = 60 cm
Volume of water left = Volume of cylinder − Volume of solid

$$= \text{Volume of cylinder} - (\text{Volume of cone} + \text{Volume of hemisphere})$$

$$= \pi r^2 h_1 - \left(\frac{1}{3}\pi r^2 h_2 + \frac{2}{3}\pi r^3\right)$$

$$= \pi(60)^2(180) - \left(\frac{1}{3}\pi(60)^2 \times 120 + \frac{2}{3}\pi(60)^3\right)$$

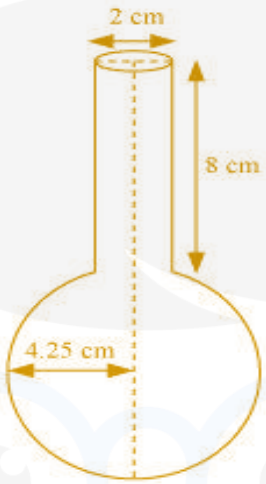
$$= \pi(60)^2\left[(180) - (40+40)\right]$$

$$= \pi(3,600)(100) = 3,60,000\pi \text{ cm}^3 = 1131428.57 \text{ cm}^3 = 1.131 \text{ m}^3$$

Question 8:

A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter o the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements, and $\pi = 3.14$.

Answer 8:



Height (h) of cylindrical part = 8 cm
Radius (r_2) of cylindrical part = 1 cm
Radius (r_1) spherical part = 4.25 cm
Volume of vessel = Volume of sphere + Volume of cylinder

$$= \frac{4}{3}\pi r_1^3 + \pi r_2^2 h$$

$$= \frac{4}{3}\pi\left(\frac{8.5}{2}\right)^3 + \pi(1)^2(8)$$

$$= \frac{4}{3}\times 3.14 \times 76.765625 + 8 \times 3.14$$

$$= 321.392 + 25.12$$

$$= 346.512$$

$$= 346.51 \text{ cm}^3$$

Hence, she is wrong.