

# Mathematics

(Chapter - 11) (*Areas Related to Circles*)

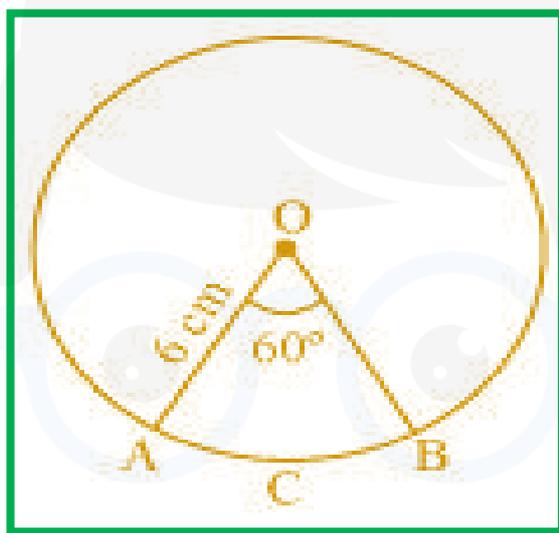
(Class - X)

## Exercise 11.1

**Question 1:**

Find the area of a sector of a circle with radius 6 cm if angle of the sector is  $60^\circ$   
[Use  $\pi = \frac{22}{7}$ ].

**Answer 1:**



Let OACB be a sector of the circle making  $60^\circ$  angle at centre O of the circle.

$$\text{Area of sector of angle } \theta = \frac{\theta}{360^\circ} \times \pi r^2$$

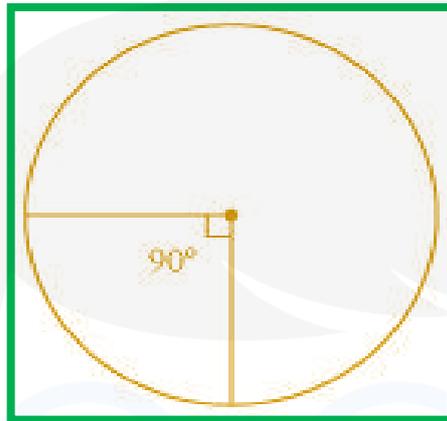
$$\text{Area of sector OACB} = \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times (6)^2$$

$$= \frac{1}{6} \times \frac{22}{7} \times 6 \times 6 = \frac{132}{7} \text{ cm}^2$$

Therefore, the area of the sector of the circle making  $60^\circ$  at the centre of the circle is  $\frac{132}{7} \text{ cm}^2$ .

**Question 2:**

Find the area of a quadrant of a circle whose circumference is 22 cm [Use  $\pi = \frac{22}{7}$ ].

**Answer 2:**

Let the radius of the circle be  $r$ .

Circumference = 22 cm

$$2\pi r = 22$$

$$r = \frac{22}{2\pi} = \frac{11}{\pi}$$

Quadrant of circle will subtend  $90^\circ$  angle at the centre of the circle.

$$\text{Area of such quadrant of the circle} = \frac{90^\circ}{360^\circ} \times \pi \times r^2$$

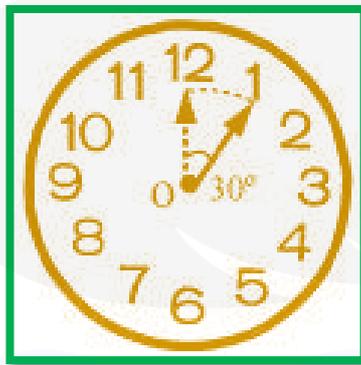
$$= \frac{1}{4} \times \pi \times \left(\frac{11}{\pi}\right)^2$$

$$= \frac{121}{4\pi} = \frac{121 \times 7}{4 \times 22}$$

$$= \frac{77}{8} \text{ cm}^2$$

**Question 3:**

The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes  $\left[Use \pi = \frac{22}{7}\right]$ .

**Answer 3:**

We know that in 1 hour (i.e., 60 minutes), the minute hand rotates  $360^\circ$ .

In 5 minutes, minute hand will rotate =  $\frac{360^\circ}{60} \times 5 = 30^\circ$

Therefore, the area swept by the minute hand in 5 minutes will be the area of a sector of  $30^\circ$  in a circle of 14 cm radius.

Area of sector of angle  $\theta = \frac{\theta}{360^\circ} \times \pi r^2$

Area of sector of  $30^\circ = \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14$

$$= \frac{22}{12} \times 2 \times 14$$

$$= \frac{11 \times 14}{3}$$

$$= \frac{154}{3} \text{ cm}^2$$

Therefore, the area swept by the minute hand in 5 minutes is  $154/3 \text{ cm}^2$ .

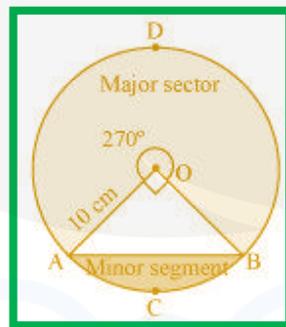
**Question 4:**

A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding:

(i) Minor segment

(ii) Major sector

[Use  $\pi = 3.14$ ]

**Answer 4:**

Let AB be the chord of the circle subtending  $90^\circ$  angle at centre O of the circle.

$$\begin{aligned} \text{(ii) Area of major sector OADB} &= \left( \frac{360^\circ - 90^\circ}{360^\circ} \right) \times \pi r^2 = \left( \frac{270^\circ}{360^\circ} \right) \pi r^2 \\ &= \frac{3}{4} \times 3.14 \times 10 \times 10 \end{aligned}$$

$$\begin{aligned} \text{Area of minor sector OACB} &= \frac{90^\circ}{360^\circ} \times \pi r^2 \\ &= \frac{1}{4} \times 3.14 \times 10 \times 10 \\ &= 78.5 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of } \triangle OAB &= \frac{1}{2} \times OA \times OB = \frac{1}{2} \times 10 \times 10 \\ &= 50 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{(i) Area of minor segment ACB} &= \text{Area of minor sector OACB} - \text{Area of } \triangle OAB \\ &= 78.5 - 50 = 28.5 \text{ cm}^2 \end{aligned}$$

**Question 5:**

In a circle of radius 21 cm, an arc subtends an angle of  $60^\circ$  at the centre. Find:

- (i) The length of the arc
- (ii) Area of the sector formed by the arc
- (iii) Area of the segment formed by the corresponding chord

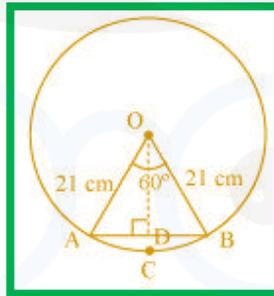
$$\left[ \text{Use } \pi = \frac{22}{7} \right]$$

**Answer 5:**

Radius ( $r$ ) of circle = 21 cm

Angle subtended by the given arc =  $60^\circ$

(i) Length of an arc of a sector of angle  $\theta = \frac{\theta}{360^\circ} \times 2\pi r$



$$\text{Length of arc ACB} = \frac{60^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 21$$

$$= \frac{1}{6} \times 2 \times 22 \times 3$$

$$= 22 \text{ cm}$$

(ii) Area of sector OACB =  $\frac{60^\circ}{360^\circ} \times \pi r^2$

$$= \frac{1}{6} \times \frac{22}{7} \times 21 \times 21$$

$$= 231 \text{ cm}^2$$

In  $\triangle OAB$ ,

$$\angle OAB = \angle OBA \quad (\text{As } OA = OB)$$

$$\angle OAB + \angle AOB + \angle OBA = 180^\circ$$

$$2\angle OAB + 60^\circ = 180^\circ$$

$$\angle OAB = 60^\circ$$

Therefore,  $\Delta OAB$  is an equilateral triangle.

$$\begin{aligned}\text{Area of } \Delta OAB &= \frac{\sqrt{3}}{4} \times (\text{Side})^2 \\ &= \frac{\sqrt{3}}{4} \times (21)^2 = \frac{441\sqrt{3}}{4} \text{ cm}^2\end{aligned}$$

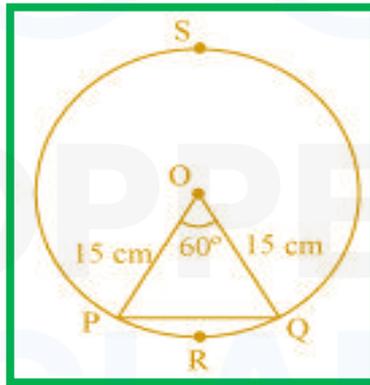
(iii) Area of segment ACB = Area of sector OACB – Area of  $\Delta OAB$

$$= \left( 231 - \frac{441\sqrt{3}}{4} \right) \text{ cm}^2$$

**Question 6:**

A chord of a circle of radius 15 cm subtends an angle of  $60^\circ$  at the centre. Find the areas of the corresponding minor and major segments of the circle. [Use  $\pi = 3.14$  and  $\sqrt{3} = 1.73$ ]

**Answer 6:**



Radius ( $r$ ) of circle = 15 cm

$$\text{Area of sector OPRQ} = \frac{60^\circ}{360^\circ} \times \pi r^2$$

$$\begin{aligned} &= \frac{1}{6} \times 3.14 \times (15)^2 \\ &= 117.75 \text{ cm}^2 \end{aligned}$$

In  $\Delta OPQ$ ,

$$\angle OPQ = \angle OQP \quad (\text{As } OP = OQ)$$

$$\angle OPQ + \angle OQP + \angle POQ = 180^\circ$$

$$2 \angle OPQ = 120^\circ$$

$$\angle OPQ = 60^\circ$$

$\Delta OPQ$  is an equilateral triangle.

$$\begin{aligned} \text{Area of } \Delta OPQ &= \frac{\sqrt{3}}{4} \times (\text{side})^2 \\ &= \frac{\sqrt{3}}{4} \times (15)^2 = \frac{225\sqrt{3}}{4} \text{ cm}^2 \\ &= 56.25\sqrt{3} \\ &= 97.3125 \text{ cm}^2 \end{aligned}$$

Area of segment PRQ = Area of sector OPRQ – Area of  $\Delta OPQ$

$$= 117.75 - 97.3125$$

$$= 20.4375 \text{ cm}^2$$

Area of major segment PSQ = Area of circle – Area of segment PRQ

$$= \pi(15)^2 - 20.4375$$

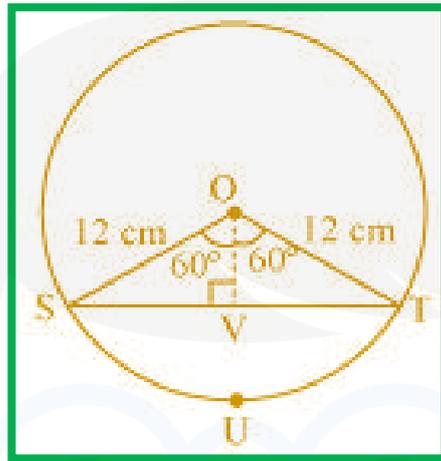
$$= 3.14 \times 225 - 20.4375$$

$$= 706.5 - 20.4375$$

$$= 686.0625 \text{ cm}^2$$

**Question 7:**

A chord of a circle of radius 12 cm subtends an angle of  $120^\circ$  at the centre. Find the area of the corresponding segment of the circle. [Use  $\pi = 3.14$  and  $\sqrt{3} = 1.73$ ]

**Answer 7:**

Let us draw a perpendicular OV on chord ST. It will bisect the chord ST.

$$SV = VT$$

In  $\triangle OVS$ ,

$$\frac{OV}{OS} = \cos 60^\circ$$

$$\frac{OV}{12} = \frac{1}{2}$$

$$OV = 6 \text{ cm}$$

$$\frac{SV}{SO} = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\frac{SV}{12} = \frac{\sqrt{3}}{2}$$

$$SV = 6\sqrt{3} \text{ cm}$$

$$ST = 2SV = 2 \times 6\sqrt{3} = 12\sqrt{3} \text{ cm}$$

$$\text{Area of } \Delta OST = \frac{1}{2} \times ST \times OV$$

$$= \frac{1}{2} \times 12\sqrt{3} \times 6$$

$$= 36\sqrt{3} = 36 \times 1.73 = 62.28 \text{ cm}^2$$

$$\text{Area of sector OSUT} = \frac{120^\circ}{360^\circ} \times \pi(12)^2$$

$$= \frac{1}{3} \times 3.14 \times 144 = 150.72 \text{ cm}^2$$

Area of segment SUT = Area of sector OSUT – Area of  $\Delta OST$

$$= 150.72 - 62.28$$

$$= 88.44 \text{ cm}^2$$

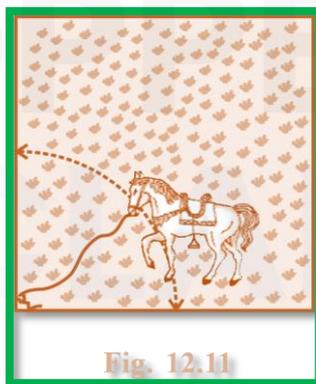
**Question 8:**

A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see the given figure). Find

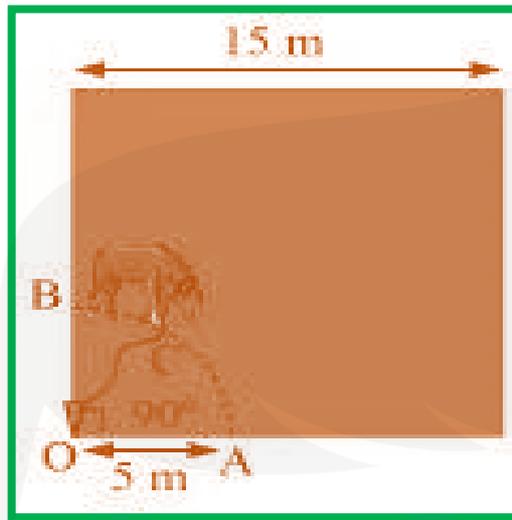
(i) The area of that part of the field in which the horse can graze.

(ii) The increase in the grazing area of the rope were 10 m long instead of 5 m.

[Use  $\pi = 3.14$ ]



**Answer 8:**



From the figure, it can be observed that the horse can graze a sector of  $90^\circ$  in a circle of 5 m radius.

**(i)** Area that can be grazed by horse = Area of sector OACB

$$= \frac{90^\circ}{360^\circ} \pi r^2$$

$$= \frac{1}{4} \times 3.14 \times (5)^2$$

$$= 19.625 \text{ m}^2$$

Area that can be grazed by the horse when length of rope is 10 m long

$$= \frac{90^\circ}{360^\circ} \times \pi \times (10)^2$$

$$= \frac{1}{4} \times 3.14 \times 100$$

$$= 78.5 \text{ m}^2$$

**(ii)** Increase in grazing area =  $(78.5 - 19.625) \text{ m}^2$

$$= 58.875 \text{ m}^2$$

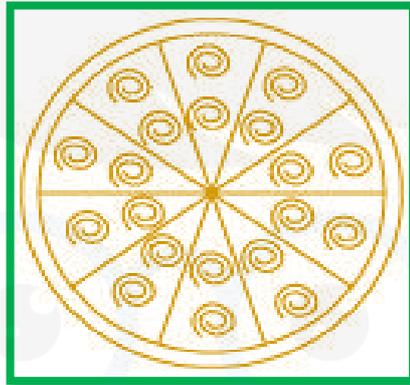
**Question 9:**

A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in figure. Find.

(i) The total length of the silver wire required.

(ii) The area of each sector of the brooch

$$\left[ \text{Use } \pi = \frac{22}{7} \right]$$

**Answer 9:**

Total length of wire required will be the length of 5 diameters and the circumference of the brooch.

$$\text{Radius of circle} = 35/2 \text{ mm}$$

$$\text{Circumference of brooch} = 2\pi r$$

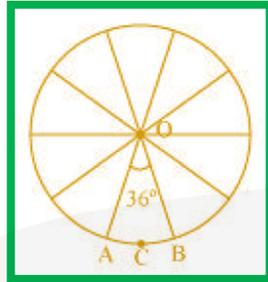
$$= 2 \times \frac{22}{7} \times \left( \frac{35}{2} \right)$$

$$= 110 \text{ mm}$$

(i) Length of wire required =  $110 + 5 \times 35$

$$= 110 + 175 = 285 \text{ mm}$$

It can be observed from the figure that each of 10 sectors of the circle is subtending  $36^\circ$  at the centre of the circle.



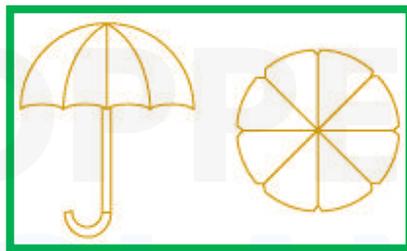
(ii) Therefore, area of each sector =  $\frac{36^\circ}{360^\circ} \times \pi r^2$

$$= \frac{1}{10} \times \frac{22}{7} \times \left(\frac{35}{2}\right) \times \left(\frac{35}{2}\right)$$

$$= \frac{385}{4} \text{ mm}^2$$

**Question 10:**

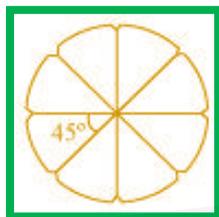
An umbrella has 8 ribs which are equally spaced (see figure). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella. [Use  $\pi = \frac{22}{7}$ ]



**Answer 10:**

There are 8 ribs in an umbrella. The area between two consecutive ribs is subtending

$$\frac{360^\circ}{8} = 45^\circ \text{ at the centre of the assumed flat circle.}$$

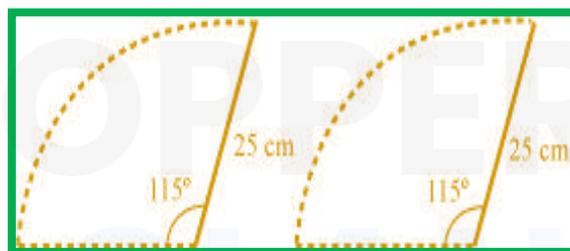


$$\begin{aligned}
 \text{Area between two consecutive ribs of circle} &= \frac{45^\circ}{360^\circ} \times \pi r^2 \\
 &= \frac{1}{8} \times \frac{22}{7} \times (45)^2 \\
 &= \frac{11}{28} \times 2025 = \frac{22275}{28} \text{ cm}^2
 \end{aligned}$$

**Question 11:**

A car has two wipers which do not overlap. Each wiper has blade of length 25 cm sweeping through an angle of  $115^\circ$ . Find the total area cleaned at each sweep of the blades. [Use  $\pi = \frac{22}{7}$ ]

**Answer 11:**



It can be observed from the figure that each blade of wiper will sweep an area of a sector of  $115^\circ$  in a circle of 25 cm radius.

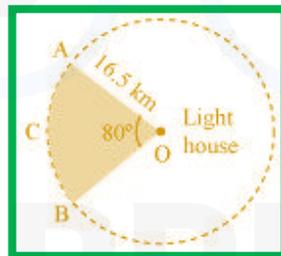
$$\begin{aligned} \text{Area of such sector} &= \frac{115^\circ}{360^\circ} \times \pi \times (25)^2 \\ &= \frac{23}{72} \times \frac{22}{7} \times 25 \times 25 \\ &= \frac{158125}{252} \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area swept by 2 blades} &= 2 \times \frac{158125}{252} \\ &= \frac{158125}{126} \text{ cm}^2 \end{aligned}$$

**Question 12:**

To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle  $80^\circ$  to a distance of 16.5 km. Find the area of the sea over which the ships warned. [Use  $\pi = 3.14$ ]

**Answer 12:**

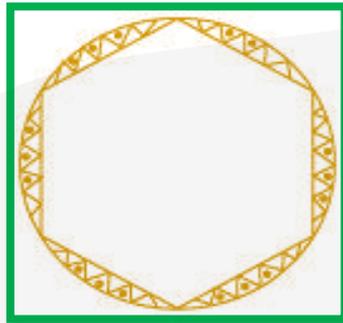


It can be observed from the figure that the lighthouse spreads light across a sector of  $80^\circ$  in a circle of 16.5 km radius.

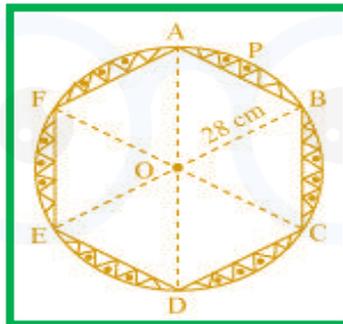
$$\begin{aligned} \text{Area of sector OACB} &= \frac{80^\circ}{360^\circ} \times \pi r^2 \\ &= \frac{2}{9} \times 3.14 \times 16.5 \times 16.5 \\ &= 189.97 \text{ km}^2 \end{aligned}$$

**Question 13:**

A round table cover has six equal designs as shown in figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs.0.35 per cm<sup>2</sup>. [Use  $\sqrt{3} = 1.7$ ]



**Answer 13:**



It can be observed that these designs are segments of the circle.

Consider segment APB. Chord AB is a side of the hexagon. Each chord will subtend

$$\frac{360^\circ}{6} = 60^\circ \text{ at the centre of the circle.}$$

In  $\triangle OAB$ ,

$$\angle OAB = \angle OBA \quad (\text{As } OA = OB)$$

$$\angle AOB = 60^\circ$$

$$\angle OAB + \angle OBA + \angle AOB = 180^\circ$$

$$2\angle OAB = 180^\circ - 60^\circ = 120^\circ$$

$$\angle OAB = 60^\circ$$

Therefore,  $\Delta OAB$  is an equilateral triangle.

$$\begin{aligned}\text{Area of } \Delta OAB &= \frac{\sqrt{3}}{4} \times (\text{side})^2 \\ &= \frac{\sqrt{3}}{4} \times (28)^2 = 196\sqrt{3} = 196 \times 1.7\end{aligned}$$

$$\begin{aligned}\text{Area of sector OAPB} &= \frac{60^\circ}{360^\circ} \times \pi r^2 \\ &= \frac{1}{6} \times \frac{22}{7} \times 28 \times 28 \\ &= \frac{1232}{3} \text{ cm}^2 \\ &= 333.2 \text{ cm}^2\end{aligned}$$

Area of segment APB = Area of sector OAPB – Area of  $\Delta OAB$

$$= \left( \frac{1232}{3} - 333.2 \right) \text{ cm}^2$$

$$\begin{aligned}\text{Therefore, area of designs} &= 6 \times \left( \frac{1232}{3} - 333.2 \right) \text{ cm}^2 \\ &= (2464 - 1999.2) \text{ cm}^2 \\ &= 464.8 \text{ cm}^2\end{aligned}$$

Cost of making 1  $\text{cm}^2$  designs = Rs 0.35

Cost of making 464.76  $\text{cm}^2$  designs =  $464.8 \times 0.35 = \text{Rs } 162.68$

Therefore, the cost of making such designs is Rs 162.68.

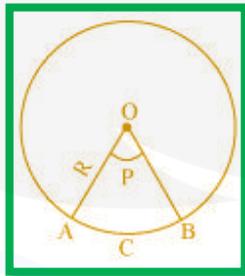
**Question 14:**

Tick the correct answer in the following:

Area of a sector of angle  $p$  (in degrees) of a circle with radius  $R$  is

- (A)  $\frac{p}{180} \times 2\pi R$       (B)  $\frac{p}{180} \times \pi R^2$       (C)  $\frac{p}{360} \times 2\pi R$       (D)  $\frac{p}{720} \times 2\pi R^2$

**Answer 14:**



We know that area of sector of angle  $\theta = \frac{\theta}{360^\circ} \times \pi R^2$

$$\begin{aligned} \text{Area of sector of angle } P &= \frac{P}{360^\circ} (\pi R^2) \\ &= \left( \frac{P}{720^\circ} \right) (2\pi R^2) \end{aligned}$$

Hence, (D) is the correct answer.

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